UNIT 1 BAND THEORY OF SOLIDS

1.1INTRODUCTION

- Band Theory of Solids is a quantum mechanical model of electrons in solids which envisages certain constrained ranges or the bands, for the energies of the electrons.
- It is also called as energy band theory of solids.
- The electron band theory of solids basically describes the quantum states that electrons can take inside metal solids.
- The overlap of the electron probability distributions of all the individual atoms in the metal solid leads to a creation of a continuous band of energies.
- The ranges of allowed energies of electrons in a solid are called allowed bands.
- Bands of energies between two such allowed bands are called forbidden bands which implies that electrons within the solid cannot be allowed to possess these energies. Band theory accounts for many of the electrical and thermal properties of solids. Solids can be categorised into conductors, semiconductors or insulators by their ability to conduct electricity.
- Electron band theory explains differences in conductivity of these solids.
- Band theory was also successful in giving us an insight into theoretical understanding of semiconductors and their physical properties.

1.2Free Electron theory of metals

In general, it was accepted that the valence electrons are involved in electrical conduction in metals and alloys. **Free electron model** is applicable to all the three categories of solids (conductors, insulatorsand semiconductors).This model is useful in explaining not only the electrical properties but also the thermal, optical and magnetic properties of solids.

There are three stages for the development of free electron theory of metalswhile making an attempt to explain theelectrical behaviour and to distinguish between the three types of solids:

(1**) Classical free electron theory:**

- The classical free electron theory was proposed by Paul Drude in 1900.
- After the discovery of electron by JJ Thomson, this free electron theory was extended by Lorentz in 1909. Hence this theory is also known as Drude & Lorentz Theory.
- According to this theory, metals consist of positive ion cores and valence electrons. The ions cores are immobile and consist of positive nucleus and the boundelectrons. The valence electrons get detached from the parent atoms during the process of formation of the metal and move randomly among these cores. Hence, they are known as **free electrons**.
- According to this theory,valence electrons become free in metals and move randomly within the metal. These free electrons are responsible for the electrical conductivity in metals and the velocities offree electrons obey the laws of classical mechanics i.e.Maxwell-Boltzmann distributionof velocities and energies. According to Maxwell-Boltzmann distribution manyelectrons can simultaneously possess the same energy (or velocity).

- In this theory, it was assumed that the free electrons move in a region of constant potential.
- This theory successfully explainedthe Ohm's law and the high electrical conductivity of metals.
- It failed to explain other features and the distinction between conductors, insulators and semiconductors

(2) **Quantum free electron theory:**

- Sommerfeld modified the Drude's classical free electron theoryanddevelopedQuantum free electron theory in 1928.
- According to this theory, the free electrons move in a regionof constant potential andobey quantum laws (Fermi-Dirac statistics).
- This theory is based on the particle character of electron anddid not take into account of its wave character.
- The theory failed to explain the distinction between conductors, insulators and semiconductors.

(3) **Band theory of solids:**

- Band theory was developed by Felix Bloch in 1928.
- According to this theory, free electrons exhibit wave character as they move between atoms in a solid.
- This theory assumed that potential varies in a periodic manner in the solid. Free electrons move in periodic potential provided by lattice.
- This theory successfully explained the classification of solids into threegroups, namely conductors, insulators and semiconductors.
- Based on band theory many of the electrical and thermal properties of solids could be successfully explained.
- It also formed the foundation of the technology of solid-state electronics.

1.3. Electrical Conduction in solids

- Consider a rectangular block 'S 'of a solid of length 'L' and cross-sectional area 'A' as shown in figure1.1.

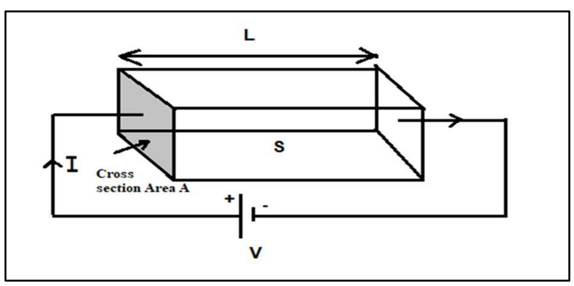


Fig1.1 Circuit for determining conductivity in a solid

- Let 'V' be the potential difference applied across the length 'L' of solid. Then Electric field 'E' produced is given by

$$E = \frac{V}{L} \qquad \text{----------------- (1)}$$

- In the presence of this electric field the free electrons are accelerated and give rise to current (I) which flows in the direction of electric field.
- The value of current I is given by

$$I = \frac{Q}{t} \qquad \text{----------------- (2)}$$

where 'Q' is the net charge transported through cross-sectional area 'A' per unit time t.

- According to Ohm's law, the magnitude of current flowing through a material is given as

$$I = \frac{V}{R} \qquad \text{----------------- (3)}$$

where 'R' is the Electrical resistance.

- When, electrons move through a material, they encounter opposition to their free movement which is seen normally in a vacuum.
- This opposition to electron motion in materials is manifested as electrical resistance.
- The resistance R is geometry dependent property and is given by

$$R = \rho\frac{L}{A} \qquad \text{----------------- (4)}$$

where 'ρ' is known as **resistivity** of material with unit Ω m (Ohm-m).

- **The reciprocal of resistivity is conductivity 'σ'.** The unit of conductivity is $\Omega^{-1}\ m^{-1}$ or mho/m or S/m.
- **It is a measure of how readily a material can transmit an electric current.**
- The Electrical conductivity is defined as the quantity of electricity flowing per unit area per unit time at a constant potential gradient and is given by

$$\sigma = \frac{1}{\rho} = \frac{L}{RA} \text{ (using equation 4)} \qquad \text{----------------- (5)}$$

- If 'n' is the number of electrons per unit volume (concentration of free electrons) and 'e' is the charge of the electrons, for the rectangular block of solid S of cross - sectional area A and length L as given in figure1.1,
- Thenthe total number of free electrons in the solid S is given as

$$N = n\,AL \qquad \text{----------------- (6)}$$

- Since the total charge present in the solid is Q = Ne = ne AL ----------------- (7)
- Hence the current 'I' through the solid is given by

$$I = \frac{Q}{t} = \frac{ne\,A\,L}{t}$$

$$\text{or} \quad I = neAv_d \qquad \text{----------------- (8)}$$

where 'v_d' is the **average drift velocity** of electrons in a solid.

- The **current density** is given by

$$J = \frac{I}{A}$$

$$\text{or} \quad J = nev_d \text{ (using equation 8)} \qquad \text{----------------- (9)}$$

From equation (3) and equations (4) and (1) we get

$$I = \frac{V}{R} = \frac{VA}{\rho L} = \sigma AE \quad (E=V/L,\ \sigma = \frac{1}{\rho})$$

$$\text{or } \frac{I}{A} = \sigma E$$

$$\text{or } J = \sigma E \qquad \text{---------------- (10)}$$

- Equation (10) is called **macroscopic form of Ohm's Law.**
- From equations (9) and (10) we get

$$\sigma = \frac{J}{E} = \frac{nev_d}{E} = ne\mu \qquad \text{---------------- (11)}$$

$$\text{where, } \mu = \frac{v_d}{E} \qquad \text{---------------- (12)}$$

- μ is called **mobility** of electrons measured in $m^2/V.s$
- It is defined as drift velocity per unit electric field. It measures how fast electrons can move in a solid due to applied electric field.

1.3.1. Thermal motion of free electrons

- Under Thermal equilibrium, the free electrons in solid are in a state of random motion.
- Free electrons keep moving randomly in the lattice structure of metal due to thermal energy. The average speed of electrons is 10^6 m/s.
- During the motion of electron, they collide with positive ion cores and get deflected.
- They move in zigzag path as shown in figure 1.2 (a).
- For every electron moving in a particular direction, there is another electron moving in the opposite direction.
- Therefore, thermal motion of free electrons does not cause flow of current through metal.

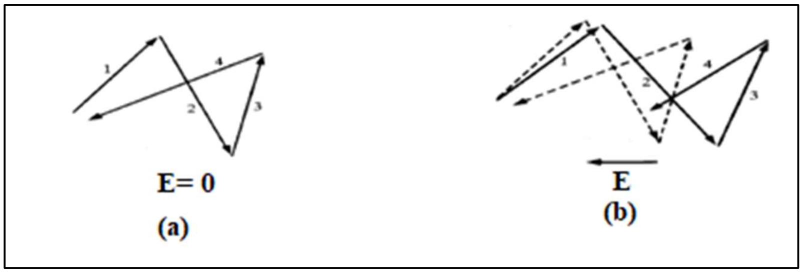


Fig.1.2 The drift motion of electron in (a) absence of electric field (b) presence of electric field. The broken line shows path of electron in absence of and the solid line shows path of electron in presence of electric field.

1.3.2. Drift motion of free electrons and Drift Velocity:

- When the ends of a piece of metal are connected to the terminals of a battery, an electric field is applied across the metal piece of metal and the equilibrium gets disturbed. The electric field accelerates the electrons.
- Electrons acquire velocity and move in a direction opposite to that of electric field as shown in fig.1.2(b).
- This directional motion of electrons due to action of electric field is called as **" Drift"** and corresponding velocity is called **drift velocityv_d.**
- During the motion of electrons, drift velocity changes continuously.
- Hence the electrons move with **mean drift velocity v_d**.
- Drift velocity is of the order of 10^{-2} m/s which is very small as compared to the thermal speed(10^6m/s).
- Thus, we define ***drift velocity as average velocity of electrons in a solid due to the application of external electric field.***
- The drift motion is directional and cause current flow in a conductor called **drift current or conduction current.**

1.4. Classification of materials

The resistivity of a solid can be determined using the circuit shown in figure 1.1. Depending on experimental values of conductivity and resistivity, materials are classified as

1. ***Conductors***: Materials having large value of conductivity of the order of 10^8S/m are called conductors. For e.g., Metals and alloys.
2. ***Insulators:*** Materials having very low value of conductivity of the order less than 10^{-12}S/mare called Insulators. For e.g., Metal oxides, glasses and plastics.
3. ***Semiconductors*****:** Materials having intermediate values of conductivity of the order of 10^6 to 10^{-6} S/m are called semi-conductors. For e.g., Silicon and Germanium.

1.5. Free electron model of solids

- Electrical conduction is one of the important properties of solids.
- It is generally accepted that the valence electrons are involved in electrical conduction in metals and alloys.
- Free electrons are responsible for conduction of electricity and heat in metals. Free electron model is applicable to all the categories of solids.
- This model explains thermal, optical, magnetic properties of solids in addition to electrical properties.

1.5.1. Classical Free electron theory of solids

Classical free electron theory of solids was first proposed by Paul Drude and was later extended by Lorentz. The theory is also called **Drude -Lorentz theory.** It is based on the following assumptions:

- Valence electrons become free in metals and move randomly within the metal.
- Free electrons move in a region of constant potential and mutual repulsion amongst the electrons does not occur.
- Velocities of electrons in a solid obey the classical Maxwell-Boltzmann distribution.

This theory successfully explains ohm's law and high electrical conductivity of metals.

- But this theory could not explain the distinction between conductors, insulators and semiconductor.
- J.J .Thomson discovered electron in 1897, then Drude developed the free electron theory of metals.
- According to this theory, **metal consists of positive ion cores and valence electrons. The ion cores are immovable which consists of positive nucleus and the electrons are bound to it.**
- **During the process of formation of metals, valence electrons get detached from the atoms and move randomly amongst these cores. Hence, they are called as free electrons.** The behavior of free electrons within the metal is considered to be similar to that of atoms in perfect gas. Hence free electrons are referred to as "**free electron gas"**.
- Potential energy of electrons inside the metal is less than the potential energy of stationary electron just outside it. Hence the movement of free electrons is restricted to the boundaries of metals.
- This energy difference produces a potential barrier which prevents the free electron to leave the surface of metal. This free electron gas is confined to potential energy box.

1.5.2. Influence of external factors on conductivity / Failure of classical free electron theory

- As per the classical free electron theory, the electrical conductivity of metals is given as

$$\sigma = ne\mu$$

where 'n' is the electron concentration (free electrons/ unit volume).

- Thus, the electrical conductivity of metals depends on electron concentration.
- Hence, according to classical free electron theory then bivalent & trivalent metals should possess much higher electrical conductivity than monovalent metals.
- This is contrary to the experimental observations that the monovalent element metals such as copper & silver are more conducting than Zinc (bivalent) & aluminum (trivalent).

- Thus, the prediction of classical free electron theory that $\sigma \propto n$ does not always hold good. Hence classical free electron theory failed to explain dependence of σ on n.
- It could also not explain the variation of conductivity and resistivity due to external factors like temperature, light and impurities in case of semiconductor and insulators.

It has been experimentally found that:

- Resistivity increases with increase in temperature for metals while for semiconductors and insulators, it decreases.
- The optical radiation does not affect the resistivity of metals, but the resistivity of semiconductors decreases.
- The resistivity of metals increases with increase in impurity concentration while that of semiconductors decreases.

1.6. Quantum free electron theory of solids

- This theory was developed by Sommerfeld in 1928.It is assumed in the classical free electron theory that electrons follow Maxwell-Boltzmann distribution of velocities and energies. According to Maxwell-Boltzmann distribution many electrons can simultaneously possess the same energy (or velocity).
- According to quantum theory, electrons obey Pauli Exclusion Principle (Pauli's Exclusion Principle states that no two electrons in the same atom can have identical values for all four of their quantum numbers. In other words, (1) no more than two electrons can occupy the same orbital and (2) two electrons in the same orbital must have opposite spins)and hence follow Fermi-Dirac distribution.
- According to quantum theory, only two electrons can occupy the same energy level. Hence even at 0K, conduction (or free) electrons occupy different discrete energy levels.
- Unlike neutral molecules, electrons are charged particles and obey Pauli's exclusion principle.
- An assembly of free electrons obeys Fermi Dirac statistics.
- This theory was based on particle nature of electrons and not on the wave nature of electrons. This theory failed to explain distinction between conductors, insulators and semiconductors.

1.7. Band theory of solids

- Felix Bloch, in 1928, formulated this theory.
- This theory is based on wave nature of electrons.
- Electrons exhibit wave character as they move between atoms in a solid.
- It is also assumed that potential varies in a periodic manner in the solid. Electrons move in a periodic potential provided by lattice.
- The theory successfully explained the classification of solids into three groups such as conductors, insulators and semiconductors.

1.8FORMATION OF ENERGY BANDS IN SOLIDS

- A single isolated atom has discrete energy levels.
- When two identical atoms are considered to be far apart, the electron energy levels in an individual atom are not affected by the presence of the other.
- As long as the atoms are widely separated, they have identical energy levels; electrons fill the levels in each atom independently.
- But when the atoms are brought closer, they begin to interact strongly and as a result, each isolated energy level will be transformed into two energy levels of similar energies.

- ***Transformation of single energy level into two or more separate energy levels is defined as the energy level splitting.***
- When two atoms come close, one energy level splits into two energy levels (Fig. 1.3b)
- When three atoms approach each other closely, the original level splits into three levels; four atoms produce four levels and so on (Fig. 1.3 *c*).
- Similarly,in generalif we consider interaction of N atoms, their isolated energy levels will be split into N energy levels. These N energy levels are so close to each other that forma near continuum (*energy band* see Fig.1.3 d).
- ***Therefore, when atoms are brought together to form a solid, their energy levels split up and form a group of closely spaced allowed energy levels of same energy value. This group of closely spaced energy levels of same energy is called Energy band(see Fig. 1.3 d).***
- The concept of energy level splitting and formation of valence band and conduction band is illustrated in Fig. 1.3 and Fig.1.4.

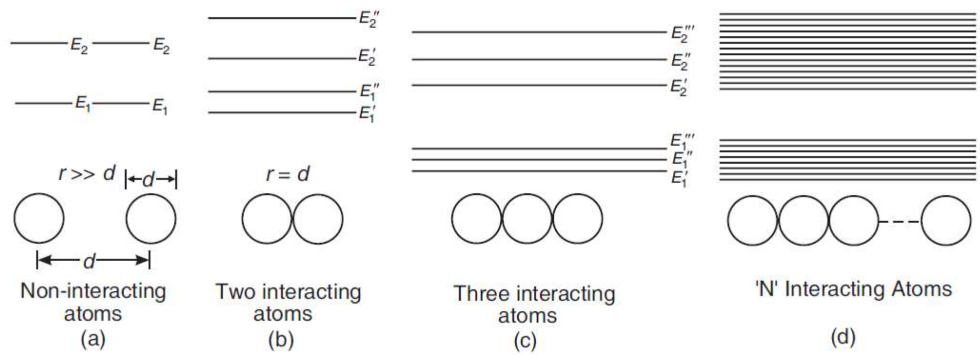


Fig.1.3. (a) Energy level splitting and formation of valence band and conduction band

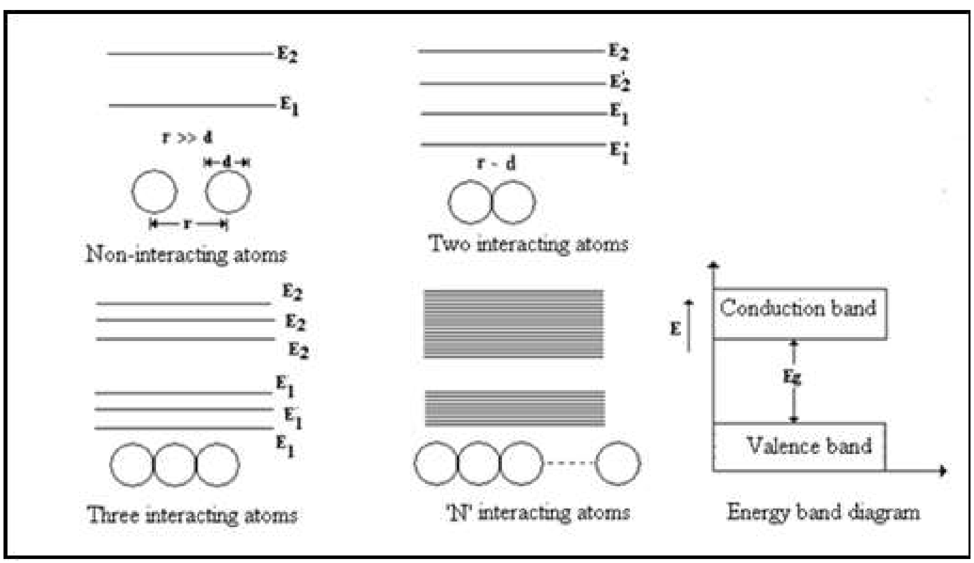


Fig.1.3. (a) Energy level splitting and formation of valence band and conduction band.

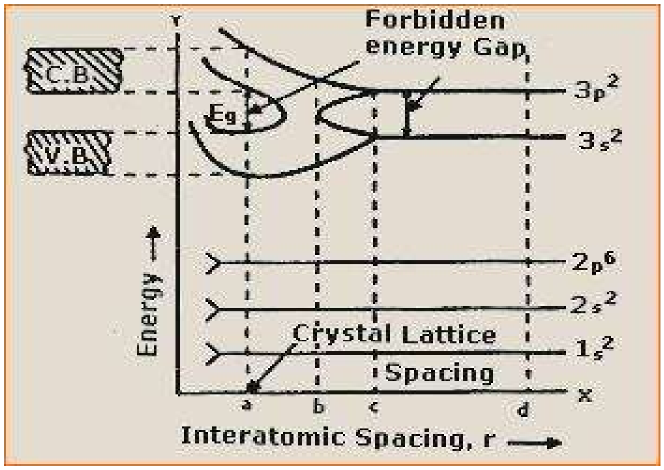


Fig.1.4. Energy level splitting and formation of valence band and conduction band(For Silicon(Z=14) $1s^2 2s^2\ 2p^6\ 3s^2 3p^2$)).

- When atoms are brought together, application of Pauli's exclusion principle becomes important.
- It states that no two electrons can have their entire quantum numbers same. Hence an energy level can accommodate at the most two electrons of opposite spin.
- The degree of splitting of energy levels depends on the depth in an atom.
- The energy levels of core electrons belonging to inner shells split to a lesser degree and hence they form a narrow core band. They are always full and do not take part in the conduction process.
- The energy levels occupied by valence electrons split more and form wider bands.
- Energy levels above the valence levels also split though they are not occupied.
- While occupying a band, electron starts from lowest energy level and fill the levels in the ascending order of energy (Aufbau's Principle).
- In general, Maximum number of energy levels that a state can have is given by $(2\ell+1)N$.
- The maximum number of electrons that each level can hold is given by**$2(2\ell+1)N$**.
- Where ℓ is orbital quantum number, 2 corresponds to two different orientations of electron spin and N is the number of interacting atoms.

- **For level s; ℓ=0**
- Number of energy levels=N
- Number of electrons=2N
- **For level p; ℓ=1**
- Number of energy levels=3N
- Number of electrons=6N
- **For level d; ℓ=2**
- Number of energy levels=5N
- Number of electrons=10N
- **For level f; ℓ=3**

  Number of energy levels=7N
  Number of electrons=14N

QUE: Explain formation of energy bands (in solids) on the basis of band theory of solids. (4)[Summer-05, 07]

1.9. VALENCE BAND, CONDUCTION BAND AND ENERGY GAP

VALENCE BAND:

- The Energy band occupied by valence electrons that are involved in covalent bonding, is called as ***valence band.***
- Depending upon the number of valence electrons this band may get partially or completely filled.
- At absolute zero, covalent bonds are complete, therefore valence band is completely filled.

CONDUCTION BAND:

- The energy band above the valence band, having free electrons responsible for electrical conduction, is called as ***conduction band.***
- At absolute zero, this energy band is empty.

ENERGY GAP:

- ***The energy interval between top of the valence band and bottom of the conduction band which is empty and forbidden is called energy gap or band gap.***
- ***It is the special characteristic of semiconductor material.***
- ***It is the minimum amount of energy required for breaking a covalent bond and to excite an electron from valence band to conduction band.***

1.10. CLASSIFICATION OF SOLIDS BASED ON BAND THEORY

- Solids can be classified into conductors, semiconductors or insulators depending upon width of Energy gap.
- Completely filled bands contain large number of electrons but do not contribute to the conductivity of the material.
- Partially filled bands are necessary for electrical conduction.
- The energy band diagram of conductors, semiconductors or insulators is shown in figure 1.5.

Conductors:

- The solids in which conduction and valence band overlap each other are called conductors. Therefore, the energy gap between valence band and conduction band is zero.
- Electrons can easily jump from lower energy band to higher one and become available for conduction.
- An application of a small amount of voltage leads to generation of large amount of current.
- Hence these solids are good electrical conductors. For e.g., Lithium, Beryllium and sodium.
- **Semiconductors**:
- The solids in which the conduction and valence bands are separated by a small energy gap of less than 2eV are called semiconductors.
- For e.g. Semiconductors like Silicon has Bandgap of 1.12 eV and Germanium has bandgap of 0.72 eV.
- A small energy gap means that a small amount of energy is required to free the electrons and move them from the valence band to the conduction band.
- The semiconductors behave like insulators at 0K, because valence electrons do not have required energy to jump to the conduction band.

- If the temperature is increased, valence electrons acquire sufficient energy to jump into the conduction band.
- Therefore, the conductivity of semiconductors increases with the increase in temperature.

Insulators:

- The solids in which the conduction band and valence bands are separated by a large energy gap of ≥ 3 eV are called insulators.
- At room temperature, the valence electrons do not have enough energy to jump into the conduction band, therefore insulator do not conduct current.
- Thus, insulators have very high resistivity and extremely low conductivity at room temperatures. For e.g. Diamond and glass.

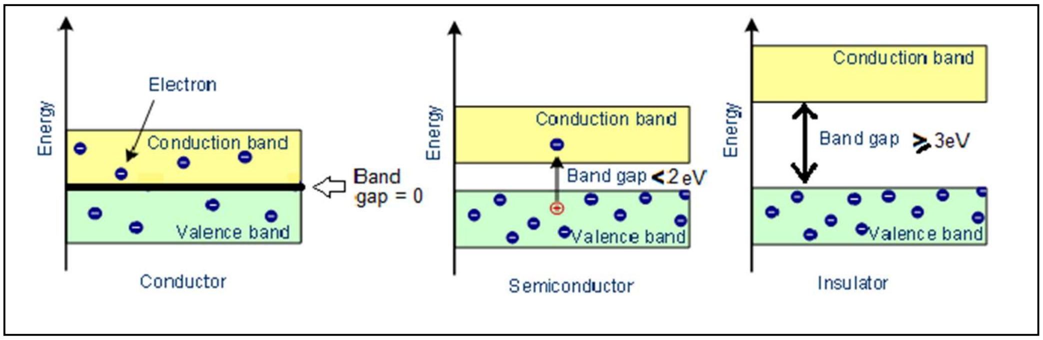


Figure1.5: Energy band diagram of conductor, semiconductor and insulator

1.10.1. DIFFERENCE BETWEEN CONDUCTORS, SEMICONDUCTORS AND INSULATORS

Table 1.1: Differences between Conductors, Semiconductors and Insulators

S.No.	Conductor	Semiconductor	Insulator
1.	**Conduction band** and **valence band** overlap with each other hence there is no energy gap between the two bands.($E_g = 0$)	**Conduction band** and **valence band** are separated by small forbidden energy gap. ($E_g \leq 2$ eV)	**Conduction band** and **valence band** are separated by large forbidden energy gap. ($E_g \geq 3$eV)
2.	Conductor has highest **conductivity.**	Semiconductor has**conductivity**between conductor and insulator.	Insulator has zero **conductivity.**
3.	It has **free electrons.**	It has **free electrons** at high temperature.	It doesn't have **free electrons.**
4.	It has **electrons** as charge carriers.	It has **electronsand holes** as charge carriers.	It doesn't have charge carriers.
5.	Its**conductivity decreases** with increase in temperature. (When we increase the temperature the vibrational motion of electrons increases and thus cause unwanted collisions which results in the increase of resistance in metals. Therefore, the mobility of electrons decreases and causes decrease in conductivity).	Its **conductivity** increases with increase in temperature.	They may exhibit small **conductivity** at high temperatures and high electric field**.**

6.	Its**conductivity decreases** with added impurity. (In most metals, the existence of impurities restricts the flow of electrons compared to pure metals. Elements which are added as alloying agents could be considered "impurities". So, alloys tend to offer less electrical conductivity than pure metal).	Its **conductivity increases** with added impurity.	No effect on **conductivity** on addition of impurities.

QUE: Discuss energy band structure of conductors, insulators and semiconductors. Give a brief account of the general properties and characteristics of semiconductor. (3)[Summer-19]

QUE: Explain classification of solids on the basis of energy band diagrams. (3) [Summer-17]

QUE: Discuss classification of solids on the basis of forbidden energy gaps. (3) [Summer-18]

QUE: Discuss energy band structures of conductors, insulators and semiconductors. (3)[Winter-13]

1.11. FERMI LEVEL

- ***Fermi level is defined as the highest filled energy level in a conductor at 0K.***
- At 0K, all the levels below Fermi level are completely filled with electrons and all the levels above Fermi levels are completely empty.
- But at high temperature there is a possibility that some of the electrons from levels below Fermi level then gets transferred (jump) to the levels above Fermi level.
- **Fermi Energy**:***Fermi energy is the maximum energy that a free electron can have in a conductor at 0K.***It is the energy associated with the Fermi level.

1.11.1. FERMI DIRAC DISTRIBUTION FUNCTION F(E)

- Fermi Dirac distribution function gives the probability that any energy level 'E' at given temperature T is occupied or not.

$$f(E) = \frac{1}{1 + \exp\left[\frac{(E-E_F)}{kT}\right]}$$

This expression governs the distribution of electrons among the energy levels as a function of temperature.

E – Energy level for which occupancy is to be determined.
E_F – Fermi level.
k – Boltzmann constant.
T – Temperature at which occupancy is to be determined.

- When $f(E)$= 0; it indicates that energy level E is completely empty.
- When $f(E)$= 1; it indicates that energy level E is completely filled.

1.11.2. VARIATION OF FERMI FUNCTION F(E) WITH TEMPERATURE AT DIFFERENT ENERGY

- The Fermi function is given by

$$f(E) = \frac{1}{1+\text{ex}\left[\frac{(E-E_F)}{kT}\right]} \quad \text{-------------- (1)}$$

At T = 0K

- **Case I**:- For **E<E$_F$**; (E – E$_F$) becomes a negative quantity and therefore Fermi distribution is given by

$$f(E) = \frac{1}{1+\exp\left[\frac{-(E-E_F)}{k\times\ 0}\right]}$$

- Since T = 0K and so 1/T = -∞

$$f(E) = \frac{1}{1+\exp[-\infty]} = \frac{1}{1+0} = 1 \quad \text{-------------- (2)}$$

- Thus, for all energy levels having ‘E’ less than E$_F$, probability for their occupation is 1. Hence they are fully occupied.

Case II: - For E > E$_F$, then

$$f(E) = \frac{1}{1+\exp\left[\frac{(E-E_F)}{k\times 0}\right]}$$

$$\text{or } f(E) = \frac{1}{1+\exp[\infty]} = \frac{1}{1+\infty} = \frac{1}{\infty} = 0 \quad \text{-------------- (4.03)}$$

- Thus all energy levels that are lyingabove E$_F$remain vacant.

Case III: -For E = E$_F$, the quantity (E-E$_F$) = 0.

$$f(E) = \frac{1}{1+\exp\left[\frac{(0)}{k\times 0}\right]} = \frac{1}{1+\exp\left[\frac{0}{0}\right]}$$

f(E) = Indeterminate

- The occupancy of Fermi energy level at 0K varies fromzero to one as shown in Figure 1.6.

- Hence, in metals, at T = 0 K, E$_F$ is the highest energy leveloccupied.

Graphical Representation:

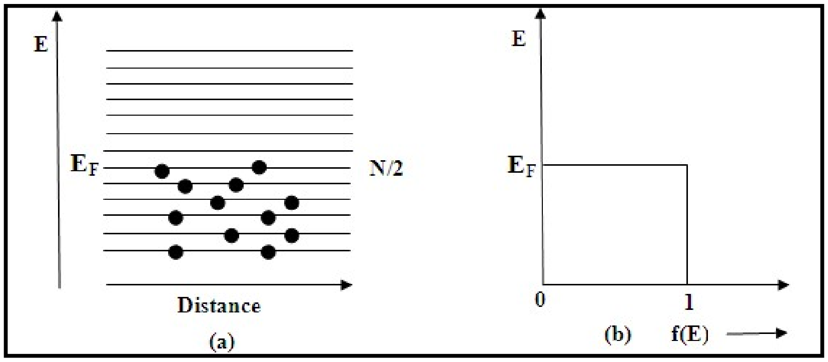


Fig.1.6: (a) Distribution of electrons in conduction band (b) Fermi function at T = 0K in a conductor

<u>For higher temperature</u>: <u>(T > 0K)</u>:-

- At higher temperature some of the electrons below Fermi level can jump to highest energy levels.
- There is possibility that the electrons from the levels below Fermi level jump to the level above Fermi level.
- Therefore, the probability of finding electrons in the levelsimmediatelybelow $E_F$willdecrease.
- On the other hand, the probability of finding electrons in the levelsimmediatelyabove $E_F$increases. This is illustrated in Figure 1.7.

∴ The probability function f(E) for Fermi level is given by,

At T > 0 K, When E = E_F.

$$f(E) = \frac{1}{1+ex^{[\frac{0}{kT}]}} = \frac{1}{1+\exp^{[0]}} = \frac{1}{1+1} = \frac{1}{2} \qquad \text{-------------- (4.04)}$$

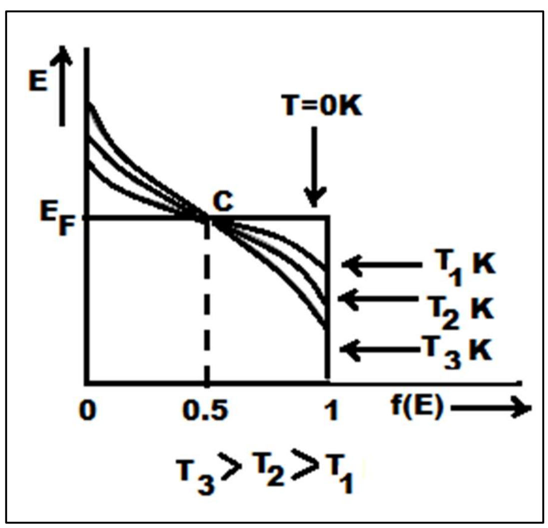


Fig.1.7: F-D distribution function versus temp at T>0K

- This implies that the probability of occupancy of Fermi energy level at any temperature above 0K is 0.5 or 50%. Therefore, we can say that Fermi energy level is the energy level, which has a probability of occupancy of 50%.
- **Fermi energy is thus the average energy possessed by electrons participating in conduction in metals at temperatures above 0K.**
- From figure 1.7, it can be seen that all the curves pass through a point (C) which is called cross over point.
- Thus f(E) is function of E and it always passes through a point (E_F,$\frac{1}{2}$) at different temperatures.

QUE: What is meant by Femi-Dirac Distribution function? Define Fermi level. (3)[Summer-13]
QUE: What is fermi function? Draw a graph showing its variation with energy at different temperatures and discuss it. (4)[Winter-17, Summer-04, 11]
QUE: What is Fermi function? Explain with the help of a diagram how it varies with change of temperature. (4) [Winter-12]
QUE:Whatis Fermi Dirac distribution function? State itssignificance.(4)[Winter-09]

1.11.3. PROBABILITY FUNCTION IS SYMMETRICAL ABOUT E_F AT ALL TEMPERATURES OTHER THAN 0K [i.e. $f(E_1) + f(E_2) = 1$]

STATEMENT:

- The probability of occupancy of energy levelE_2 lying ΔE above E_F is equal to probability of non- occupancy of energy levelE_1 lying ΔE below E_F. This can be mathematically represented as

$$f(E_2) = 1 - f(E_1)$$

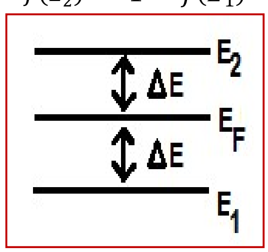


Fig. 1.8: Equally spaced energy level on either side of Fermi level

PROOF:

- Let E_1& E_2 are the two energy levels, which are at distance ΔE on the either side of fermi level as shown in fig.1.8.

$$\therefore E_1 = E_f + \Delta E$$
$$E_2 = E_f - \Delta E$$

- We know that,

$$f(E_2) = f(E_F + \Delta E) = \frac{1}{1 + \exp\left[\frac{(E_F + \Delta E - E_F)}{kT}\right]}$$

$$f(E_2) = f(E_F + \Delta E) = \frac{1}{1+\exp\left[\frac{(\Delta E)}{kT}\right]} \text{-------------(4.05)}$$

- Let an E_l be the energy level lying ΔE below the Fermi level E_F i.e. $E_l = E_F - \Delta E$. The probability of E_1being unoccupied $=1 - f(E_1)$.

$$1 - f(E_1) = 1 - f(E_F - \Delta E) = 1 - \frac{1}{1 + \exp\left[\frac{(E_F - \Delta E - E_F)}{kT}\right]}$$

$$=1 - \frac{1}{1 + \exp\left[\frac{(-\Delta E)}{kT}\right]}$$

$$1 - f(E_1) = \frac{1 + \exp\left[\frac{(-\Delta E)}{kT}\right] - 1}{1 + \exp\left[\frac{(-\Delta E)}{kT}\right]}$$

$$1 - f(E_1) = \frac{\exp\left[\frac{(-\Delta E)}{kT}\right]}{1 + \exp\left[\frac{(-\Delta E)}{kT}\right]}$$

$$\text{Dividing by } \exp\left[\frac{(-\Delta E)}{kT}\right] \text{ we get}$$

$$1 - f(E_1) = \frac{\exp\left[\frac{(-\Delta E)}{kT}\right] \Big/ \exp\left[\frac{(-\Delta E)}{kT}\right]}{\frac{1}{\exp\left[\frac{(-\Delta E)}{kT}\right]} + 1} = \frac{1}{\exp\left[\frac{(\Delta E)}{kT}\right] + 1} = \frac{1}{1 + \exp\left[\frac{(\Delta E)}{kT}\right]} \qquad \text{------------ (4.06)}$$

- From equation (1) and equation (2) we get

$$\text{or } 1 - f(E_1) = f(E_2)$$

$$\therefore \; f(E_1) + f(E_2) = 1$$

- This statement indicates that probability of an energy level E_2 being occupied above E_F is same as the probability of an energy level E_l being vacant below E_F and hence, it is proved that the probability function is symmetrical about E_F at all temperatures.

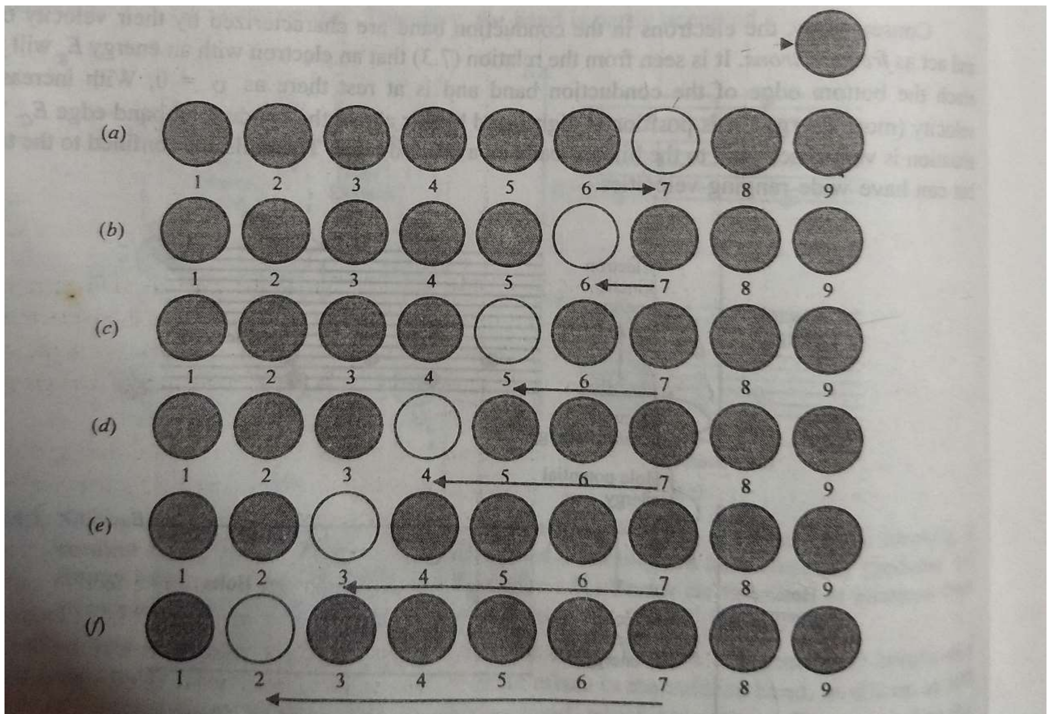
QUE: Show that probability function f (E) is symmetrical about fermi level.
QUE:The probability function for the two energy states which are equally spaced on the either side of fermi level, add upto unity.
QUE:If $E_1 = E_f + \Delta E$ and $E_2 = E_f - \Delta E$, then show that $f(E_1) + f(E_2) = 1$.

1.12CONCEPT OF EFFECTIVE MASS

- Experimentally it is observed that mass of electrons in some solids are less and, in some solids, larger than the mass of free electron.
- The experimentally determined electron mass in solids is called effective mass.
- It is denoted by m*.
- Change in the mass of electron in solids is due to interaction between atoms of solids and drifting of electrons.

1.13. Concept of hole:

- When a small amount of external energy is applied to a semiconductor, the electrons in a valence band move to the conduction band leaving a vacancy behind in the valence band. This vacancy is called as hole.
- The electric charge of hole is same as that of electron but has opposite polarity.
- When a covalent bond somewhere in the solid breaks, this vacancy gets filled by electron and a hole is created at another place.
- In this way, position of vacancy changes within the crystal.
- In other words, the 'hole' moves from one place to other within the crystal lattice.
- The movement of hole causes electrical current.
- The current in a semiconductor is due to movement of electrons in conduction band and holes in valence bands.



3.5. Analogy of motion of a vacancy. As the particles move toward right, the vacancy moves toward left. Note that the motion of a large number of particles in one direction is equivalent to the motion of a single vacancy in the opposite direction.

1.14CLASSIFICATION OF SEMICONDUCTORS

Semiconductors can be broadly classified as

(i) Intrinsic (Pure) semiconductors and

(ii) Extrinsic (Doped) semiconductors

1.14.1. INTRINSIC SEMICONDUCTORS

- ***Chemically pure semiconductors in which electrical conduction is due to thermally excited electrons and holes are known as intrinsic semiconductors. For eg.Germanium and silicon.***
- In these semiconductors, conductivity is mainly due to electron-hole pairs generated due to thermal agitation.
- The electrons reaching the conduction band, due to thermal excitation leave equal number of holes in valence band.

- Hence number of free electrons in conduction band is always equal to the number of holes in the valence band.

QUE:What do you mean by intrinsic semiconductor?

CHARACTERISTIC FEATURES

- They are crystalline in nature.
- They form covalent bonds with their neighboring atoms.
- These materials are tetravalent with four valence electrons in the outer most shell.
- At 0K, they do not have free electrons for conduction.
- Energy gap is very small, less than 2 eV.
- Electrons and holes are generated in pairs.
- The number of free electrons in conduction band is always equal to the number of holes in the valence band.
- At 0K, no free electron is available to participate in the process of conduction.
- Fermi energy level lies exactly at the middle of the energy gap.
- They exhibit high resistivity and low conductivity.
- Conductivity of intrinsic semiconductors increases with temperature.

1.14.1. CONDUCTIVITY IN INTRINSIC SEMICONDUCTORS

- Let us consider a sample of semiconductor across which a d.c. voltage source is connected as shown in following Fig.1.9.
- The electric field E due to the potential difference V causes a current I due to the electrons moving toward the positive terminal and holes towards the negative terminal of the source.
- Thus, the current in the semiconductor is bipolar, composed of two currents due to - (i) electrons drifting in the conduction band and (ii) holes drifting in the valance band.

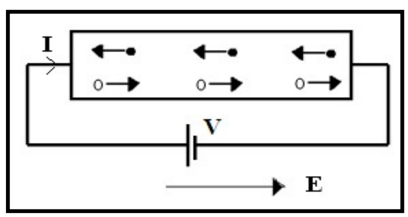


Fig 1.9: Conductivity in an intrinsic semiconductor

- The conductivity of intrinsic semiconductors depends on the intrinsic carrier concentration, mobility of electrons and holes and also varies with temperature.

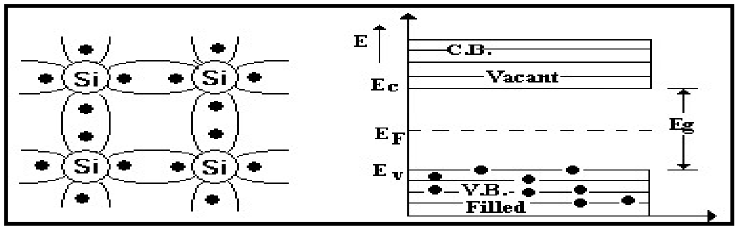
1.14.2. ENERGY LEVEL DIAGRAM OF INTRINSIC SEMICONDUCTOR

To describe current conduction and energy band diagram in intrinsic silicon let us consider silicon as example.

At T=0K

- At 0K, all valence electrons of each silicon atom are involved in covalent bond with neighboring atoms and has no free electron is available to participate in the process of conduction.
- Under these conditions, valence band is completely filled and conduction band is completely empty as shown in Fig.1.10.

- Electrons in the valence band cannot cross the forbidden energy gap and hence the material behaves as an insulator.

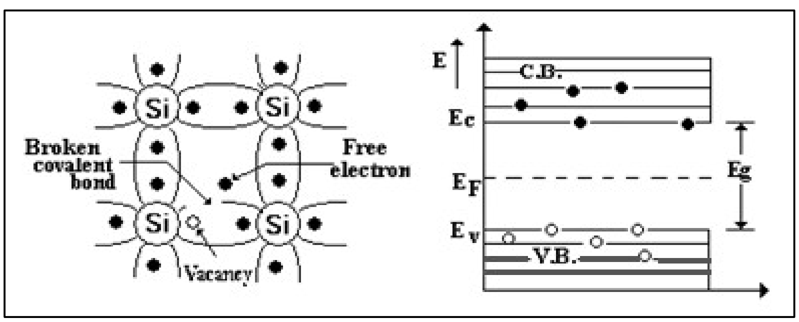


(a) (b)

Fig. 1.10: (a) 2D representation of silicon crystal (b) Energy band diagram of silicon at T = 0K

AT T>0K

- As temperature increases, electrons acquire sufficient energy to break the covalent bonds due to the supply of thermal energy and jumps from valance band to conduction band after gaining energy equal to E_g.
- Whenever one electron is made available for conduction, a hole is formed in valance band at the same time as shown in Fig.1.11.
- If n= number of electrons, p= number of holes and n_i= intrinsic carrier concentration or density. Hence, n = p = ni,
- As the concentration of electrons in conduction band and valence band are equal, Fermi level E_F is located exactly at the middle of forbidden gap (E_g).



(a) (b)

Fig. 1.11: (a) 2D representation of silicon crystal (b) Energy band diagram of silicon at T > 0K.

QUE : Explain in brief the concept of Fermi level. Derive an expression for Fermi energy in intrinsic semiconductor. (4)[Summer-01,15] **OUE:Whatis fermi function?**

1.14.3 EXPRESSION OF FERMI LEVEL /ENERGY IN INTRINSIC SEMICONDUCTOR

$$E_F = \frac{E_g}{2}$$

Thus, fermi level lies at the centre of energy gap in an intrinsic semiconductor.

1.15 EXTRINSIC SEMICONDUCTORS

- The electronic properties and the conductivity of an intrinsic semiconductor can be changed in a controlled manner by adding very small quantities of other elements called ***dopants***.

- This can be achieved by adding impurities of III or IV group elements to the melt and then allowing it to solidify into the crystal.
- This process is called ***doping*** and these doped semiconductors are called extrinsic semiconductors.

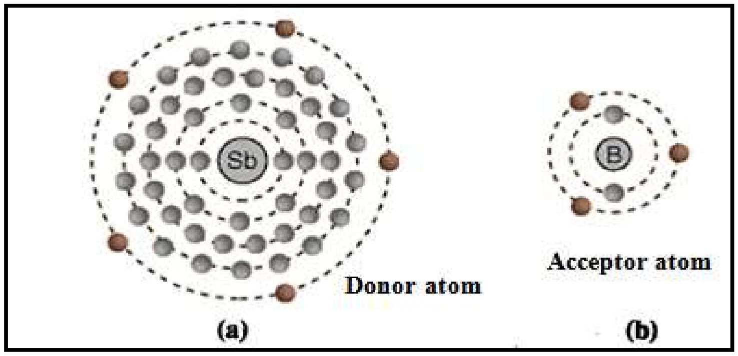


Fig.1.12: Illustration of valence electrons in (a) pentavalent and (b) trivalent impurities.

Extrinsic Semiconductors are further classified into two categories:

- ***N-type semiconductor → Pure semiconductor + group V element [***pentavalent impurity***]***
- ***P-type semiconductor → Pure semiconductor + group III element [***trivalent impurity***]***

- Impurity atoms with 5 valence electrons produce N-type semiconductors by contributing extra electrons,
- Impurity atoms with 3 valence electrons produce P-type semiconductors by producing a "hole" or electron deficiency.

1.15.1. N-TYPE SEMICONDUCTOR

- ***An n-type semiconductor is produced when a pure semiconductor is doped with a pentavalent impurity (having 5-electrons in the outermost orbit) such as Phosphorus,Arsenic or Antimony.***
- ***The impurity atom has one excess unpaired electron as shown in figure1.13.***
- At room temperature thermal energy is sufficient to make this electron free.
- This impurity is called donor impurity, as it donates electrons.
- The addition of donor impurity increases the number of electrons in the semiconductor.
- **Hence in N type semiconductor, electrons are majority charge carriers and holes are minority charge carriers.**

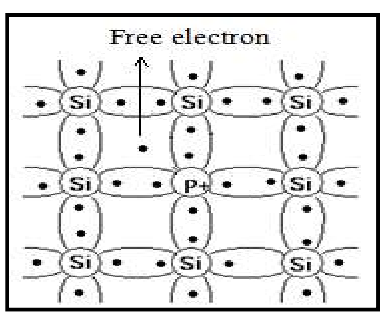


Fig.1.13: N-type semiconductor

1.15.2. ENERGY BAND DIAGRAM OF N-TYPE SEMICONDUCTOR

- Figure 1.14 shows the Energy band diagram of N-type semiconductor.
- At 0K, the donor level is located below the bottom of conduction band. The donor level is filled by donor atoms and fermi level E_F is located between the bottom of conduction band anddonor level.
- Thus, at 0K, the donor atoms are not ionized, the conduction band is emptywhile the valence band is full and the material behaves as an insulator(Fig.1.14a).
- While at higher temperature (T > 0K), the electrons from the donor level and from the valence band jump toconductionbandand material shows the conductivity(Fig.1.14 b).
- Conduction band contains large number of electrons donated by impurity atoms in addition to electrons jumping from valence band.
- The impurity atoms become positive ions (they are ionized). They gain a positive charge due to donation of electrons. The donor level is now empty as it is devoid of electrons. It contains only positive ions.
- The fermi level $E_F$shifts below the donor level as shown in fig.1.14(b).

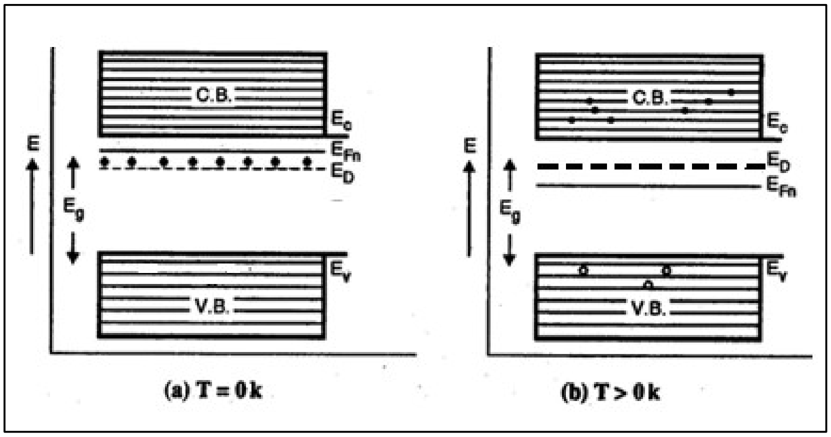


Fig.1.14: Energy band diagram of N-type semiconductor at (a) At 0K (b) At T > 0K.

1.15.3 P-TYPE SEMICONDUCTOR

- ***A p-type semiconductor is produced when a pure semiconductor is doped with a trivalent impurity (three electrons in outer most orbit) such as Boron, Aluminium, Gallium or Indium.***
- *Them impurity atom is deficient of one electron. The absence of one electron is treated as hole as shown in fig.1.15.*
- The impurity is called acceptor impurity, as it accepts one electron from neighboring semiconductor atoms if small amount of energy is supplied.
- *Then hole will be created in the neighboring semiconductor atom.*
- *The impurity atom supplied holes which are ready to accept electrons.*
- *The number of holes is thus more than electrons in P-type semiconductor.*
- ***Therefore, holes are majority charge carriers and electrons are minority charge carriers in P-type semiconductor. The conductivity is mainly due to holes in this semiconductor.***

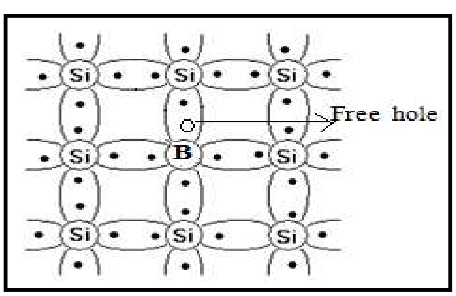


Fig1.15: P-type semiconductor

1.15.4 ENERGY BAND DIAGRAM OF P-TYPE SEMICONDUCTOR

- ***The energy band diagram of P-type semiconductor is shown in fig.1.16.***
- At 0 K, the acceptor levels are devoid of electrons and are hence empty.The valence band is full and conduction band is empty, the material behaves as an insulator (Fig.1.16a).
- The acceptor level is located above the valence band and fermi level E_F is between acceptor level and top of valence band.
- While at higher temperature (T > 0K), the electrons from valence band jump to acceptor level and also into the conduction band leaving holes in valence band, hence material shows the conductivity (Fig.1.16b).
- The acceptor level is now filled with electrons. It contains ionized atoms carrying negative charge. The fermi level $E_F$shifts above the acceptor level.
- In P type semiconductor, holes are majority charge carriers and electrons are minority charge carriers.

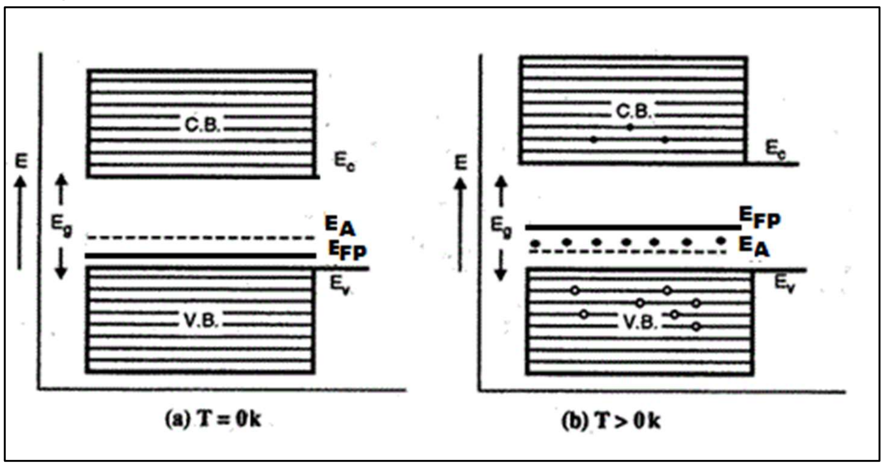


Fig.1.16: Energy band diagram of P-type semiconductor at (a) At 0K (b) At T > 0K

QUE: Draw energy band diagram for N- type semiconductor at 0 K and T K.
(4)[Winter-14, Summer-16]
QUE: Draw energy band diagram for P- type semiconductor at room temperature.
(3)[Winter-11]

1.15.5 DIFFERENCE BETWEEN INTRINSIC & EXTRINSIC SEMICONDUCTOR

Table 1.2: Differences between intrinsic semiconductors and extrinsic semiconductors

Intrinsic Semiconductor.	**Extrinsic Semiconductor**
1. It is a **pure** Semiconductor.	1. It is **impure** (doped) Semiconductor.
2. No. of electrons (n) and no. of holes (p) are equal**.(n = p)**	2. No. of electrons (n) and no. of holes (p) are not equal**.(n ≠ p)**

3. Cannot be used to manufacture devices.	3. Can be used to manufacture devices.
4. Fermi level always lies at the middle of forbidden energy gap.(**$E_F = E_g/2$**)	4. Fermi level does not lie at the middle of forbidden energy gap. (**$E_F \neq E_g/2$**)

QUE: Distinguish between intrinsic and extrinsic semiconductors.

1.15.6 DIFFERENCE BETWEEN N-TYPE & P-TYPE SEMICONDUCTOR

Table 1.3: Differences between N-type Semiconductor and P-type semiconductor

N-type Semiconductor	P-type Semiconductor
1. It is doped with **pentavalent** impurity.	1. It is doped with **trivalent** impurity.
2. No. of electrons (n) is greater than **no. of holes** (p).**(n ≥ p)**	**2. No. of electrons** (n) is less than **no. of holes** (p).**(n ≤ p)**
3. Fermi level lies near conduction band.	**3. Fermi level** lies near valence band.

ImportantFormulae

1. Resistivity ρ = 1/ σ where σ = conductivity (Ω^{-1} m^{-1})
2. Electrical conductivity σ = n e μ where μ is mobility of electrons
3. Mobility of electrons $\mu = \frac{\sigma}{ne}$
4. Current density $J = \frac{I}{A} = \frac{I}{\pi r^2}$
5. Drift velocity $v_d = \frac{J}{n\,e}$ where J = Current density and n= concentration of free electrons per unit volume or free electron density.
6. Drift velocity $v_d = \mu E$ where E is Electric field strength
7. Boltzmann constant $k = 1.38 \times 10^{23}$ J/K = $\frac{1.38\times10^{23}}{1.602\times10^{-19}} = 8.6 \times 10^{-5}$ eV/K
8. Probability that an energy level E is filled by an electron

$$f(E) = \frac{1}{1 + \exp\left[\frac{(E-E_F)}{kT}\right]}$$

9. Probability that an energy level E is not occupied by an electron = 1 - f(E)

1.A uniform silver wire has a resistivity of 1.54×10^{-8} Ω m at room temperature. Calculate the mobility of electrons in silver wire assuming that there is 5.8×10^{28} conduction electrons /m^3.

Solution:

Given: In Silver, Resistivity $\rho = 1.54\times10^{-8}$ Ω m

Concentration of electrons $n = 5.8 \times 10^{28}$ /m^3

Mobility μ =?

Electrical conductivity $\sigma = n\,e\,\mu = \frac{1}{\rho}$

$$\text{Mobility } \mu = \frac{1}{\rho n e}$$

$$= \frac{1}{1.54\times10^{-8}\times5.8\times10^{28}\times1.602\times10^{-19}}$$

$$= 6.98\times 10^{-3}\, m^2/V.s$$

2.Calculate the current density in copper wire of diameter 0.16 cm which carries a steady current of 10 A.

Solution:

Given: Diameter of Copper wire d = 0.16 cm = 0.16×10^{-2} m

Radius of Copper wire = d/2 = 0.08×10^{-2} m

Current I = 10 A

Current density J =?

Current density J = $\frac{I}{A} = \frac{I}{\pi r^2}$

$= \frac{10}{3.14\times (0.08\times 10^{-2})^2} = 4.976\times\ 10^6\ A/m^2$

3.Calculate the drift velocity of the free electrons with a mobility of 3.5 × $10^{-3}\ m^2/V.s$ in copper for an electric field strength of 0.5 V m^{-1}.

Solution:

Given: Mobility of electrons $\mu = 3.5 \times 10^{-3} m^2/V.s$

Electric field strength E = 0.5 V/m

Drift velocity v_d =?

Drift velocity $v_d = \mu E = 3.5\times\ 10^{-3}\ \times 0.5$

$= 1.75\times\ 10^{-3}$ m/s

4. Find the drift velocity of electrons in a copper wire whose cross-sectional area is 1 mm when the wire carries a current of 10 A. The density of free electrons in copper is $8.5\times\ 10^{28}$ electrons/m^3. Assume that each copper atom contributes one electron to the electron gas.

Solution:

Given: Area of cross-section A = 1 mm = 10^{-3} m

Current I = 10 A

Free electron density = $8.5\times\ 10^{28}\ electrons/m^3$

Drift velocity v_d =?

Drift velocity $v_d = \frac{J}{n\,e} = \frac{\frac{I}{A}}{n\,e}$

$= \frac{I}{A\,n\,e}$

$= \frac{10}{10^{-3}\times 8.5\times 10^{28}\ \times 1.602\times 10^{-19}}$

$= 7.343\times\ 10^{-7}\ m/s$

5.Find the mobility of electrons in copper assuming that each atom contributes one free electron for conductivity. (Resistivity of Copper = 1.7X10^{-6} Ω-cm, free electron concentration in Copper = 8.5 ×10^{28} /m^3).

Solution:

Given: Resistivity of Copper 'ρ' =1.7×10^{-6} Ω-cm = 1.7×10^{-8} Ω-m

Free electron concentration in Copper 'n' = 8.5×10^{28} /m^3

Mobility of electrons μ =?

Mobility $\mu = \frac{1}{\rho n e}$

$= \frac{1}{1.7 \times 10^{-8} \times 8.5 \times 10^{28} \times 1.602 \times 10^{-19}}$

$= 4.32 \times 10^{-3}\, m^2/V.s$

6. A semiconductor wafer is 0.5 mm thick. A potential of 100mV is applied across the wafer. If the drift velocity is 40 m/s across it, find the electron mobility.

Solution:

Given: Thickness of Wafer 'd' = 0.5 mm = 5 x 10^{-4} m

Potential 'V' = 100 mV =100 x 10^{-3}V= 0.1 V

Drift velocity $v_d = 40\ m/s$

Mobility μ =?

Drift velocity $v_d = \mu E = \mu \frac{V}{d}$

$\therefore\ \mu = \frac{v_d \times d}{V} = \frac{40 \times 5 \times 10^{-4}}{0.1}$ = 0.2 m^2/V.s

7. The conductivity of silver at 20°C is $6.8 \times 10^7 \Omega^{-1}$ m^{-1}. Calculate the mobility of electrons in silver assuming that there are 5.8×10^{28} conduction electrons /m^3.

Solution:

Given: Electrical conductivity $\sigma = 6.8 \times 10^7 \Omega^{-1}$ m^{-1}

Free electron density n = 5.8×10^{28} electrons /m^3

Mobility of electrons μ =?

Mobility of electrons $\mu = \frac{\sigma}{ne}$

$= \frac{6.8 \times 10^7}{5.8 \times 10^{28} \times 1.602 \times 10^{-19}} = 7.318 \times 10^{-3}\, m^2/V.s$

8.What is the probability that a quantum state whose energy is 0.10 eV (i) above and (ii) below Fermi energy will be occupied? Assume T as 800K.

Ans: Given: $E-E_F$ =0.10 eV

T= 800K

Solution: Case (i): Above Fermi energy level

$$f(E) = \frac{1}{1+\exp\left[\frac{(E-E_F)}{kT}\right]} = \frac{1}{1+\exp\left[\frac{(0.10)}{8.6 \times 10^{-5} \times 800}\right]} = 0.19 = 19\%$$

Case (ii): Below Fermi energy level

$$f(E) = \frac{1}{1+\exp\left[-\frac{(E-E_F)}{kT}\right]} = \frac{1}{1+\exp\left[\frac{-(0.10)}{8.6 \times 10^{-5} \times 800}\right]} = 0.81 = 81\%$$

1-f(E) = 1-0.81 =0.19 =19%

9. Evaluate the Fermi function for energy kT above the Fermi energy.

Solution:

$$f(E) = \frac{1}{1 + \exp\left[\frac{(E-E_F)}{kT}\right]}$$

For energy kT above the Fermi energy, $E - E_F = kT$

Therefore, $f(E) = \frac{1}{1+\exp\left[\frac{(E-E_F)}{kT}\right]} = \frac{1}{1+\exp\left[\frac{(kT)}{kT}\right]} = \frac{1}{1+\exp[1]} = \frac{1}{1+2.7183} = 0.2689$

10. Use the Fermi distribution function to obtain the value of F(E) for E – E_F = 0.01 eV at 200K.

Solution:

Given: E – E_F = 0.01 eV = 0.01 × 1.6×10^{-19}C = 1.6×10^{-21} J

T = 200K

$f(E) = ?$

Boltzmann constant k = 1.38×10^{-23}J/K

$$f(E) = \frac{1}{1 + \exp\left[\frac{(E-E_F)}{kT}\right]}$$

$$= \frac{1}{1+\exp\left[\frac{(0.01 \times 1.6\times10^{-1})}{1.38\times 10^{-23} \times200}\right]}$$

$$= \frac{1}{1+\exp[0.5797]}$$

$$= \frac{1}{1+1.7855} = 0.3589$$

11. In a solid consider the energy level lying 0.01eV below Fermi level. What is the probability of this level not being occupied by an electron?

Solution:

Given: E_F - E = 0.01 eV or (E – E_F) = - 0.01 eV

At room temperature, Thermal energy kT = 0.026 eV

Probability of a level being not occupied by an electron, 1-f(E) =?

$$f(E) = \frac{1}{1 + \exp\left[\frac{(E-E_F)}{kT}\right]}$$

$$= \frac{1}{1+\exp\left[\frac{-0.01}{0.026}\right]} = 0.595$$

1-f(E) =1- 0.595 = 0.405

12. Calculate the probabilities for an electronic state to be occupied at 20°C if energy of these states is 0.11 eV (i) above and (ii) below Fermi level.

Solution:

Given: E-E_F =0.11 eV

T= 20+273 = 293 K

Solution: Case (i): Above Fermi energy level

$$f(E) = \frac{1}{1+\exp\left[\frac{(E-E_F)}{kT}\right]} = \frac{1}{1+\exp\left[\frac{(0.11)}{8.6\times10^{-5}\times293}\right]} = 0.0126$$

Case (ii): Below Fermi energy level

$$f(E) = \frac{1}{1+\exp\left[-\frac{(E-E_F)}{kT}\right]} = \frac{1}{1+\exp\left[\frac{-(0.11)}{8.6\times10^{-5}\times293}\right]} = 0.987$$

13. Find the temperature at which there is 1% probability that a state exists with energy 0.5 eV above Fermi energy.

Solution :

Given : Probability f (E) = 1% =1/100

$E - E_F = 0.5$ eV

Boltzmann constant $k = 8.6 \times 10^{-5}$ eV/K
Temperature T = ?

$$f(E) = \frac{1}{1+\exp\left[\frac{(E-E_F)}{kT}\right]}$$

$$\frac{1}{100} = \frac{1}{1+\exp\left[\frac{(0.5)}{8.6\times10^{-5}\times T}\right]}$$

$$or\ 100 = 1+\exp\left[\frac{(0.5)}{8.6\times10^{-5}\times T}\right]$$

$$99 = 1+\exp\left[\frac{5813.95}{T}\right]$$

$$\text{or}\quad 99 = \exp\left[\frac{5813.95}{T}\right]$$

$$\text{or}\ \ln 99 = \frac{5813.95}{T}$$

$$\text{or}\quad T = \frac{5813.95}{4.595} = 1265.27\ \text{K}$$

14. For Copper at 1000K, find the energy at which the probability F(E) that a conduction electron state will be occupied is 90%. The fermi energy is 7.06 eV.

Solution:

Given: T = 1000K

F(E) = 90% = 0.90

E_F = 7.06eV

Boltzmann constant k= 8.6×10^{-5}eV/K

Energy E =?

$$f(E) = \frac{1}{1+\exp\left[\frac{(E-E_F)}{kT}\right]}$$

$$0.90 = \frac{1}{1 + \exp\left[\frac{(E-E_F)}{kT}\right]}$$

$$\text{or } 1 + \exp\left[\frac{(E-E_F)}{kT}\right] = \frac{1}{0.90}$$

$$\exp\left[\frac{(E-E_F)}{kT}\right] = \frac{1}{0.90} - 1$$

$$\text{or } \exp\left[\frac{(E-E_F)}{kT}\right] = 0.11$$

$$\text{or } \frac{(E-E_F)}{kT} = \ln 0.11$$

or $E - E_F = kT \ln 0.11 = 8.6 \times 10^{-5} \times 1000 \times (-2.207)$

or $E - E_F = -0.189$

or $E - 7.06 = -0.189$ or $E = 6.87\text{eV}$

15. Estimate the temperature at which 5% probability of electrons having an excess energy of 0.1 eV above Fermi energy level of Gold.

Solution:

Given: f(E)=5%=0.05

$(E-E_F) = 0.1$ eV

Boltzmann constant $k = 1.38 \times 10^{-23}$J/K

$$f(E) = \frac{1}{1 + \exp\left[\frac{(E-E_F)}{kT}\right]}$$

$$0.05 = \frac{1}{1 + \exp\left[\frac{(E-E_F)}{kT}\right]}$$

$$\text{or } 1 + \exp\left[\frac{(E-E_F)}{kT}\right] = \frac{1}{0.05}$$

$$\exp\left[\frac{(E-E_F)}{kT}\right] = \frac{1}{0.05} - 1$$

$$\text{or } \exp\left[\frac{(E-E_F)}{kT}\right] = 19$$

$$\text{or } \frac{(E-E_F)}{kT} = \ln 19$$

$$\frac{(E-E_F)}{kT} = 2.9444$$

$$T = \frac{0.1x1.6x10^{-19}}{1.38\quad ^{-23}x2.9444}$$

T=393.842 K

16. At what temperature we can expect a 10% probability that electrons in silver have an energy which is a 1% above Fermi energy ? The fermi energy of silver= 5.5eV.

Solution :

Given : Probability f (E) = 10% =0.1

$$E - E_F = 1\%E_F = 5.5eV/100 = 0.055eV$$

Boltzmann constant $k = 8.6 \times 10^{-5}$ eV/K
Temperature T = ?

$$f(E) = \frac{1}{1 + \exp\left[\frac{(E-E_F)}{kT}\right]}$$

$$\frac{10}{100} = \frac{1}{1 + \exp\left[\frac{(0.055)}{8.6\times10^{-5}\times T}\right]}$$

$$or\ 10 = 1 + \exp\left[\frac{(0.055)}{8.6 \times 10^{-5} \times T}\right]$$

$$10 = 1 + \exp\left[\frac{639.53}{T}\right]$$

$$\text{or} \quad 9 = \exp\left[\frac{639.53}{T}\right]$$

$$\text{or } \ln 9 = \frac{639.53}{T}$$

$$\text{or } T = \frac{639.53}{2.197} = 291.09 \text{ K}$$

17.The fermi energy of silver is 5.5eV.Calculate the fraction of free electrons at room temperature located up to a width of kT on either side of fermi level.

Solution :

Given : E_F = 5.5 eV

The fraction of electrons that occupy level higher than fermi level$E_F$is given by

$$\frac{kT}{E_F} = \frac{0.026\ eV}{5,5\ eV} = 0.0047$$

Therefore,the fraction of free electrons at room temperature located up to a width of kT on either side of fermi level = 2×0.0047 = 0.01.

18.A specimen of silicon has a cross section area 2x2 mm^2, one volt is impressed across the bar result in current of 8 mA. Determine the drift velocity of free electrons. The number of electrons per unit volume in silicon is 1.92 x 10^{21} /m^3.

Given: Area of cross-section A = 2x2 mm^2=2x10^{-3}x2x10^{-3}= 4x10^{-6}m^2

Current I = 8mA=8x10^{-3}

Free electron density = 1.92 x 10^{21}$electrons/m^3$

Drift velocity v_d =?

$$\text{Drift velocity } v_d = \frac{J}{n\,e} = \frac{\frac{I}{A}}{n\,e}$$

$$= \frac{I}{A\,n\,e}$$

$$= \frac{8x10^{-3}}{4x10^{-6}\times 1.92\times 10^{21} \times 1.602\times 10^{-19}}$$

$$= 6.5 m/s$$

Question Bank

Q1. What does Free electron theory in metals suggests? Explain.
Q2. State and derive expression for of conductivity of a metal.
Q3. Define drift velocity. Write an expression for drift velocity in terms of mobility and electric field.
Q4. Write an expression for relation between drift velocity and current density.
Q5. Explain the formation of energy bands in solids on the basis of band theory of solids. **[(4)S-05, S-7]**
Q6. Discuss energy band structures of conductors, insulators and semiconductors. **[(3)W-13]**
Q7. Explain classification of solids on the basis of energy band diagram. **[(3)S-17]**
Q8. Discuss the classification of solids on the basis of forbidden energy gap. **[(3)S-18]**
Q9.What is meant by Femi-Dirac Distribution function? Define Fermi level. **[(3)S-13]**
Q10. What is fermi function? Draw a graph showing its variation with energy at different temperatures and discuss it. **[(4)W-17, S-04, S-11]**
Q11.What is Fermi function? Explain with the help of a diagram how it varies with change of temperature. **[(4) W-12]**
Q12.What is Fermi Dirac distribution function? State its significance. **[(4)W-09]**
Q13.What is an intrinsic semiconductor?
Q14.What is extrinsic semiconductor**?**
Q15.Draw energy band diagram for N- type semiconductor at 0 K and T K.
Q16.Draw energy band diagram for P- type semiconductor at 0 K and T K.
Q17.Draw energy band diagram for intrinsic semiconductor at 0 K and T K.
Q.18.What iseffective mass of electron in solids ?
Q19.What is bound and free electrons?
Q20.What is the basic assumptions of classical free electron theory.
Q21.Discuss the important postulates of free electron theory.
Q22.What is drift velocity, drift current and mobility of electrons?
Q23. Derive an expression for the electrical conductivity of a metal using free electron theory.
Q24. Elaborate the factors affecting conductivity of materials.
Q25. Explain how the materials are classified on the basis of conductivity.
Q26.The mobility of electrons in copper is $34.8 cm^2/V.s.$ If an electric field of intensity 500 V/m is applied across the copper bar, what is the drift velocity of electrons? [Ans.1.74m/s]
Q.27.A specimen of silicon has a cross section area of 2x2 mm^2. One volt impressed across the bar results in a current of 8mA. Determine the drift velocity of free electrons. The number of electrons per unit volume in silicon is $1.92 \times 10^{21}/m^3$.[Ans.6.5 m/s]
Q28. Calculate the probability that an energy level 3kT above fermi level is occupied by electron. [Ans: 0.047]
Q.29.In a solid consider the energy level lying 0.01eV above Fermi level. What is the probability of this level being occupied by an electron at 300K? [Ans: 0.045]
Q.30.In a solid consider the energy level lying 0.01eV above Fermi level. What is the probability of this level not being occupied by an electronat 300K? [Ans: 0.595]
Q.31.In a solid consider the energy level lying 0.01eV above Fermi level. What is the probability of this level being occupied by an electron at 200K? [Ans: 0.359]
Q.32. In a solid consider the energy level lying 0.01eV below Fermi level. What is the probability of this level being occupied by an electron at 300K? [Ans: 0.595]

Q.33.Find the temperature at which there is 1% probability that a state exists with energy 2 eV is occupied. Given that fermi energy is 1.5 eV. [Ans: 1262K]