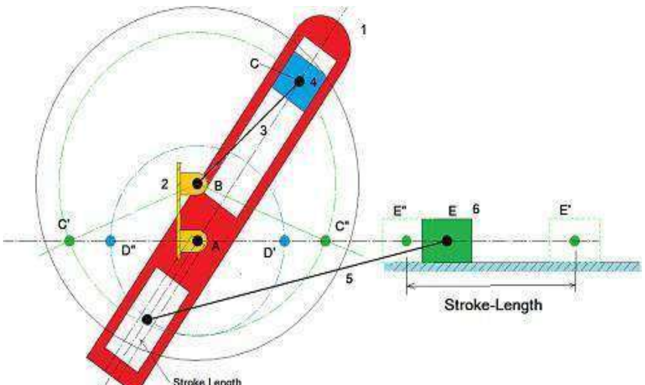
UNIT 1

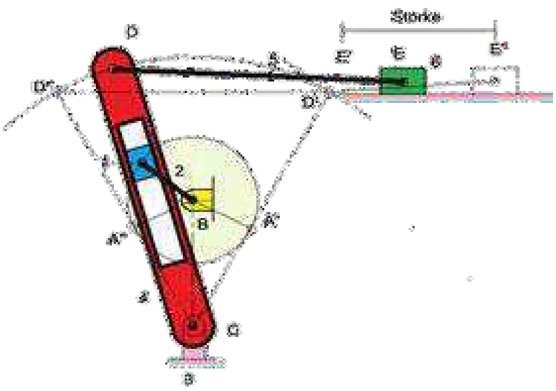
MECHANISMS & MACHINES

1

Machines and Mechanisms



Whitworth Quick-Return Mechanism



Crank-Rocker Quick-Return Mechanism for Shaping Machine

Course Contents

Machine and Mechanism:

- **Mechanism:**

– If a number of bodies are assembled in such a way that the motion of one causes constrained and predictable motion to the others, it is known as a *mechanism.*

- **Machine:**

– A *machine* is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work.

- **Analysis:**

– *Analysis* is the study of motions and forces concerning different parts of an existing mechanism.

- **Synthesis:**

– *Synthesis* involves the design of its different parts.

Types of constrained motion:

Completely constrained motion:

– When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion.

– For example, the piston and cylinder (in a steam engine) form a pair and the motion of the piston is limited to a definite direction (*i.e.* it will only reciprocate) relative to the cylinder irrespective of the direction of motion of the crank.

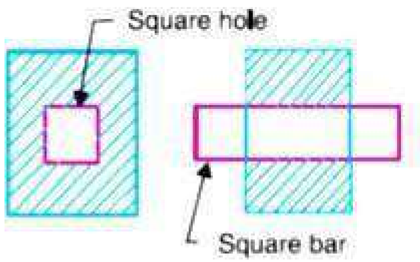


Fig. 1.1

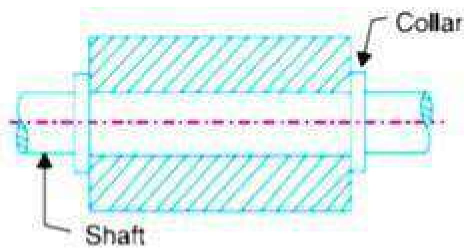


fig. 1.2

– The motion of a square bar in a square hole, as shown in Fig. 1.1, and the motion of a shaft with collars at each end in a circular hole, as shown in Fig. 1.2, are also examples of completely constrained motion.

Incompletely constrained motion:

– When the motion between a pair can take place in more than one direction, then the motion is called an incompletely constrained motion. The change in the direction of impressed force may alter the direction of relative motion between the pair. A circular bar or shaft in a circular hole, as shown in Fig. 1.3, is an example of an incompletely constrained motion as it may either rotate or slide in a hole. These both motions have no relationship with the other.

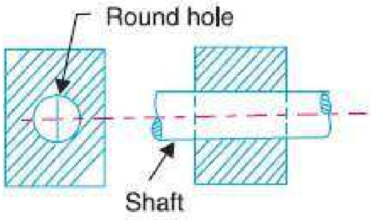


Fig. 1.3

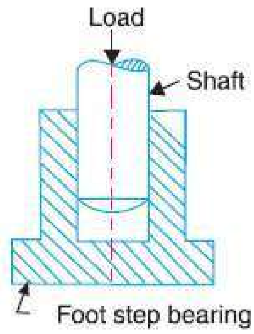


FIG. 1.4

Successfully constrained motion:

- When the motion between the elements, forming a pair, is such that the constrained motion is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion. Consider a shaft in a foot-step bearing as shown in Fig. 1.4.
- The shaft may rotate in a bearing or it may move upwards. This is a case of incompletely constrained motion. But if the load is placed on the shaft to prevent axial upward movement of the shaft, then the motion of the pair is said to be successfully constrained motion. The motion of an I.C. engine

Types of Links:

- A mechanism is made of a number of resistant bodies out of which some may have motions relative to the others. A resistant body or a group of resistant bodies with rigid connections preventing their relative movements is known as a link.
- A link may also define as a member or a combination of members of a mechanism, connecting other members and having motion relative to them.
- Links may be classified into binary, ternary and quaternary.

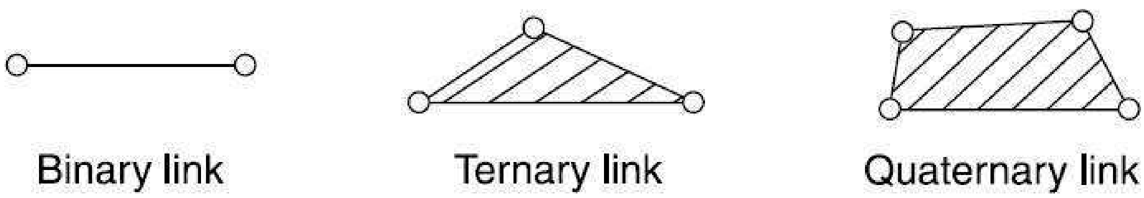


FIG. 1.4 Types of link

Kinematic Pair:

- When two kinematic links are connected in such a way that their motion is either completely or successfully constrained, these two links are said to form a kinematic pair.
- Kinematic pairs can be classified according to:

Kinematic pairs according to nature of contact:

a. Lower Pair:

- A pair of links having surfaced or area contact between the members is known as a lower pair. The contact surfaces of two links are similar.
- Examples: Nut turning on a screw, shaft rotating in a bearing.

b. Higher Pair:

- When a pair has a point or line contact between the links, it is known as a higher pair. The contact surfaces of two links are similar.
- Example: Wheel rolling on a surface, Cam and Follower pair etc.

Kinematic pairs according to nature of contact:

a. Closed Pair:

- When the elements of a pair are held together mechanically, it is known as a closed pair. The two elements are geometrically identical; one is solid and full and the other is hollow or open. The latter not only envelops the former but also encloses it. The contact between the two can be broken only by destruction of at least one of the members.

b. Unclosed Pair:

- When two links of a pair are in contact either due to force of gravity or some spring action, they constitute an unclosed pair. In this, the links are not held together mechanically, e.g. cam and follower pair.

Kinematic pairs according to Nature of Relative Motion:

a. Sliding pair:

- When two links have a sliding motion relative to another; the kinematic pair is known as sliding pair.

b. Turning pair:

- When one link is revolve or turn with respect to the axis of first link, the kinematic pair formed by two links is known as turning pair.

c. Rolling pair:

- When the links of a pair have a rolling motion relative to each other, they form a rolling pair.

d. Screw pair:

- If two mating links have a turning as well as sliding motion between them, they form a screw pair.

e. Spherical pair:

- When one link in the form of sphere turns inside a fixed link, it is a spherical pair.

Types of Joint:

– The usual types of joints in a chain are:

  - Binary Joint
  - Ternary Joint
  - Quaternary Joint

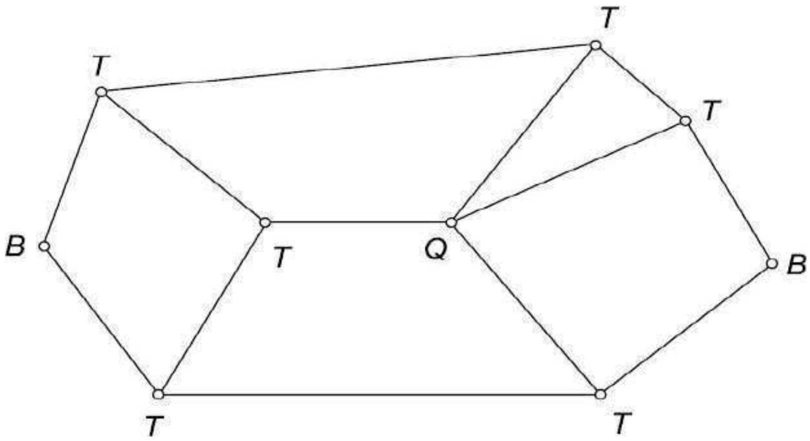


Fig1.5. Types of joint

a. **Binary Joint:**
   - If two links are joined at the same connection, it is called a binary joint. For example, in fig. at joint B

b. **Ternary Joint:**
   - If three links joined at a connection, it is known as a ternary link. For example point T in fig.

c. **Quaternary Joint:**
   - If four links joined at a connection, it is known as a quaternary link. For example point Q in fig.

Degrees of Freedom:

– An unconstrained rigid body moving in space can describe the following independent motion:

   a. Translational motion along any three mutually perpendicular axes x, y and z.

   b. Rotational motion about these axes

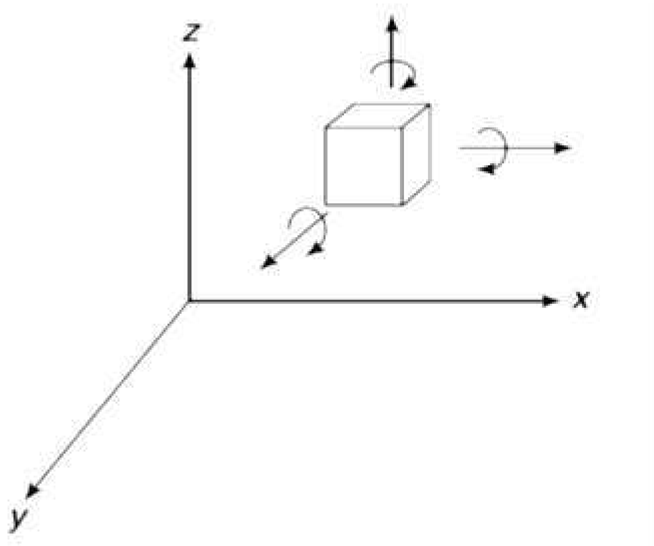


Fig.1.6 Degrees of freedom

- A rigid body possesses six degrees of freedom.
- Degrees of freedom of a pair is defined as the number of independent relative motions, both translational and rotational, a pair can have.
- DOF = 6 – Number of Restraints

Kinematic chain

- Kinematic chain is defined as the combination of kinematic pairs in which each links forms a part of two kinematic pairs and the relative motion between the links is either completely constrained or successfully constrained.
- Examples: slider-crank mechanism
- For a kinematic chain

$$N = 2P - 4 = 2(j + 2)/3$$

- Where N = no. of links, P = no. of Pairs and j = no. of joints
- When,

LHS > RHS, then the chain is locked
LHS = RHS, then the chain is constrained
LHS < RHS, then the chain is unconstrained

Kutzbach Criterion

- DOF of a mechanism in space can be determined as follows:
- In mechanism one link should be fixed. Therefore total no. of movable links are in mechanism is (N-1)
- Any pair having 1 DOF will impose 5 restraints on the mechanism, which reduces its total degree of freedom by 5P1.
- Any pair having 2 DOF will impose 4 restraints on the mechanism, which reduces its total degree of freedom by 4P2
- Similarly, the other pairs having 3, 4 and 5 degrees of freedom reduce the degrees of freedom of mechanism. Thus,
- Thus,

$$F = 6(N-1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - 1P_5 - 0P_6$$

- Hence,

$$F = 6(N-1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - 1P_5$$

- The above equation is the general form of **Kutzbach criterion.** This is applicable to any type of mechanism including a spatial mechanism.

Grubler's criterion

- If we apply the Kutzbach criterion to planer mechanism, then equation of Kutzbach criterion will be modified and that modified equation is known as Grubler‟s Criterion for planer mechanism.
- Therefore in planer mechanism if we consider the links having 1 to 3 DOF, the total number of degree of freedom of the mechanism considering all restraints will becomes,

$$F = 3\,(N-1) - 2\,P_1 - 1\,P_2$$

- The above equation is known as **Grubler's criterion** for planer mechanism.
- Sometimes all the above empirical relations can give incorrect results, e.g. fig (a) has 5 links, 6 turning pairs and 2 loops. Thus, it is a structure with zero degree of freedom.

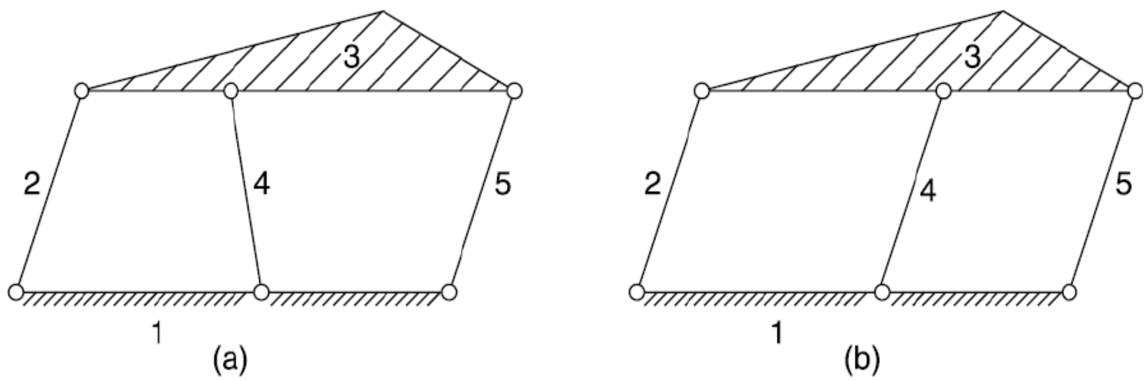


Fig. 1.7

- However, if the links are arranged in such a way as shown in fog. (b), a double parallelogram linkage with one degree of freedom is obtained. This is due to the reason that the lengths of links or other dimensional properties are not considered in these empirical relations.
- Sometimes a system may have one or more link which does not introduce any extra constraint. Such links are known as redundant links and should not be counted to find the degree of freedom. For example fig. (B) has 5 links, but the function of the mechanism is not affected even if any one of the links 2, 4 and 5 are removed. Thus, the effective number of links in this case is 4 with 4 turning pairs, and thus 1 degree of freedom.
- In case of a mechanism possessing some redundant degree of freedom, the effective degree of freedom is given by,

$$F = 3\,(N - 1) - 2\,P_1 - 1P_2 - F_r$$

- Where F = no. of redundant degrees of freedom

The Four-Bar chain

- A four bar chain is the most fundamental of the plane kinematic chains. It is a much proffered mechanical device for the mechanisation and control of motion due to its simplicity and versatility. Basically, it consists of four rigid links which are connected in the form of a quadrilateral by four pin-joints.
- When one of the link fixed, it is known as mechanism or linkage. A link that makes complete revolution is called the crank. The link opposite to the fixed link is called coupler, and the forth link is called a lever or rocker if it oscillates or another crank if it rotates.
- It is impossible to have a four-bar linkage if the length of one of the link is greater than the sum of other three. This has been shown in fig.

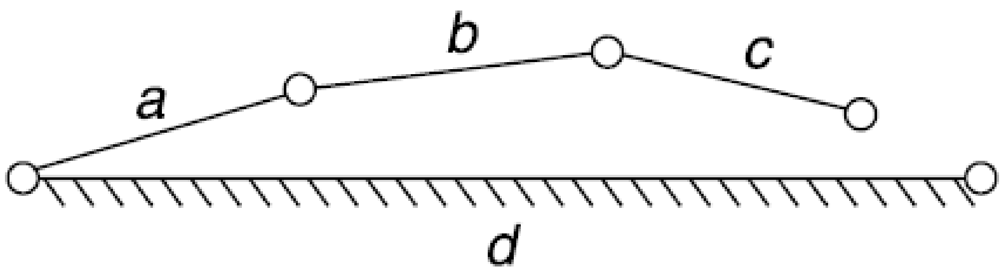


Fig. 1.7 Four bar chain

Grashof's law:

- We have already discussed that the kinematic chain is a combination of four or more kinematic pairs, such that the relative motion between the links or elements is completely constrained The simplest and the basic kinematic chain is a four bar chain or quadric cycle chain, as shown in Fig. 5.18. It consists of four links, each of them forms a turning pair at A, B, C and D. The four links may be of different lengths.
- According to Grashof"s "s law for a four bar mechanism, the sum of the shortest and longest link lengths should not be greater than the sum of the remaining two link lengths if there is to be continuous relative motion between the two links.

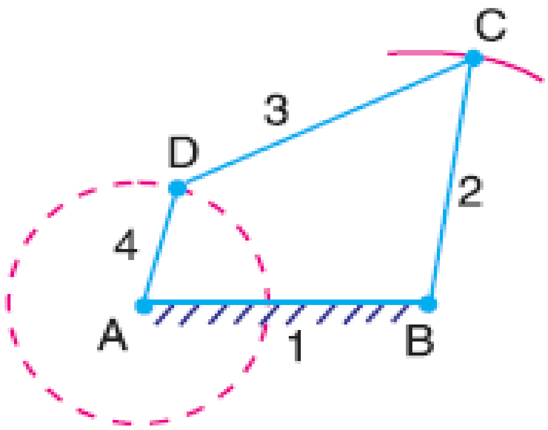


Fig. 1.8 Grashof's law

- A very important consideration in designing a mechanism is to ensure that the input crank makes a complete revolution relative to the other links. The mechanism in which no link makes a complete revolution will not be useful. In a four bar chain, one of the links, in particular the shortest link, will make a complete revolution relative to the other three links, if it satisfies the Grashof"s law. Such a link is known as crank or driver. In Fig.5.18, AD (link 4) is a crank.
- The link BC (link 2) which makes a partial rotation or oscillates is known as lever or rocker or follower and the link CD (link 3) which connects the crank and lever is called connecting rod or coupler. The fixed link AB (link 1) is known as frame of the mechanism.

Inversion of Mechanism:

- When the number of links in kinematic chain is more than three, the chain is known as mechanism. When one link of the kinematic chain at a time is fixed, give the different mechanism of the kinematic chain. The method of generating different mechanism by fixing a link is called the inversion of mechanism.
- The number of inversion is equal to the numbers of links in the kinematic chain.
- The inversion of mechanism may be classified as:
  - **a.** Inversion of four-bar chain
  - **b.** Inversion of single slider crank chain
  - **c.** Inversion of double slider crank chain

Inversion of Four-Bar chain

First inversion: coupled wheel of locomotive

- The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links is shown in Fig.

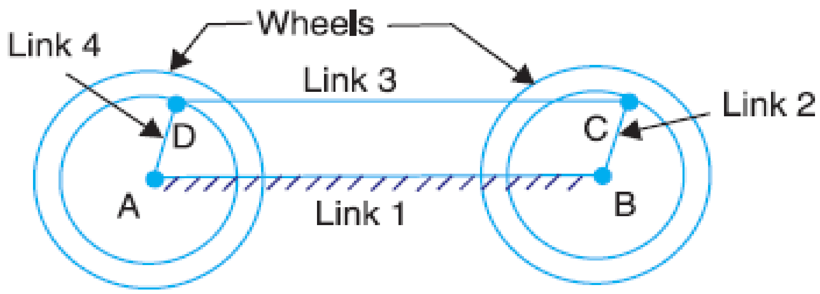


Fig. 1.9 coupled wheel of locomotive

- In this mechanism, the links *AD* and *BC* (having equal length) act as cranks and are connected to the respective wheels. The link *CD* acts as a coupling rod and the link *AB* is fixed in order to maintain a constant centre to Centre distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

Second inversion: Beam Engine

- A part of the mechanism of a beam engine (also known as cranks and lever mechanism) which consists of four links is shown in Fig. 1.10.
- In this mechanism, when the crank rotates about the fixed centre A, the lever oscillates about a fixed centre D. The end E of the lever CDE is connected to a piston rod which reciprocates due to the rotation of the crank.

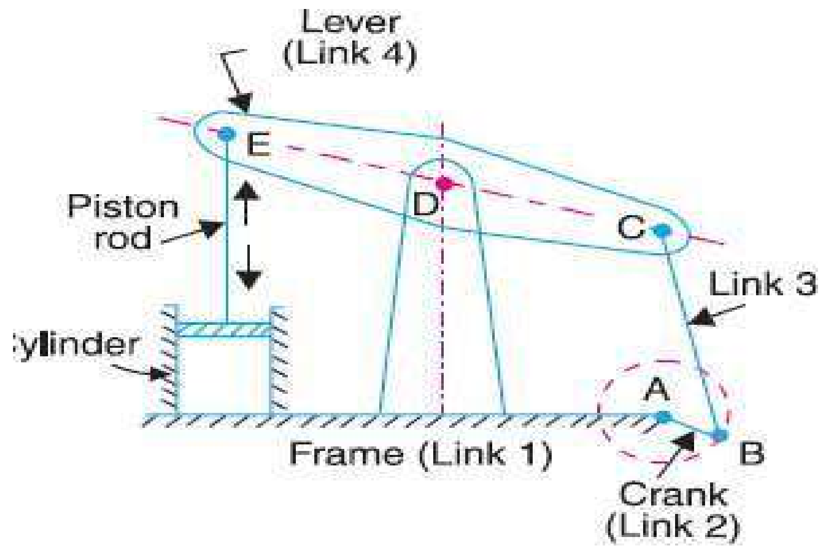


Fig. 1.10 beam engine

– In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.

Third inversion: watts indicator mechanism

– A Watt‟s indicator mechanism (also known as Watt's straight line mechanism or double lever mechanism) which consists of four links is shown in Fig.

– The four links are: fixed link at A, link AC, link CE and link BFD. It may be noted that BF and FD form one link because these two parts have no relative motion between them. The links CE and BFD act as levers.

– The displacement of the link BFD is directly proportional to the pressure of gas or steam which acts on the indicator plunger. On any small displacement of the mechanism, the tracing point E at the end of the link CE traces out approximately a straight line.

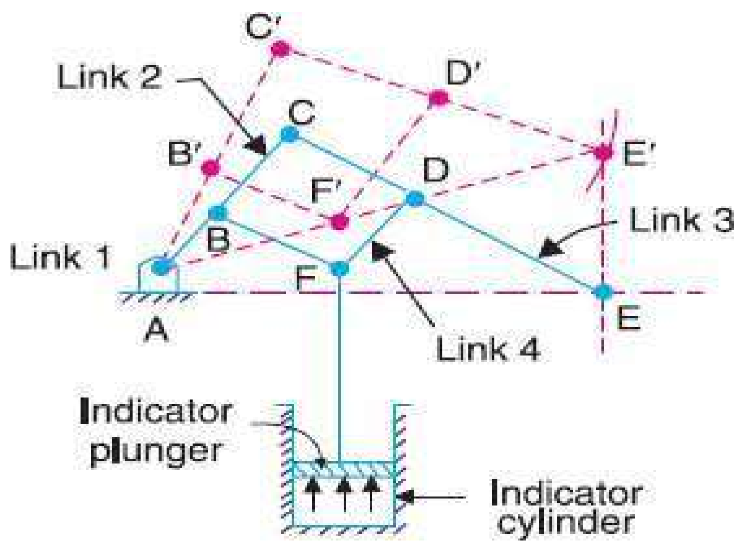


Fig. 1.11 watts indicator mechanism

The slider-crank chain

- When one of the turning pairs of a four-bar chain is replaced by a sliding pair, it becomes a single slider-crank chain or simply a slider-crank chain.
- It is also possible to replace two sliding pairs of a four-bar chain to get a double slider-crank chain. In a slider-crank chain, the straight line path of the slider may be passing through the fixed pivot O or may be displaced.
- The distance e between the fixed pivot O and the straight line path of the slider is called the offset and the chain so formed an offset slider-crank chain.
- Different mechanisms obtained by fixing different links of a kinematic chain are known as its inversions.

First inversion

- This inversion is obtained when link 1 is fixed and links 2 and 4 are made the crank and slider respectively. (fig.a)
- **Applications:**
  - a Reciprocating engine
  - b Reciprocating compressor

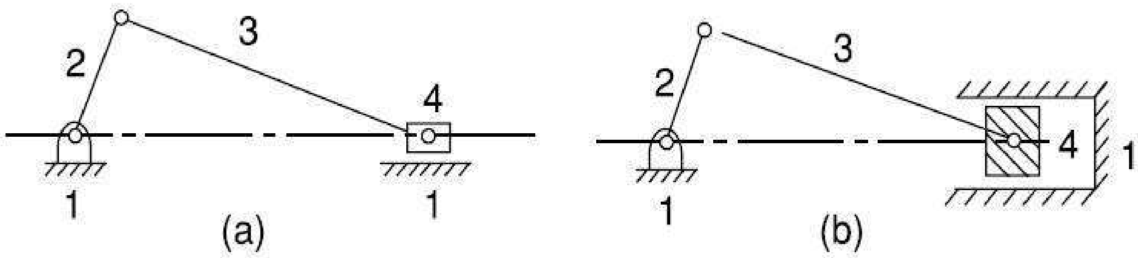


Fig. 1.12 first inversion

Second inversion

- Fixing of the link 2 of a slider-crank chain results in the second inversion.
- **Applications:**
  - a Whitworth quick-return mechanism
  - b Rotary engine

Third Inversion

- By Fixing of the link 3 of the slider-crank mechanism, the third inversion is obtained. Now the link 2 again acts as a crank and the link 4 oscillates.
- **Applications:**
  - a **Oscillating cylinder engine**
  - b Crank and slotted-lever mechanism

Fourth Inversion

- If the link 4 of the slider-crank mechanism is fixed, the fourth inversion is obtained. Link 3 can oscillates about the fixed pivot B on the link 4. This makes the end A of the link 2 to oscillate about B and the end O to reciprocate along the axis of the fixed link 4.

- **Application: Hand Pump**

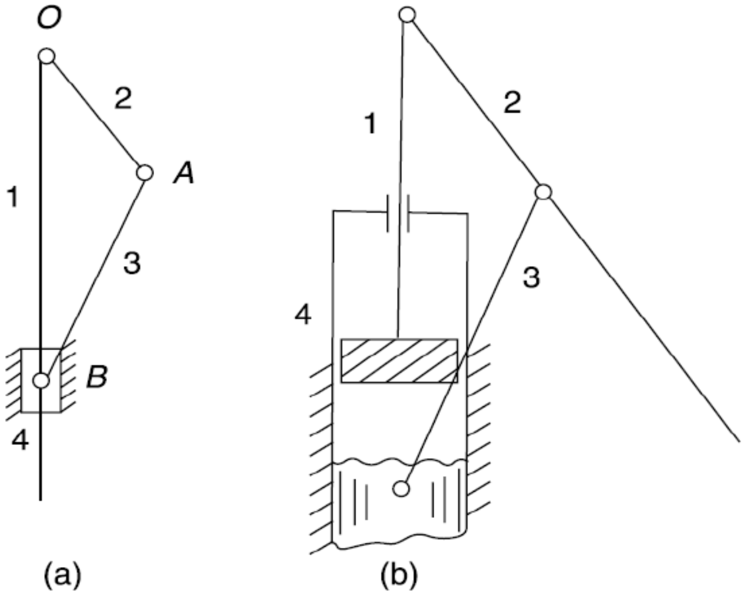


Fig. 1.13 hand pump

- Fig.1.13 shows a hand-pump. Link 4 is made in the form of a cylinder and a plunger fixed to the link 1 reciprocates in it.

Whitworth Quick-Return Mechanism:

- This mechanism used in shaping and slotting machines.
- In this mechanism the link CD (link 2) forming the turning pair is fixed; the driving crank CA (link 3) rotates at a uniform angular speed and the slider (link 4) attached to the crank pin at a slides along the slotted bar PA (link 1) which oscillates at D.
- The connecting rod PR carries the ram at R to which a cutting tool is fixed and the motion of the tool is constrained along the line RD produced.

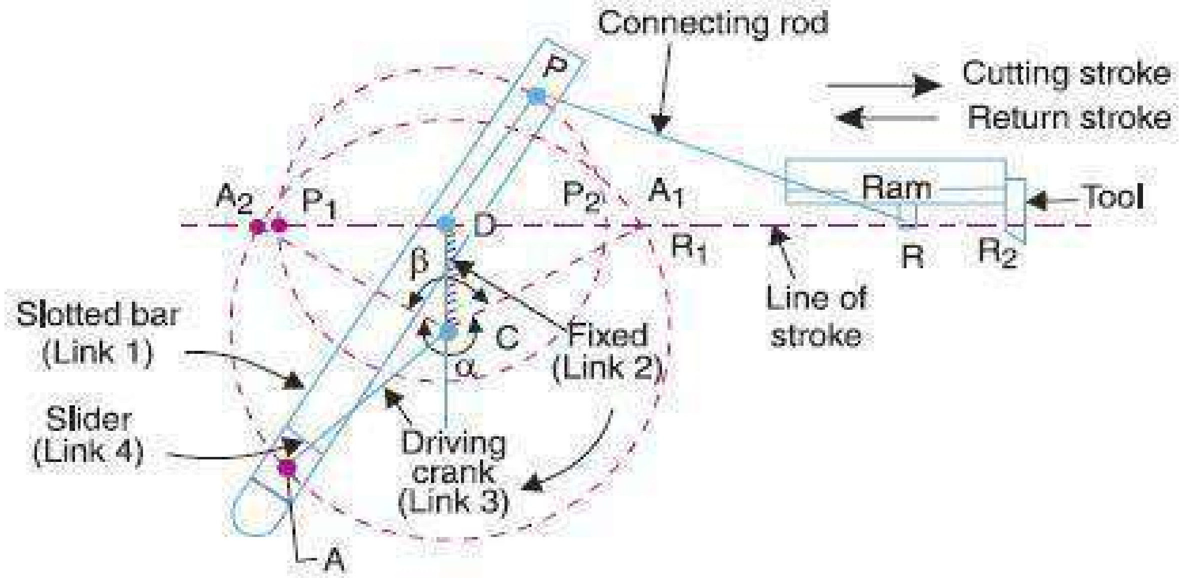


Fig. 1.14 Whitworth quick returns mechanism

– The length of effective stroke = 2 PD. And mark P1R1 = P2 R2 = PR.

$$\frac{\text{time of cutting stroke}}{\text{time of return}} = \frac{\alpha}{\beta} = \frac{\alpha}{360° - \alpha} = \frac{360° - \beta}{\beta}$$

Rotary engine

– Sometimes back, rotary internal combustion engines were used in aviation. But now- a-days gas turbines are used in its place.

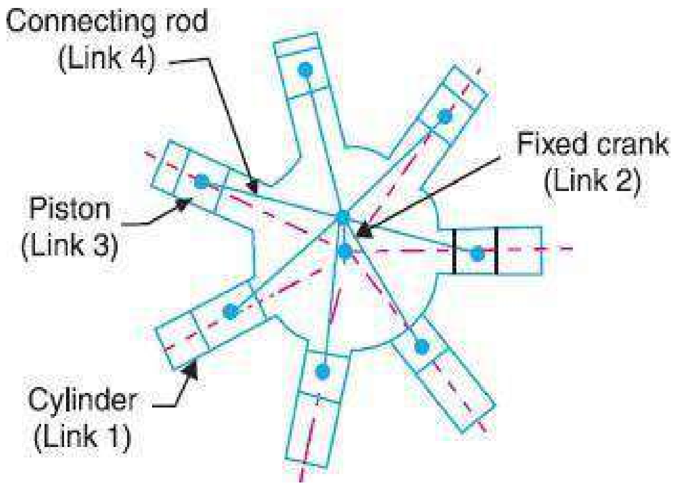


Fig. 1.15 rotary engine

– It consists of seven cylinders in one plane and all revolves about fixed center D, as shown in Fig. 5.25, while the crank (link 2) is fixed. In this mechanism, when the connecting rod (link 4) rotates, the piston (link 3) reciprocates inside the cylinders forming link 1.

Oscillating cylinder engine

– The arrangement of oscillating cylinder engine mechanism, as shown in Fig. Is used to convert reciprocating motion into rotary motion.

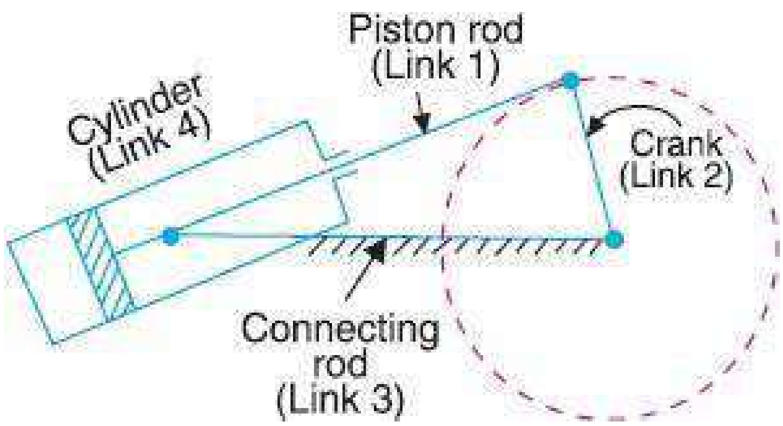


Fig. 1.16 oscillating cylinder engine

– In this mechanism, the link 3 forming the turning pair is fixed. The link 3 corresponds to the connecting rod of a reciprocating steam engine mechanism. When the crank (link 2) rotates, the piston attached to piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at *A*.

Crank and slotted-lever Mechanism

– This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines.

– In this mechanism, the link AC (i.e. link 3) forming the turning pair is fixed, as shown in Fig. The link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank CB revolves with uniform angular speed about the fixed center C. A sliding block attached to the crank pin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivoted point A.

– A short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke R1R2. The line of stroke of the ram (i.e. R1R2) is perpendicular to AC produced.

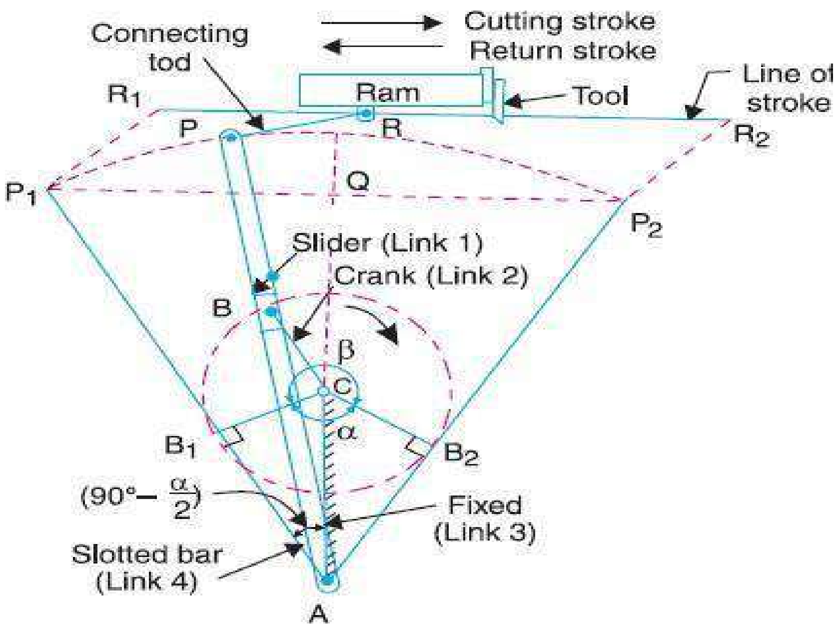


Fig.1.17 *Crank and slotted lever mechanism*

– In the extreme positions, *AP*1 and *AP*2 are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position *CB*1 to *CB*2 (or through an angle β) in the clockwise direction. The return stroke occurs when the crank rotates from the position *CB*2 to *CB*1 (or through angle α) in the clockwise direction. Since the crank has uniform angular speed, therefore,

$$\frac{\text{time of cutting stroke}}{\text{time o} \quad \text{return}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta} = \frac{360^\circ - \alpha}{\alpha}$$

Example based on Degrees of Freedom:

1 **For the kinematic linkages shown in following fig. calculate the following:**

The numbers of binary links (N_b)

The numbers of ternary links (Nt)

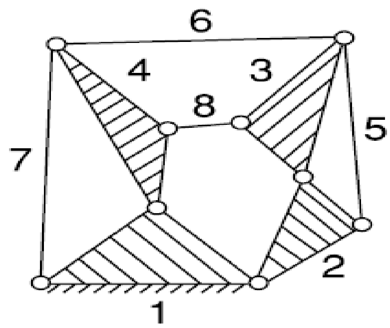
The numbers of other (quaternary) links (No)

The numbers of total links (n)

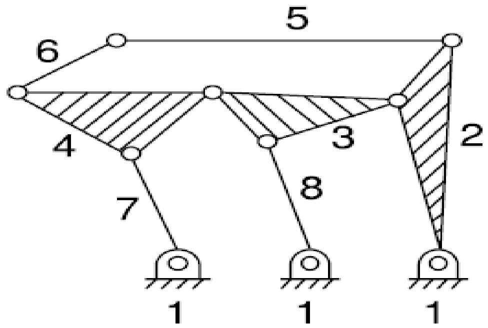
The numbers of loops (L)

The numbers of joints or pairs (P1)

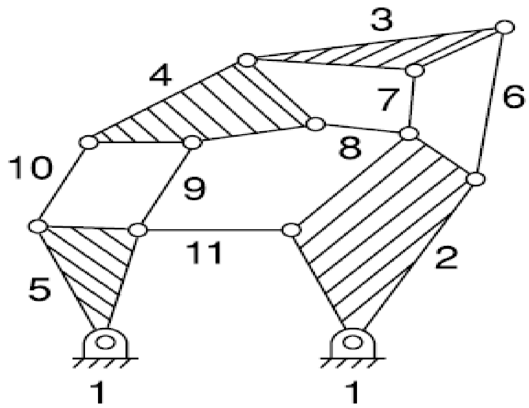
The numbers of degrees of freedom (F)



(a)



(b)



(c)

a $N_b = 4$; $N_t = 4$; $N_0 = 0$; $N = 8$; $L = 4$; $P_1 = 11$ (by counting) $P_1 = (N + L - 1) = 11$

$F = 3 (N - 1) - 2P_1$

$F = 3 (8 - 1) - 2 \times 11 = -1$ or,

v $F = N - (2 L + 1)$

$F = 8 - (2 \times 4 + 1) = -1$

b $N_b = 4$; $N_t = 4$; $N_0 = 0$; $N = 8$; $L = 3$; $P_1 = 10$ (by counting) $P_1 = (N + L - 1) = 10$

$F = 3(N - 1) - 2P_1$
$F = 3(8 - 1) - 2\times10 = 1$
or, $F = N - (2L + 1)$
$F = 8 - (2\times3 + 1) = 1$

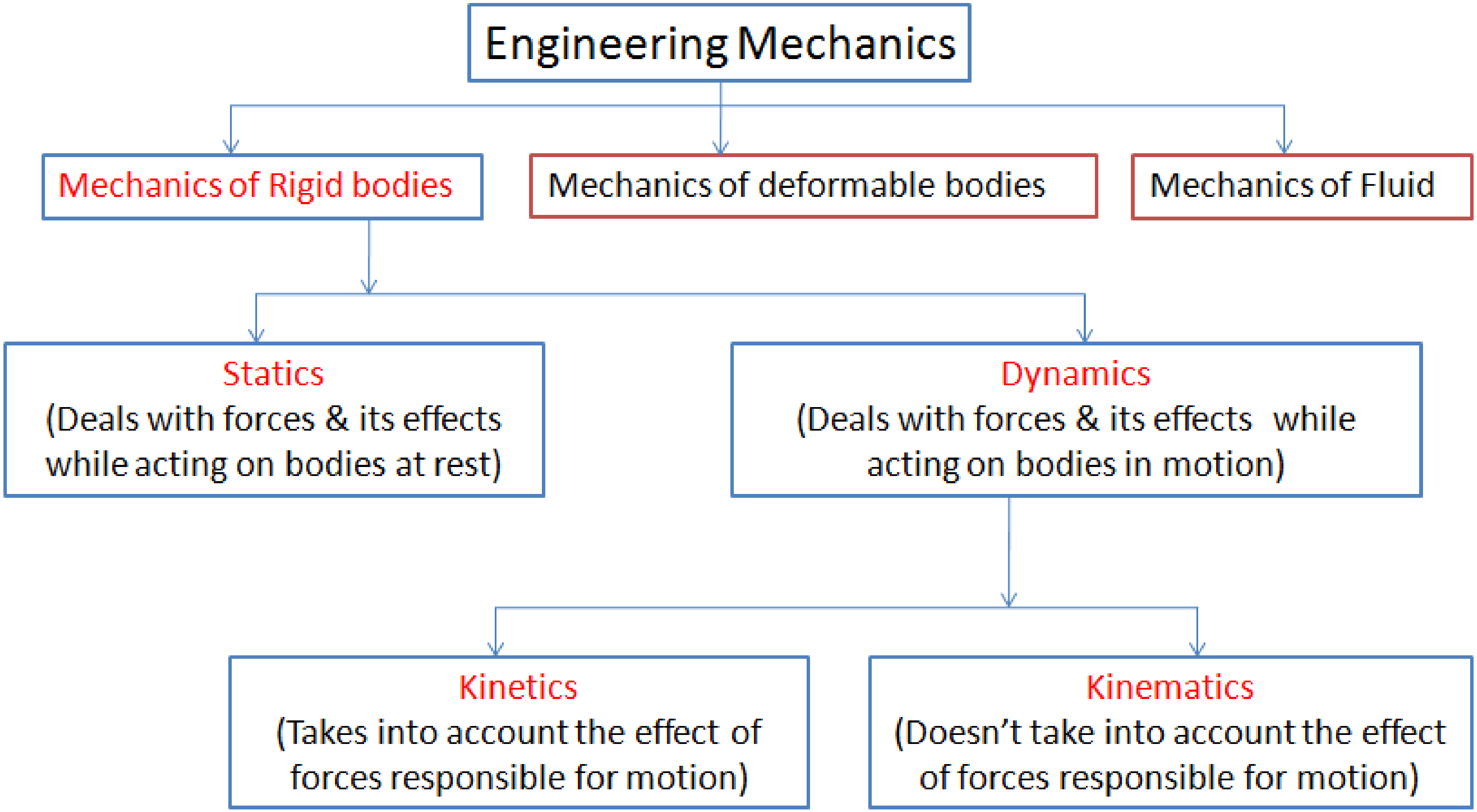
c $N_b = 7$; $N_t = 2$; $N_0 = 2$; $N = 11$; $L = 5$; $P_1 = 15$ (by counting) $F = N - (2L + 1)$
$F = 11 - (2\times5 + 1) = 0$
Therefore the linkage is a structure.

LECTURE 1

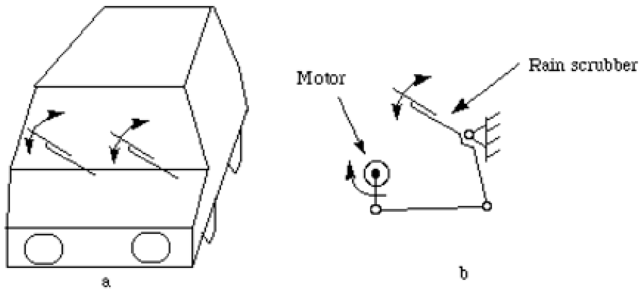
Mechanisms

MRCET CAMPUS
UGC Autonomous

DEPARTMENT OF MECHANICAL ENGINEERING


Engineering Mechanics
Mechanics of Rigid bodies
Mechanics of deformable bodies
Mechanics of Fluid
Statics
(Deals with forces & its effects while acting on bodies at rest)
Dynamics
(Deals with forces & its effects while acting on bodies in motion)
Kinetics
(Takes into account the effect of forces responsible for motion)
Kinematics
(Doesn't take into account the effect of forces responsible for motion)

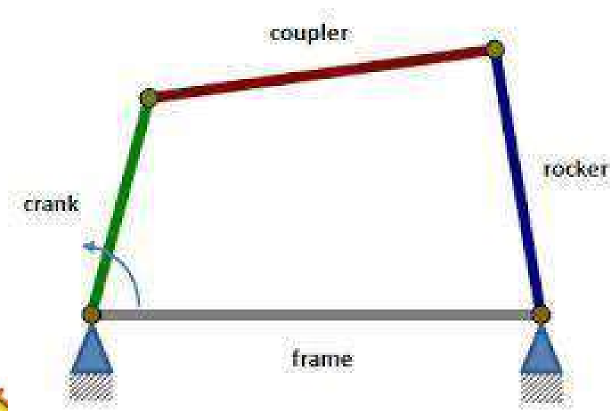
BASICS

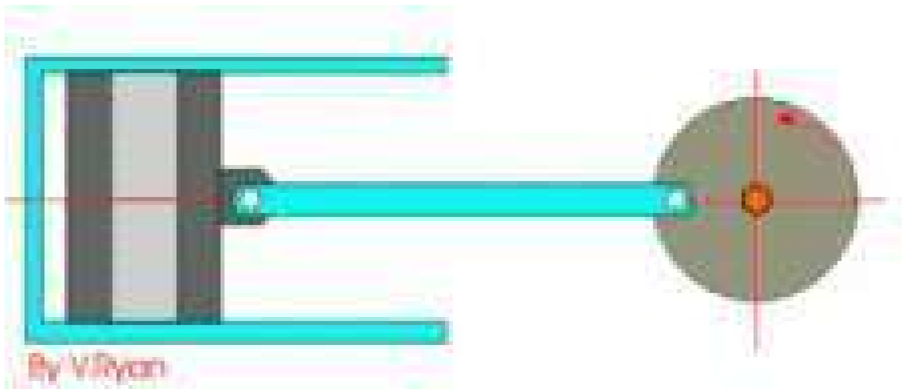


Windshield wiper

Mechanism:

- A number of bodies are assembled in such a way that the motion of one causes constrained and predictable motion to the others.
- A mechanism transmits and modifies a motion.
- Example: 4 bar mechanism, Slider crank mechanism





BASICS

Machine: (Combinations of Mechanisms)

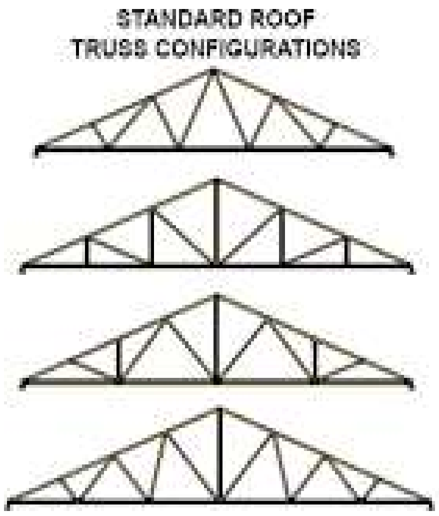
Transforms energy available in one form to another to do certain type of desired useful work.

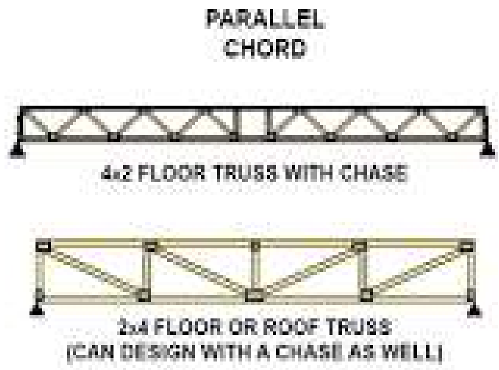
Lathe Machine

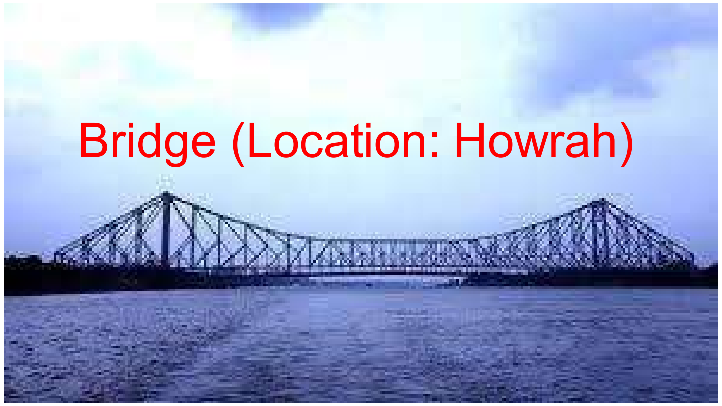
BASICS

Structure:

- Assembly of a number of resistant bodies meant to take up loads.
- No relative motion between the members







Truss

LECTURE 2

KINEMATIC LINK AND CLASSIFICATION OF LINKS

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BASICS

Kinematic Link (element): It is a Resistant body i.e. transmitting the required forces with negligible deformation.

Types of Links

1. Rigid Link

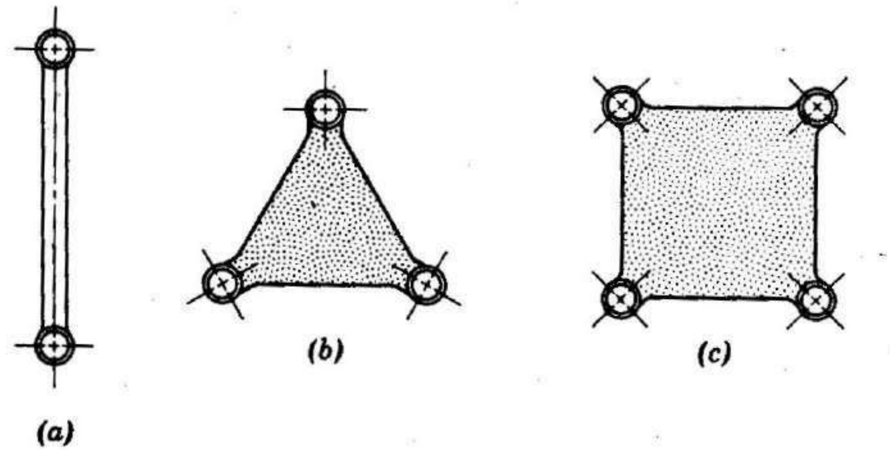
Doesn't undergo deformation. Example: Connecting rod, crank

2. Flexible Link

Partially deformed link. Example: belts, Ropes, chains

3. Fluid Link

Formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only. Example: Jacks, Brakes



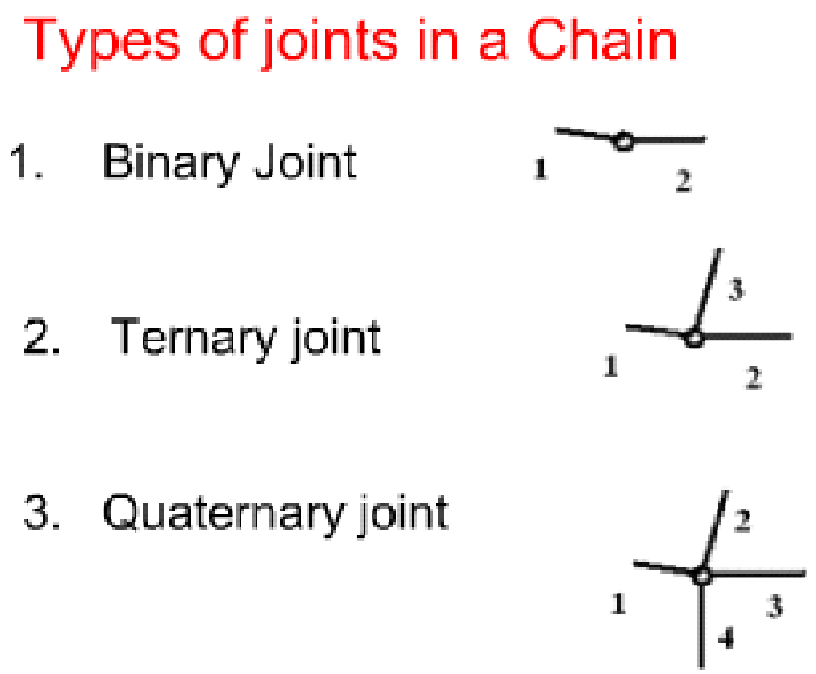
Binary link (2 vertices) **Ternary link (3 vertices)** **Quaternary link (4 vertices)**

BASICS

Kinematic Joint: Connection between two links by a pin

Types of Joints:

- Binary Joint (2 links are connected at the joint)
- Ternary Joint (3 links are connected)
- Quaternary Joint. (4 links are connected)

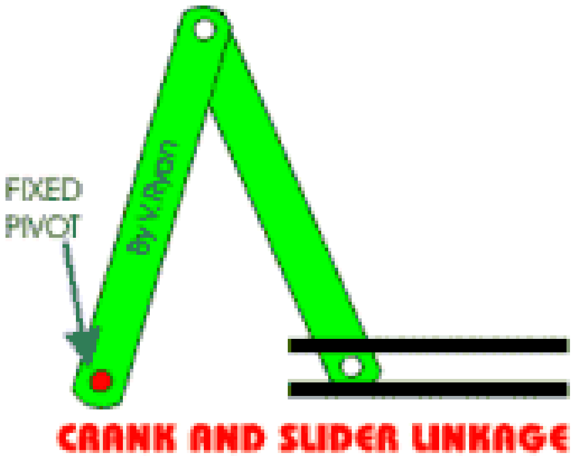


Note: if 'l' number of links are connected at a joint, it is equivalent to (l-1) binary joints.

BASICS

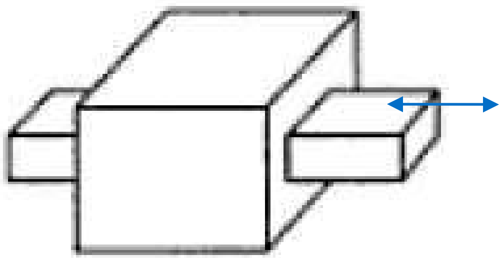
Kinematic Pair:

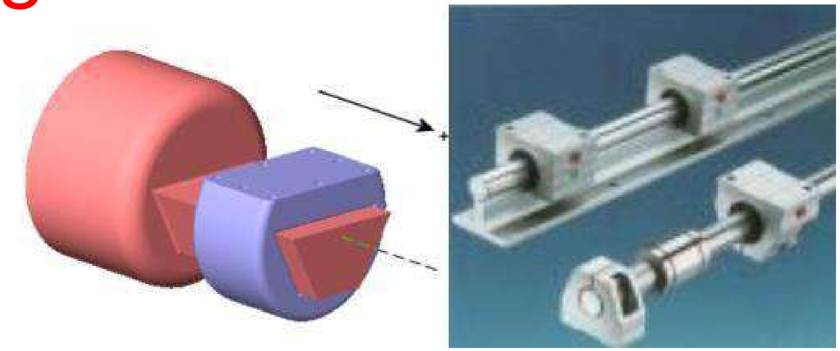
- The two links (or elements) of a machine, when in contact with each other, are said to form a pair.
- If the relative motion between them is completely or successfully constrained (i.e. in a definite direction), the pair is known as kinematic pair



KINEMATIC PAIRS ACCORDING TO THE RELATIVE MOTION

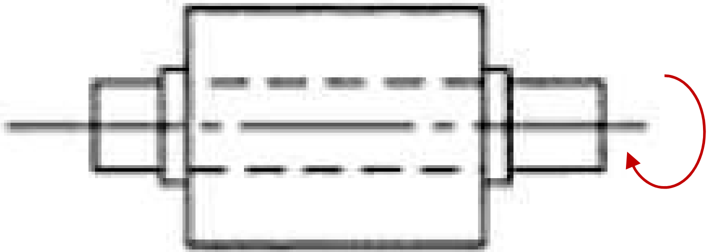
1. Sliding Pair





Rectangular bar in a rectangular hole

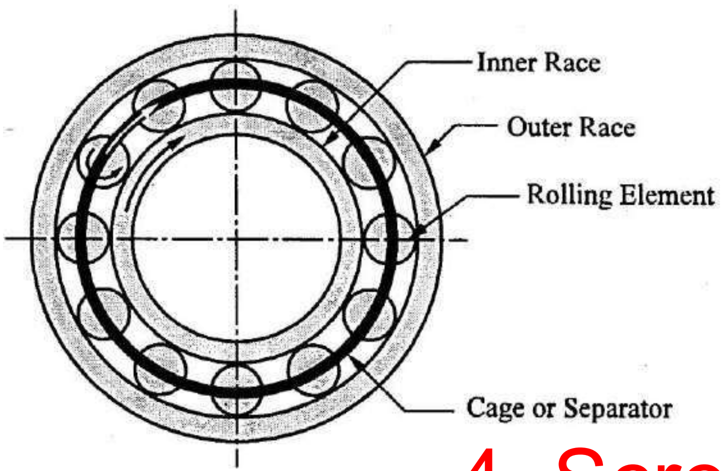
2. Turning or Revolving Pair



Collared shaft revolving in a circular hole

KINEMATIC PAIRS ACCORDING TO THE RELATIVE MOTION

3. Rolling Pair



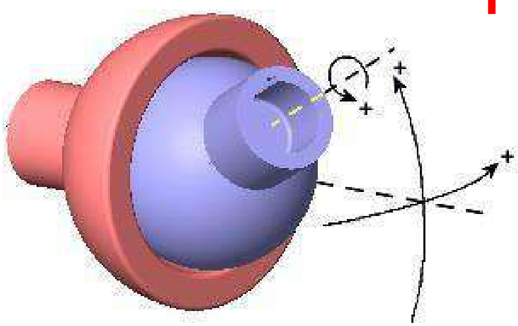
Links of pairs have a rolling motion relative to each other.

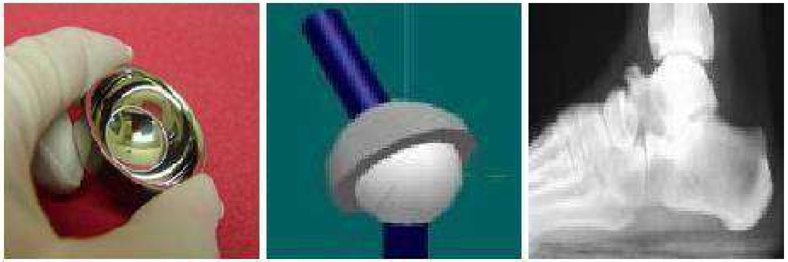
4. Screw or Helical Pair

if two mating links have a turning as well as sliding motion between them.

KINEMATIC PAIRS ACCORDING TO THE RELATIVE MOTION

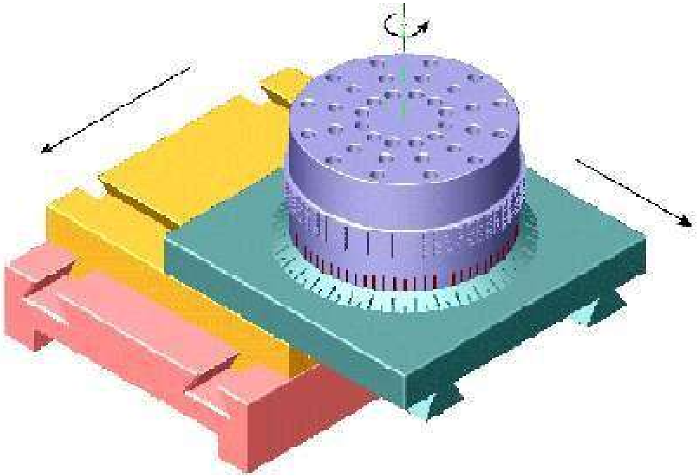
5. Spherical Pair





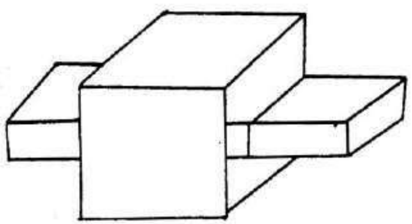
When one link in the form of a sphere turns inside a fixed link

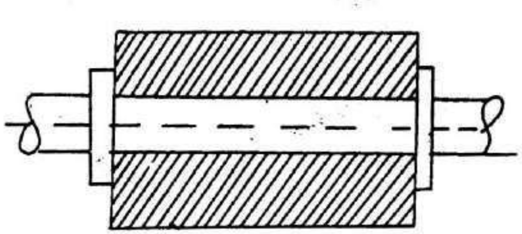
6. Planar Pair



KINEMATIC PAIRS ACCORDING TO TYPE OF CONTACT

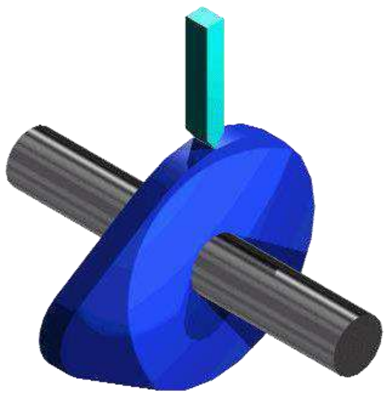
1. Lower Pair





The joint by which two members are connected has surface (Area) contact

2. Higher Pair



The contact between the pairing elements takes place at a point or along a line.

Toothed gearing, belt and rope drives, ball and roller bearings and cam and follower are the examples of higher pairs

LECTURE 3

CONSTRAINED MOTION AND CLASSIFICATION

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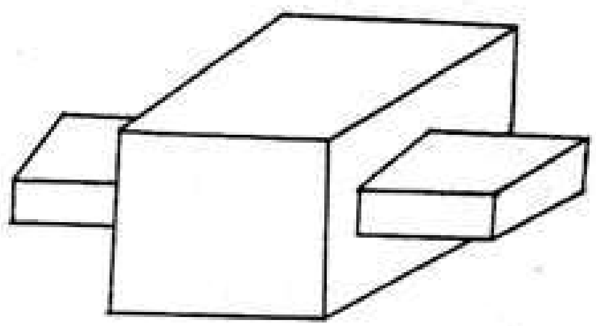
KINEMATIC PAIRS ACCORDING TO TYPE OF CONSTRAINT

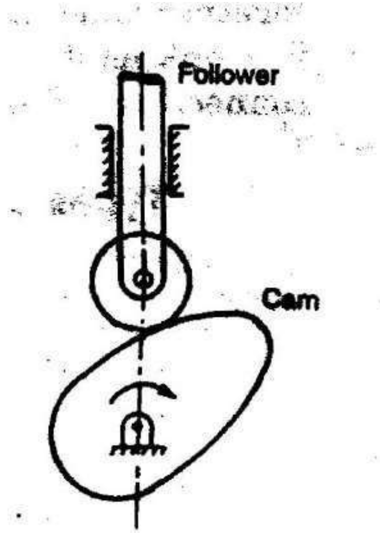
1. Closed Pair



Two elements of pair are held together mechanically to get required relative motion.

Eg. All lower pairs

2. Unclosed Pair

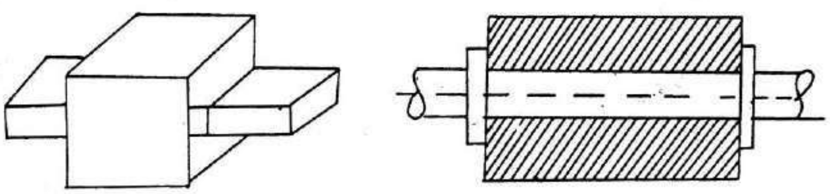


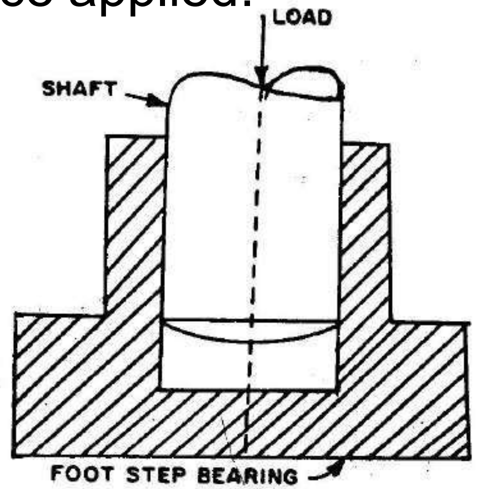
- Elements are not held mechanically.
- Held in contact by the action of external forces.

Eg. Cam and spring loaded follower pair

CONSTRAINED MOTION

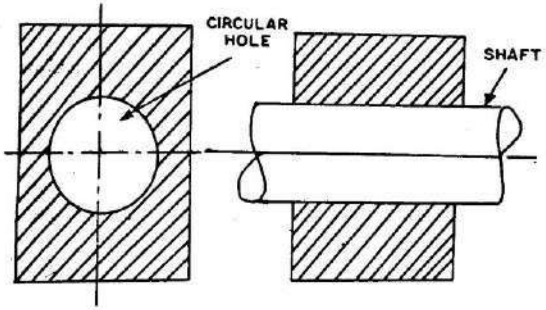
1. Completely constrained Motion:
Motion in definite direction irrespective of the direction of the force applied.





2. Successfully (partially) constrained Motion:

- Constrained motion is not completed by itself but by some other means.
- Constrained motion is successful when compressive load is applied on the shaft of the foot step bearing



3. Incompletely constrained motion:
Motion between a pair can take place in more than one direction.

Circular shaft in a circular hole may have rotary and reciprocating motion. Both are independent of each other.

KINEMATIC CHAIN

Group of links either joined together or arranged in a manner that permits them to move relative (i.e. completely or successfully constrained motion) to one another.

Example: 4 bar chain

The following relationship holds for kinematic chain

$$l = 2p - 4$$

$$j = \frac{3}{2} l - 2$$

Where

p = number of lower pairs

l = number of links

j = Number of binary joints

KINEMATIC CHAIN

$$l = 2p - 4$$

$$j = \frac{3}{2}l - 2$$

If LHS > RHS, Locked chain or redundant chain; no relative motion possible.

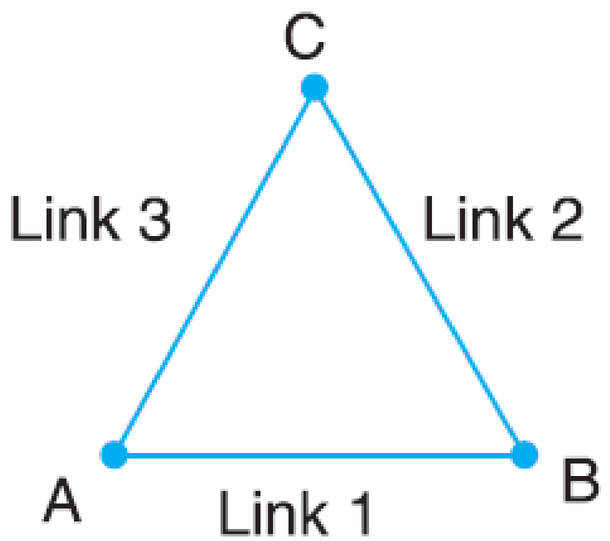
LHS = RHS, Constrained chain .i.e. motion is completely constrained

LHS < RHS, unconstrained chain. *i.e. the relative motion is not completely constrained.*

NUMERICAL EXAMPLE-1

Determine the nature of the chain

(K2:U)



$l = 3 \quad p = 3 \quad j = 3$

From equation

$$l = 2p - 4$$
$$= 2 \times 3 - 4 = 2$$

L.H.S. > R.H.S.

$$j = \frac{3}{2} l - 2$$
$$= \frac{3}{2} \times 3 - 2 = 2.5$$

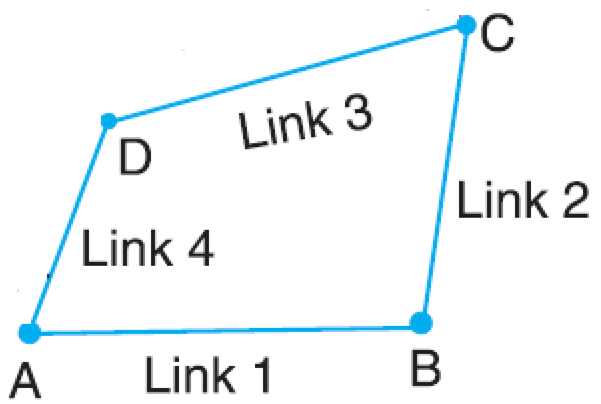
L.H.S. > R.H.S.

Therefore it is a locked Chain

EXERCISE

Determine the nature of the chains given below (K2:U)

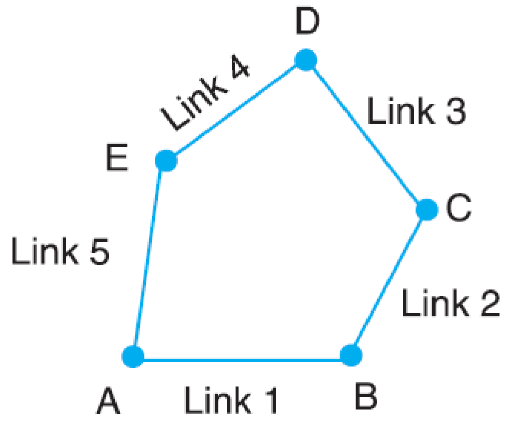
Hint: Check equations $l = 2p - 4$, $j = \frac{3}{2}l - 2$



$l = 4, p = 4$, and $j = 4$

L.H.S. = R.H.S.

constrained kinematic chain



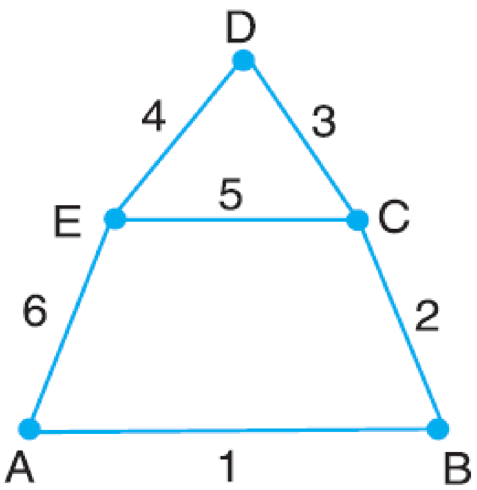
$l = 5, p = 5$, and $j = 5$

L.H.S. < R.H.S.

unconstrained chain

NUMERICAL EXAMPLE-2

Determine the nature of the chain (K2:U)



- l = 6
- j = 3 Binary joints (A, B & D) + 2 ternary joints (E & C)
- We know that, 1 ternary joint = (3-1) = 2 Binary Joints
- Therefore, j = 3 + (2*2) = 7

$$j = \frac{3}{2} l - 2$$

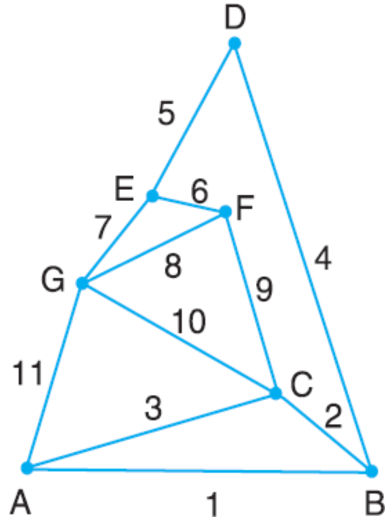
$$= \frac{3}{2} \times 6 - 2 = 7$$

L.H.S. = R.H.S.

Therefore, the given chain is a kinematic chain or constrained chain.

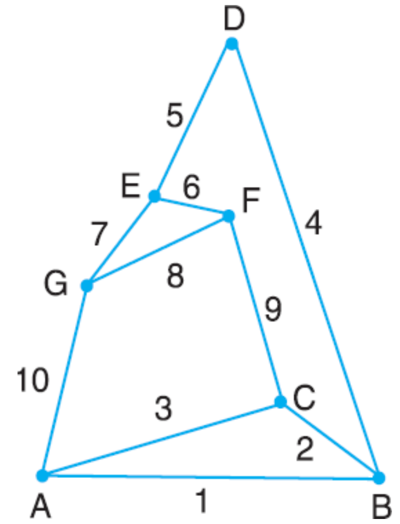
EXERCISE

Determine the number of joints (equivalent binary) in the given chains (K2:U)



Number of Binary Joints = 1 (D)
No. of ternary joints = 4 (A, B, E, F)
No. of quaternary joints = 2 (C & G)

Therefore, $j = 1 + 4\,(2) + 2\,(3) = 15$



No. of Binary Joints = 1 (D)
No. of ternary joints = 6 (A, B, C, E,F, G)

$j = 1 + 6\,(2) = 13$

KINEMATIC CHAIN

- For a kinematic chain having higher pairs, each higher pair is taken equivalent to two lower pairs and an additional link.
- In this case to determine the nature of chain, the relation given by A.W. Klein, may be used

$$j + \frac{h}{2} = \frac{3}{2} l - 2$$

where j = Number of binary joints,

h = Number of higher pairs, and

l = Number of links.

LECTURE 4

MECHANISM AND MACHINES

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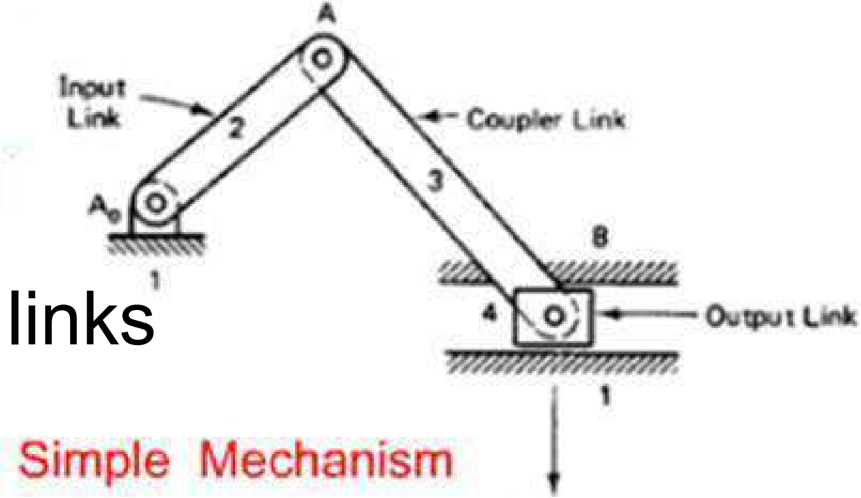
CLASSIFICATION OF MECHANISMS

Mechanism:

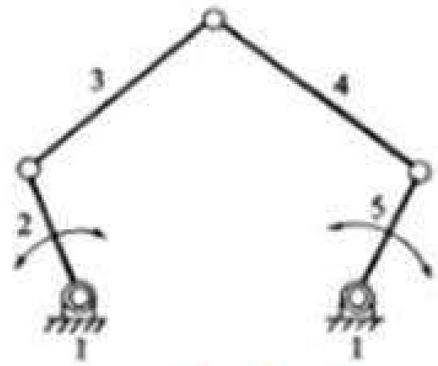
When one of the links of a kinematic chain is fixed, the chain is called Mechanism.

Types:

- Simple - 4 Links
- Compound - More than 4 links



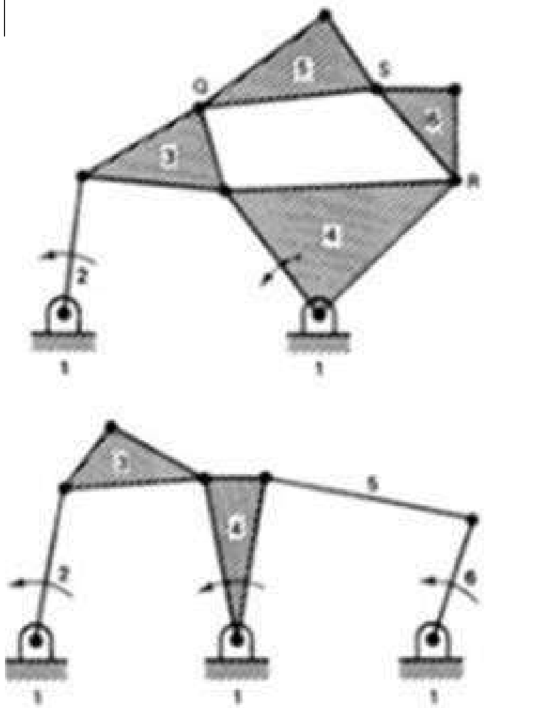
Simple Mechanism



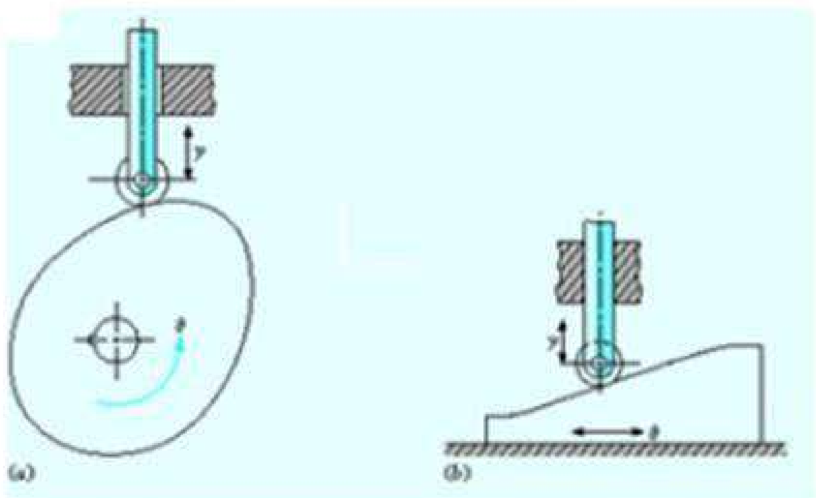
Compound Mechanism

Classification of mechanisms

- Complex - Ternary or Higher order fl Links
- Planar - All links lie in the same plane



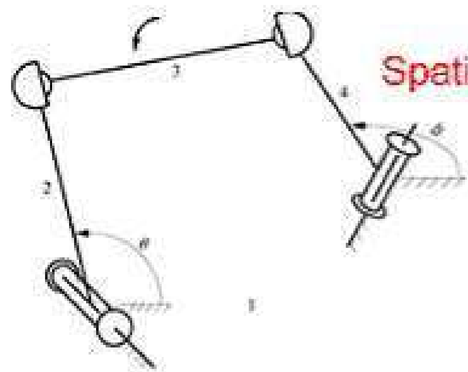
Complex Mechanism



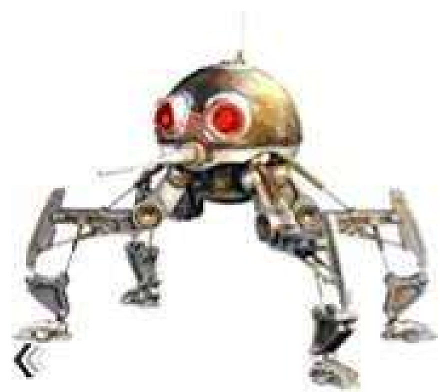
Planar Mechanism

Classification of mechanisms

- Spatial - Links of a mechanism lie in different planes



Spatial Mechanism



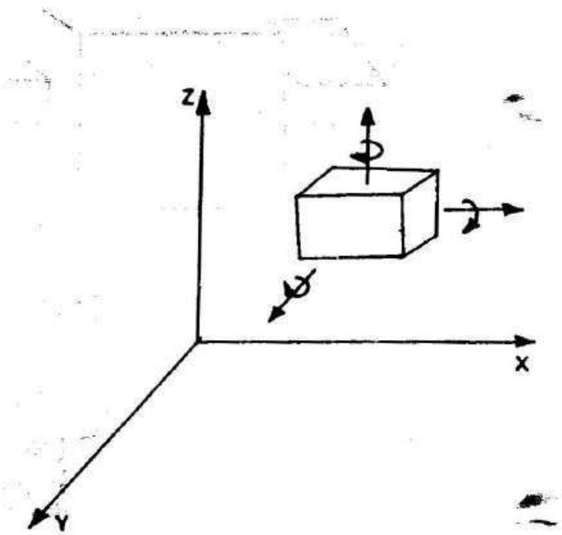
Parallel robot

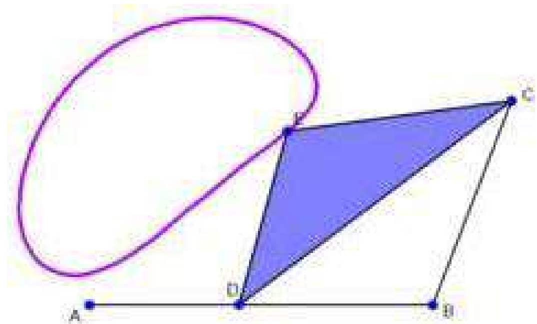
Machine

When a mechanism is required to transmit power or to do some particular type of work, it then becomes a **machine**. In such cases, the various links or elements have to be designed to withstand the forces (both static and kinetic) safely.

DEGREES OF FREEDOM (DOF) / MOBILITY

It is the number of **independent coordinates** required to describe the **position of a body**.

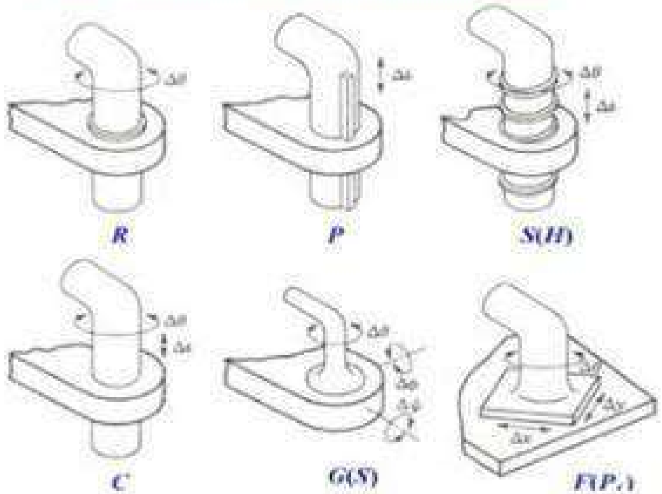




4 bar Mechanism has 1 DoF as the angle turned by the crank AD is fully describing the position of the every link of the mechanism

DOF

The Lower Pairs Joints



Pair	Symbol	Pair Variable	Degree of Freedom	Relative Motion
Revolute	R	$\Delta\theta$	1	Circular
Prism	P	Δs	1	Rectilinear
Screw	$S(H)$	$\Delta\theta$ or Δs ($\Delta s = h\Delta\theta$)	1	Helical
Cylinder	C	$\Delta\theta$ and Δs	2	Cylindric
Sphere	$G(S)$	$\Delta\theta, \Delta\phi, \Delta\psi$	3	Spheric
Flat	$F(P_l)$	$\Delta x, \Delta y, \Delta\theta$	3	Planar

DEGREES OF FREEDOM/MOBILITY OF A MECHANISM

It is the number of inputs (number of independent coordinates) required to describe the configuration or position of all the links of the mechanism, with respect to the fixed link at any given instant.

KUTZBACH CRITERION

For mechanism having plane motion

$$\text{DoF} = n = 3(l-1) - 2j - h$$

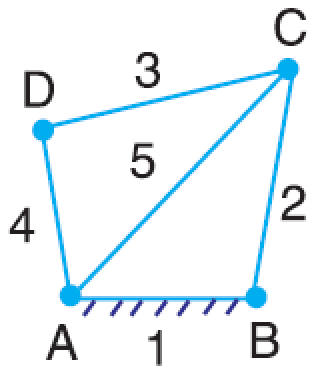
l = number of links

j = number of binary joints or lower pairs (1 DoF pairs)

h = number of higher pairs (i.e. *2 DoF pairs)*

NUMERICAL EXAMPLE -1 &2

Determine the DoF of the mechanism shown below:

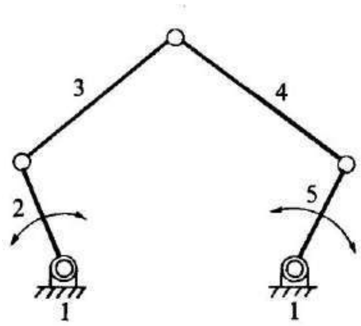


$$n = 3\,(l - 1) - 2\,j - h$$ **Kutzbach Criterion**

l = 5 ; j = 2+ 2* (3-1) = 6 ; h = 0

$$n = 3\,(5 - 1) - 2 \times 6 = 0$$

DoF = 0, means that the mechanism forms a structure



l = 5 ; j = 5; h = 0

n = 3 (5-1) – 2*5 – 0 = 2

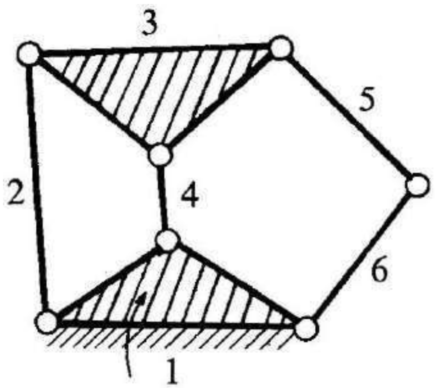
Two inputs to any two links are required to yield definite motions in all the links.

NUMERICAL EXAMPLE -3 &4

Determine the Dof for the links shown below:

$$n = 3\,(l - 1) - 2\,j - h$$

Kutzbach Criterion

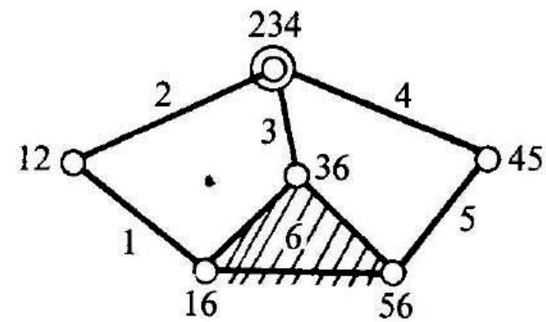


l = 6 ; j = 7 ; h = 0

n = 3 (6-1) – 2 (7) – 0 = 1

Dof = 1

i.e., one input to any one link will result in definite motion of all the links.



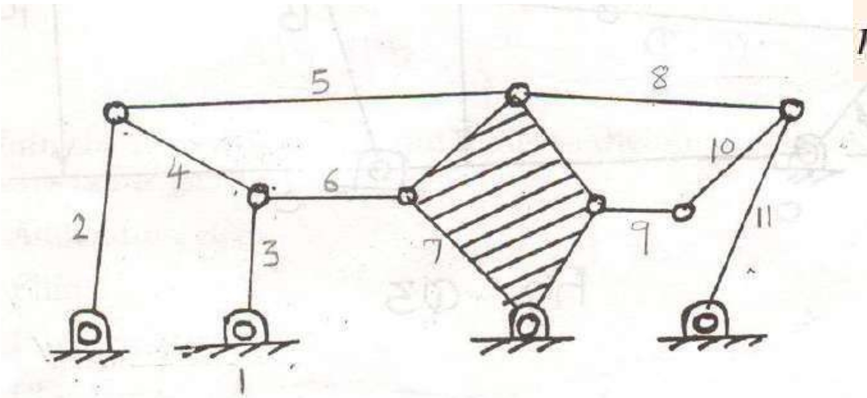
Note: at the intersection of 2, 3 and 4, two lower pairs are to be considered

l = 6 ; j = 5 + 1 (3-1) = 7 ; h = 0

n = 3 (6-1) – 2 (7) – 0 = 1

Dof = 1

NUMERICAL EXAMPLE - 5



$$n = 3\ (l - 1) - 2\ j - h$$

Kutzbach Criterion

l = 11 ; j = 7 + 4 (3-1) = 15 ; h = 0

n = 3 (11-1) – 2 (15) – 0 = 0

Dof = 0

Here, j= 15 (two lower pairs at the intersection of 3, 4, 6; 2, 4, 5; 5, 7, 8; 8, 10, 11) and h = 0.

Summary

Dof = 0, Structure

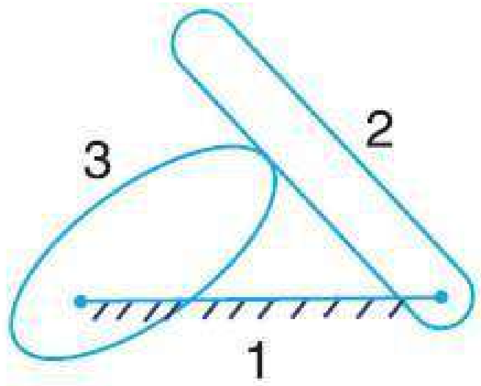
Dof = 1, mechanism can be driven by a single input motion

Dof = 2, two separate input motions are necessary to produce constrained motion for the mechanism

Dof = -1 or less, redundant constraints in the chain and it forms a statically indeterminate structure

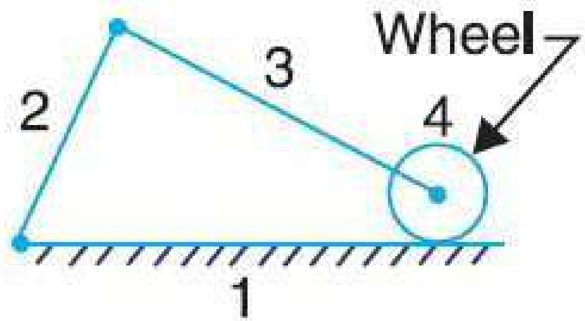
KUTZBACH CRITERION FOR HIGHER PAIRS

$$n = 3\,(l - 1) - 2\,j - h$$



$l = 3, j = 2$ and $h = 1$

$$n = 3\,(3 - 1) - 2 \times 2 - 1 = 1$$

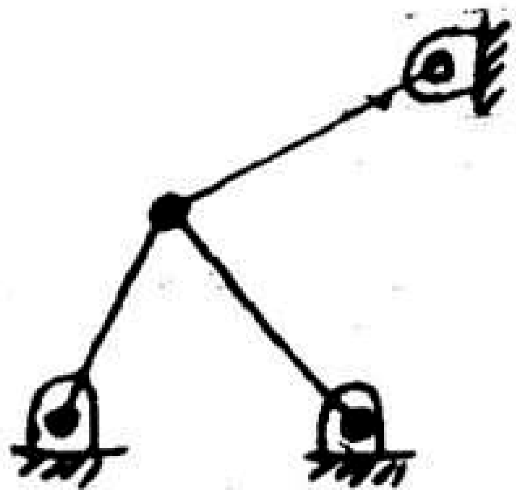


$l = 4,\ j = 3$ and $h = 1$

$$n = 3\,(4 - 1) - 2 \times 3 - 1 = 2$$

KUTZBACH CRITERION

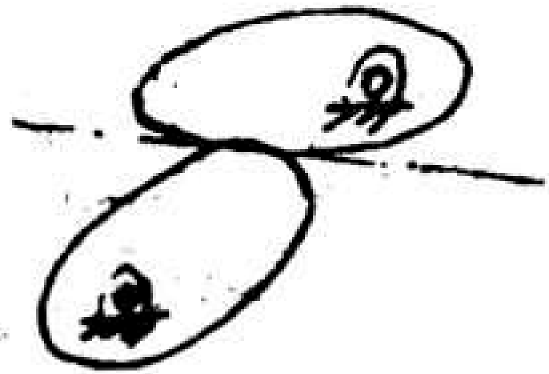
$$n = 3(l - 1) - 2j - h$$



l = 4, j = 5, h=0

n = 3 (4-1) – 2 (5) – 0 = -1

Indeterminate structure



l = 3, j = 2, h=1

n = 3 (3-1) – 2 (2) – 1 = 1

GRUBLER'S CRITERION FOR PLANE MECHANISMS

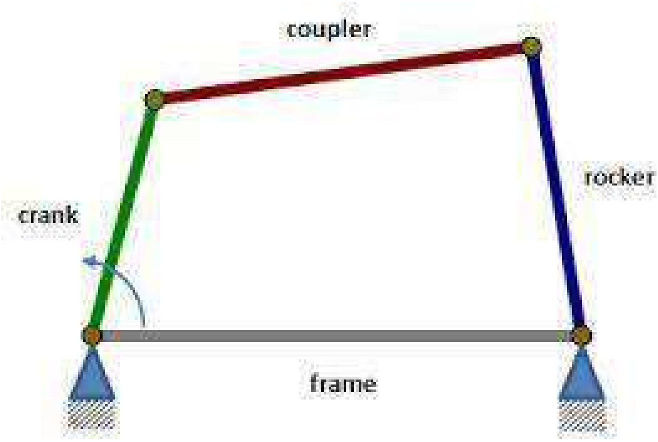
Kutzbach Criterion $n = 3\,(l - 1) - 2\,j - h$

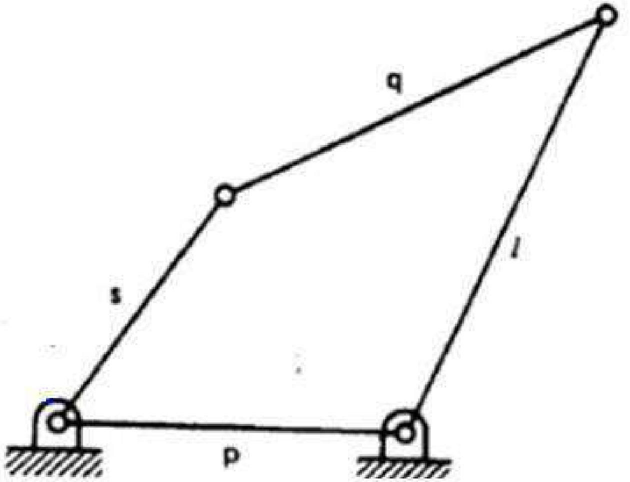
Grubler's criterion applies to mechanisms having 1 DoF.

Substituting n = 1 and h=0 in Kutzbatch equation, we can have Grubler's equation.

$$1 = 3\,(l - 1) - 2\,j \qquad \text{or} \qquad 3l - 2j - 4 = 0$$

GRASHOF'S LAW

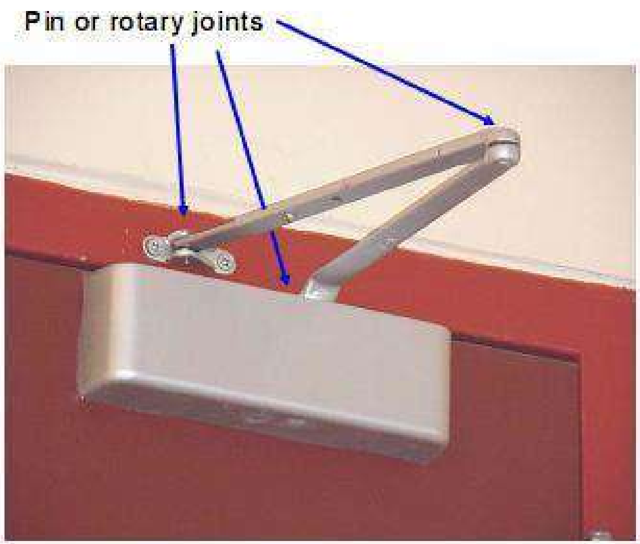


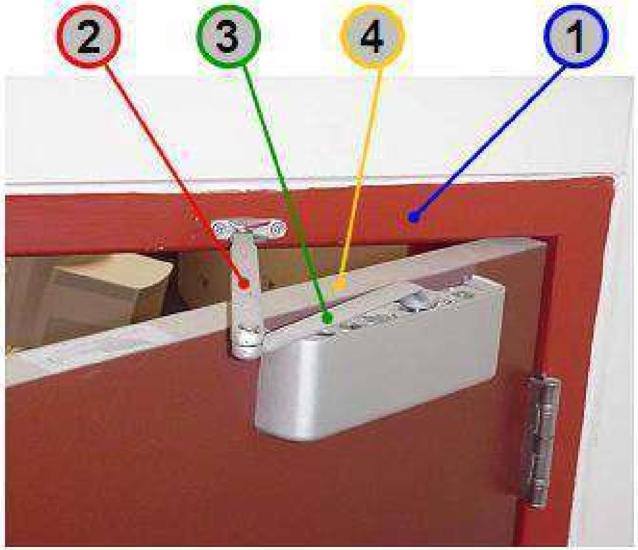


$$l + s \leq p + q$$

According to **Grashof 's law for a four bar mechanism,** the sum of the shortest and longest link lengths <u>should not be greater than</u> the sum of the remaining two link lengths if there is to be continuous relative motion between the two links.

Example: 4 bar door damper linkage





1	= Wall	or	Link 1	This is the grounded (held still)
2	= Bar 2	or	Link 2	
3	= Bar 3	or	Link 3	
4	= Door	or	Link 4	

LECTURE 5

INVERSION OF MECHANISM

LIVE TO LEARN & LEARN TO SHARE

MRCET CAMPUS

UGC Autonomous

DEPARTMENT OF MECHANICAL ENGINEERING

INVERSIONS OF MECHANISM

- A mechanism is one in which one of the links of a kinematic chain is fixed.

- Different mechanisms can be obtained by fixing different links of the same kinematic chain.

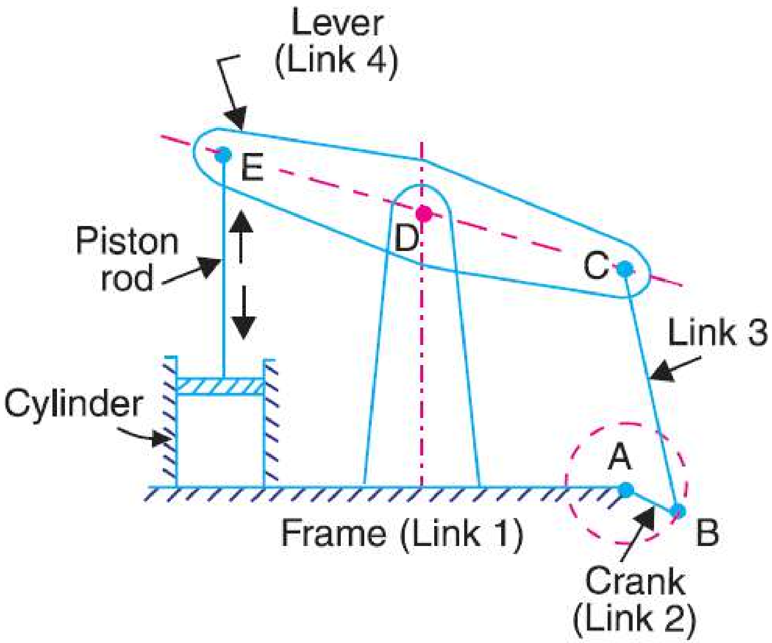
- It is known as inversions of the mechanism.

INVERSIONS OF FOUR BAR CHAIN

- Beam engine (crank and lever mechanism)
- Coupling rod of a locomotive (Double crank mechanism)
- Watt's indicator mechanism (Double lever mechanism)

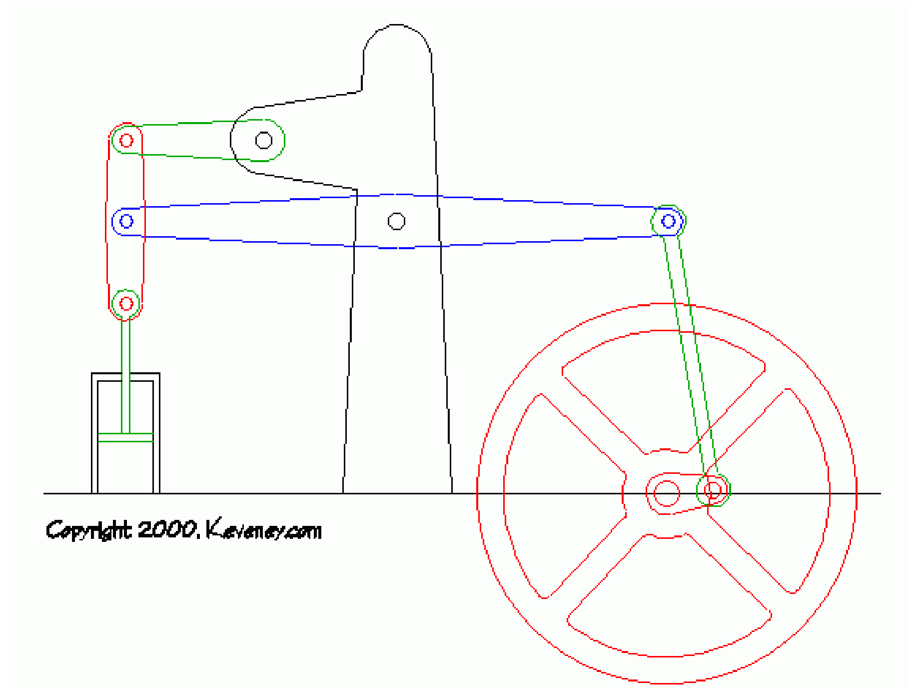
INVERSIONS OF FOUR BAR CHAIN

1. Beam engine (crank and lever mechanism)



Beam engine.

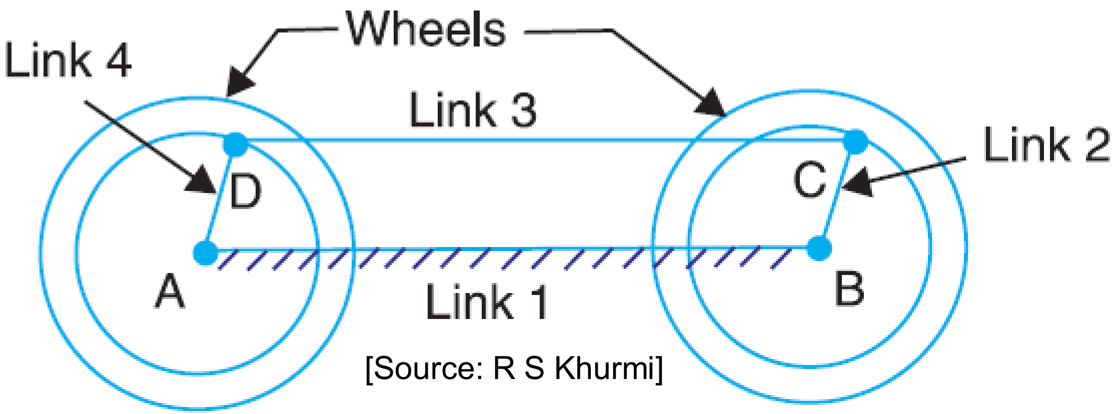
[Source: R S Khurmi]



The purpose of this mechanism is to convert rotary motion into reciprocating motion.

INVERSIONS OF FOUR BAR CHAIN

2. Coupling rod of a locomotive (Double crank mechanism).

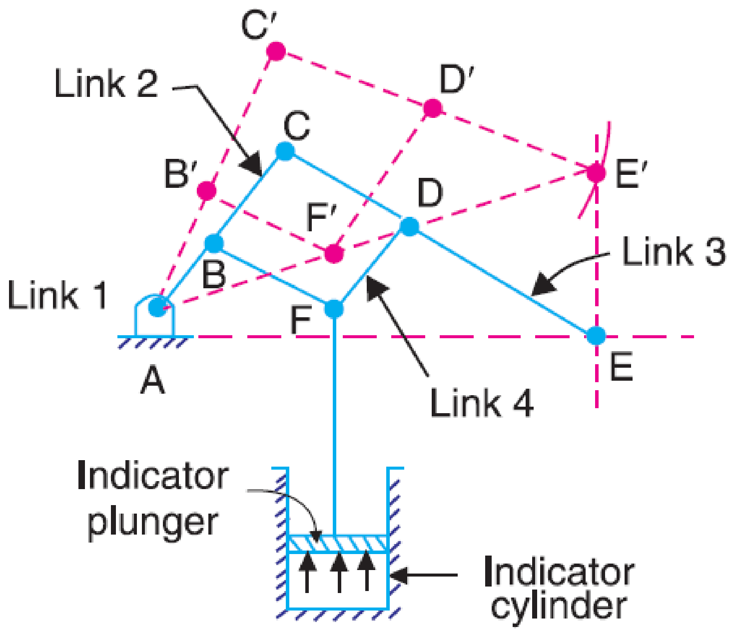


[Source: R S Khurmi]

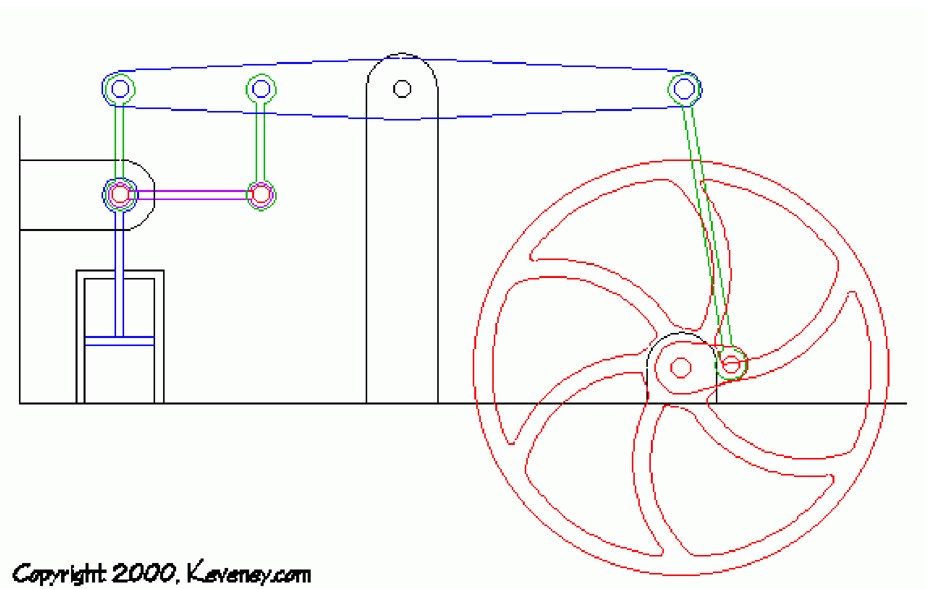
- links *AD and BC (having equal length) act as cranks and are connected* to the respective wheels.
- The link CD acts as a coupling rod and the link AB is fixed in order to maintain a constant centre to centre distance between them.
- This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

INVERSIONS OF FOUR BAR CHAIN

3. Watt's indicator mechanism (Double lever mechanism)



Watt's indicator mechanism.
[Source: R S Khurmi]



On any small displacement of the mechanism, the tracing point *E at the end of the link CE* traces out approximately a straight line

LECTURE 7

INVERSION OF SINGLE SLIDER CRANK CHAINS

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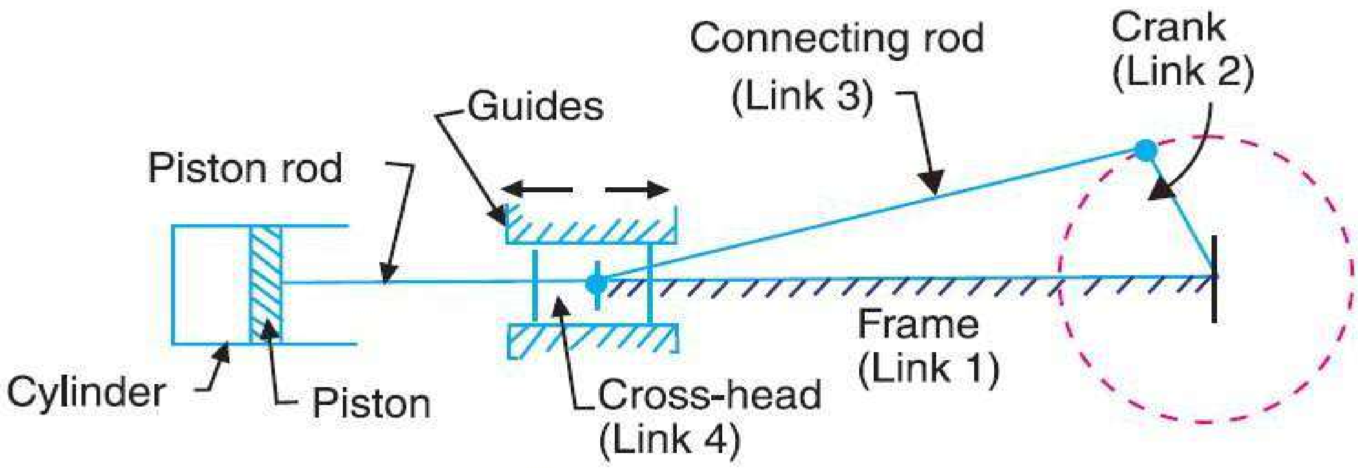
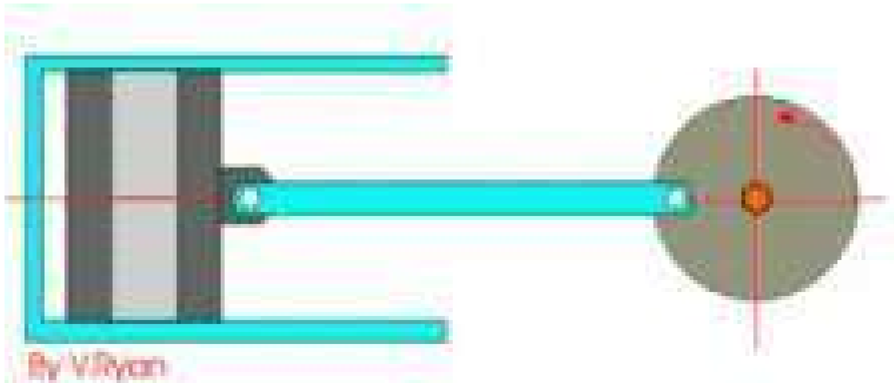
SINGLE SLIDER CRANK CHAIN

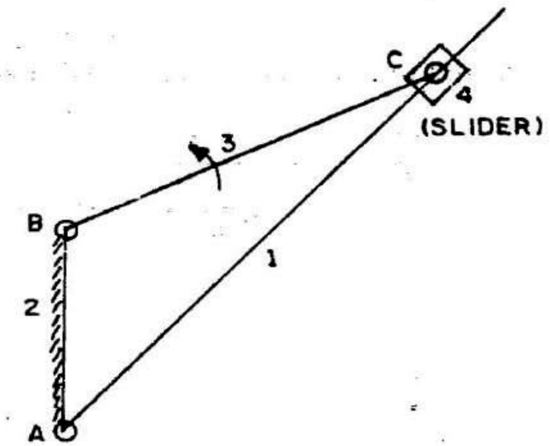


Single slider crank chain

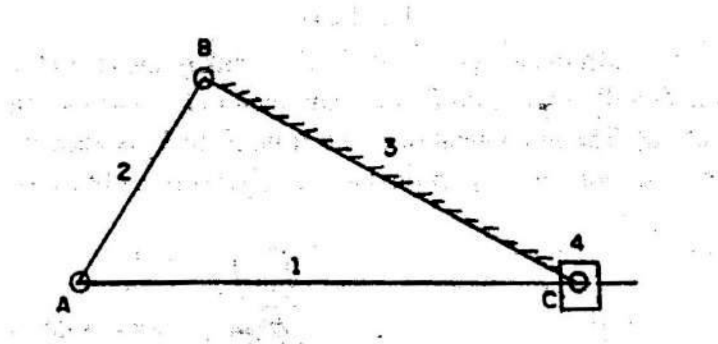
Links 1-2, 2-3, 3-4 = Turning pairs; Link 4-1 = Sliding pair



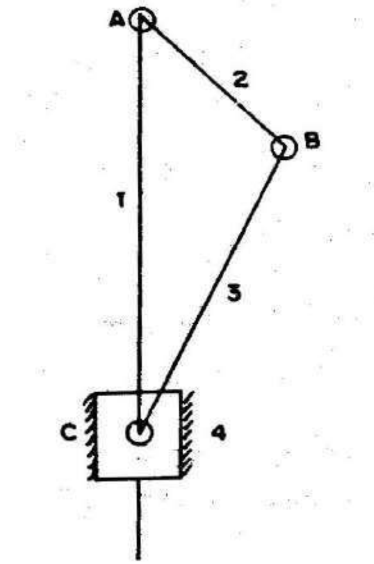
INVERSIONS OF SINGLE SLIDER CRANK CHAIN



crank fixed



connecting rod fixed

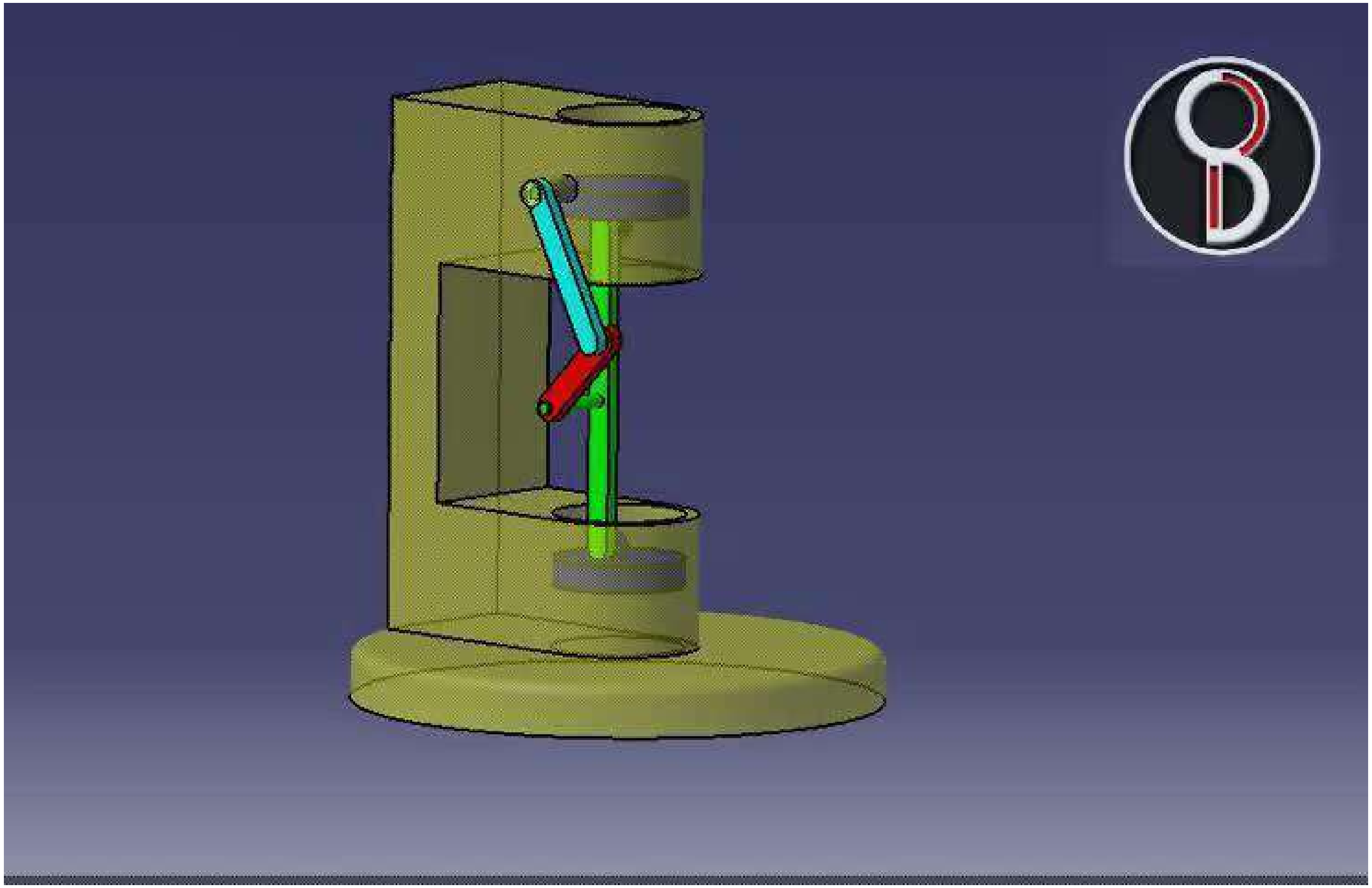


slider fixed

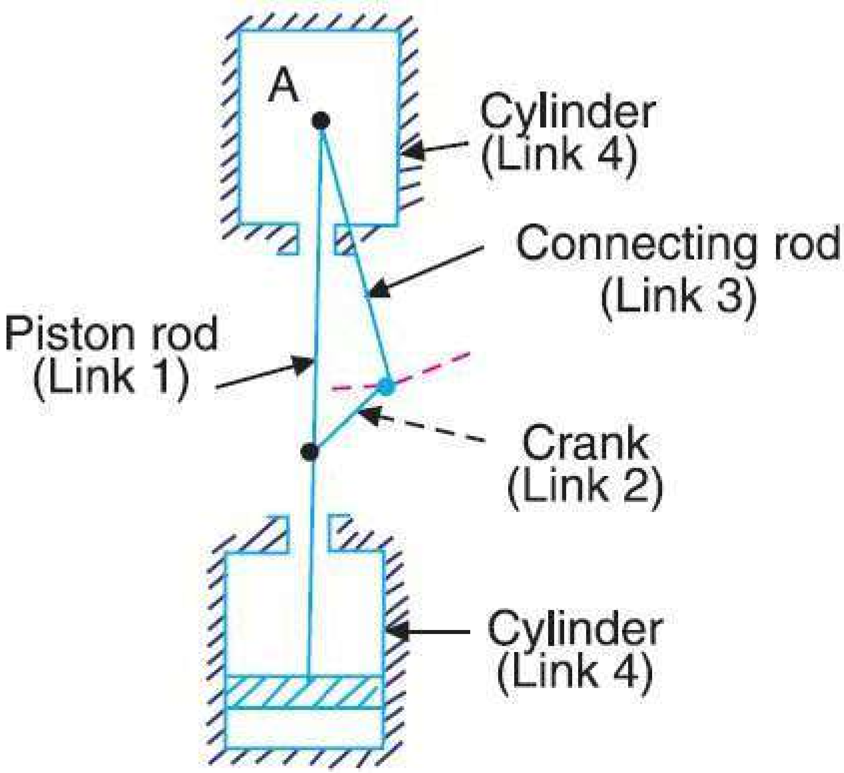
INVERSIONS OF SINGLE SLIDER CRANK CHAIN

- Pendulum pump or Bull engine
- Oscillating cylinder engine
- Rotary internal combustion engine (or) Gnome engine
- Crank and slotted lever quick return motion mechanism
- Whitworth quick return motion mechanism

PENDULUM PUMP OR BULL ENGINE



PENDULUM PUMP OR BULL ENGINE



[Source: R S Khurmi]

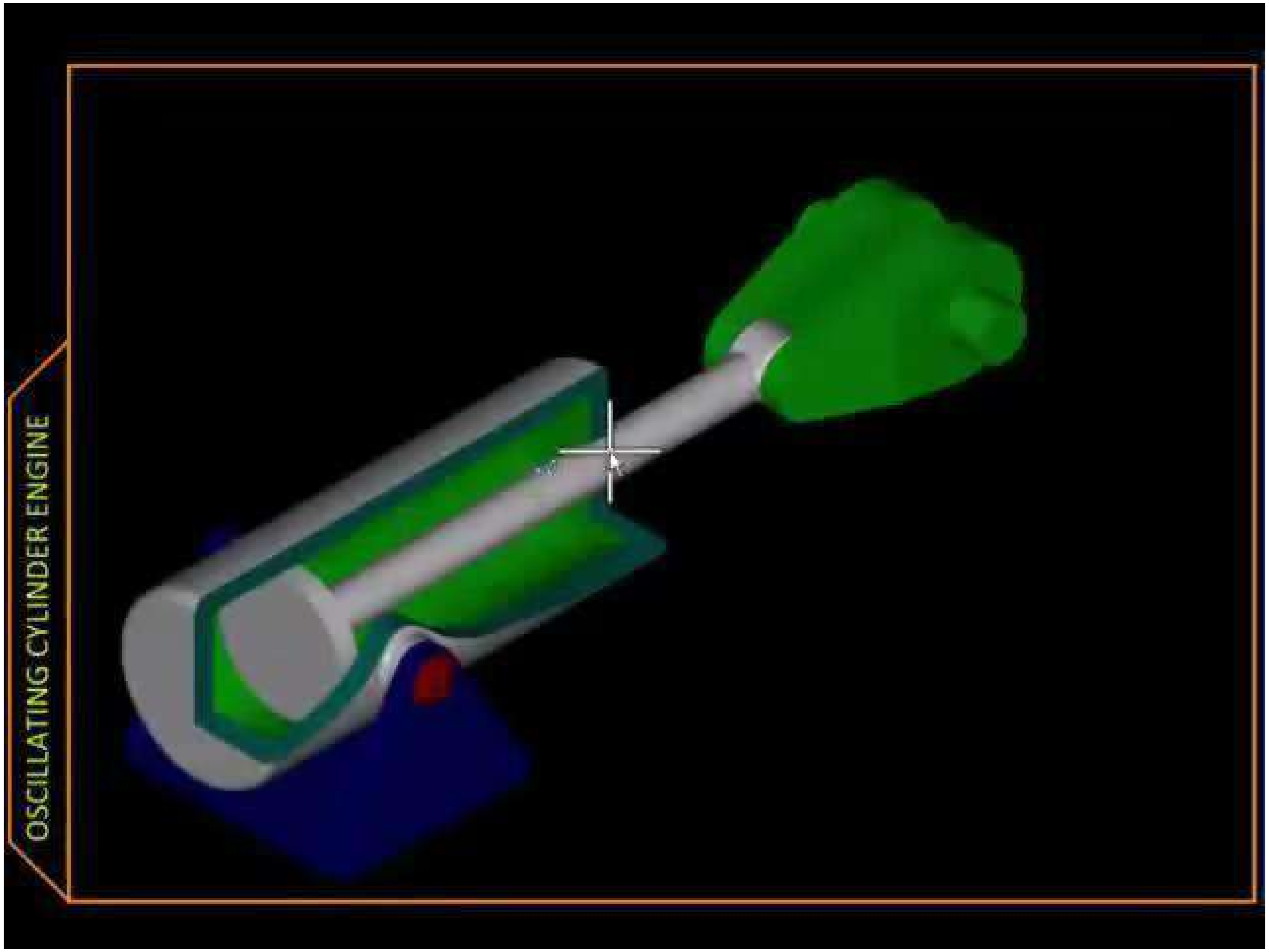
This inversion is obtained by fixing the cylinder or link 4 (i.e. sliding pair)

when the crank (link 2) rocks, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at A.

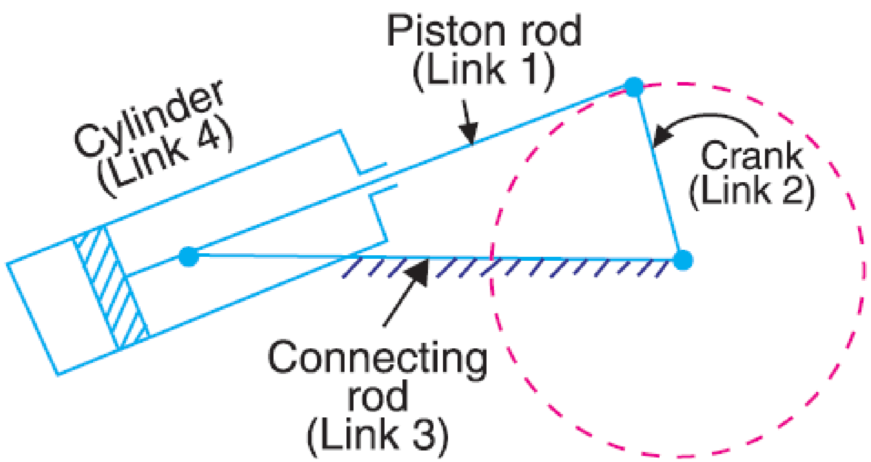
The piston attached to the piston rod (link 1) reciprocates.

It supplies water to a boiler.

OSCILLATING CYLINDER ENGINE



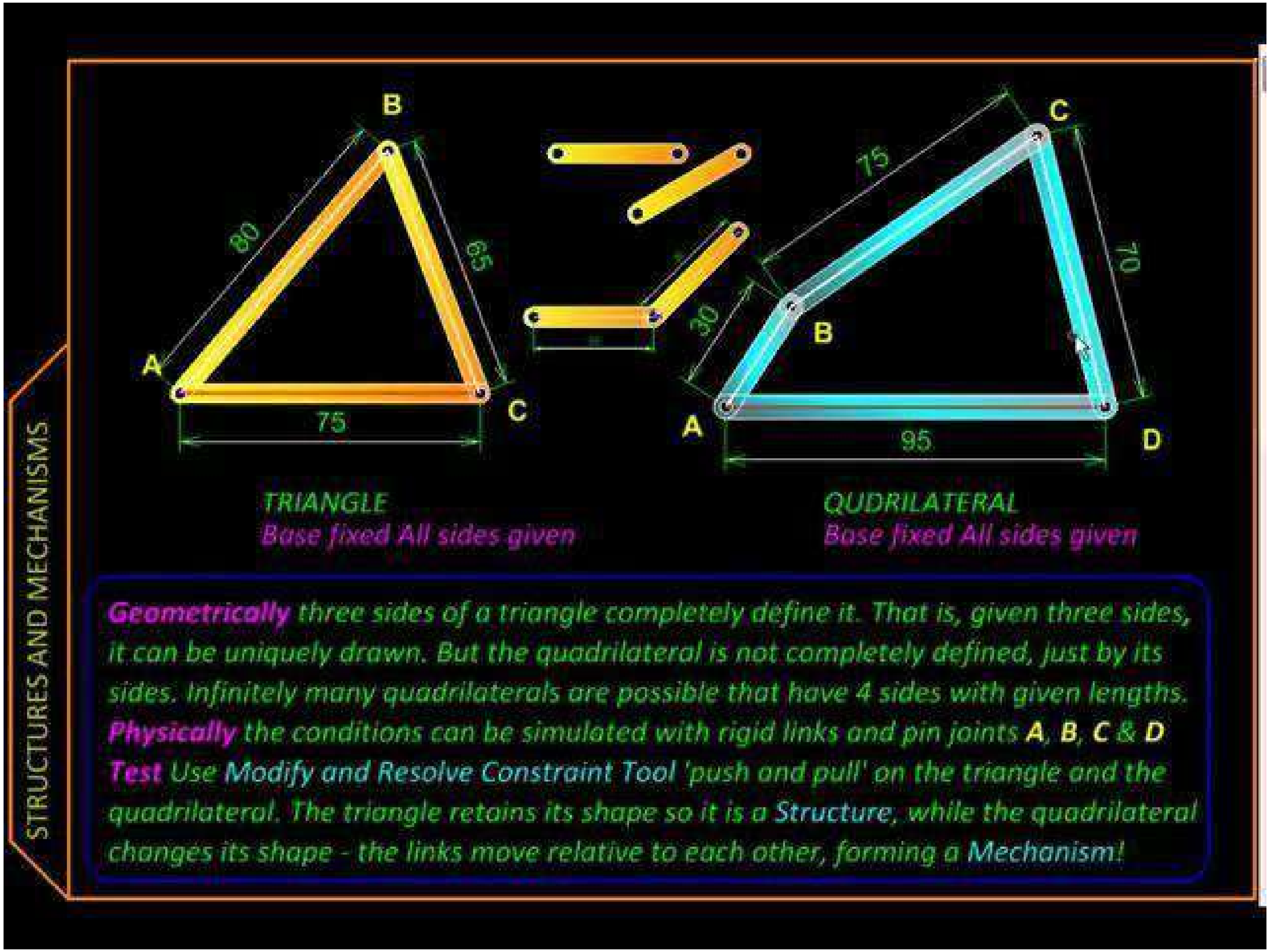
OSCILLATING CYLINDER ENGINE



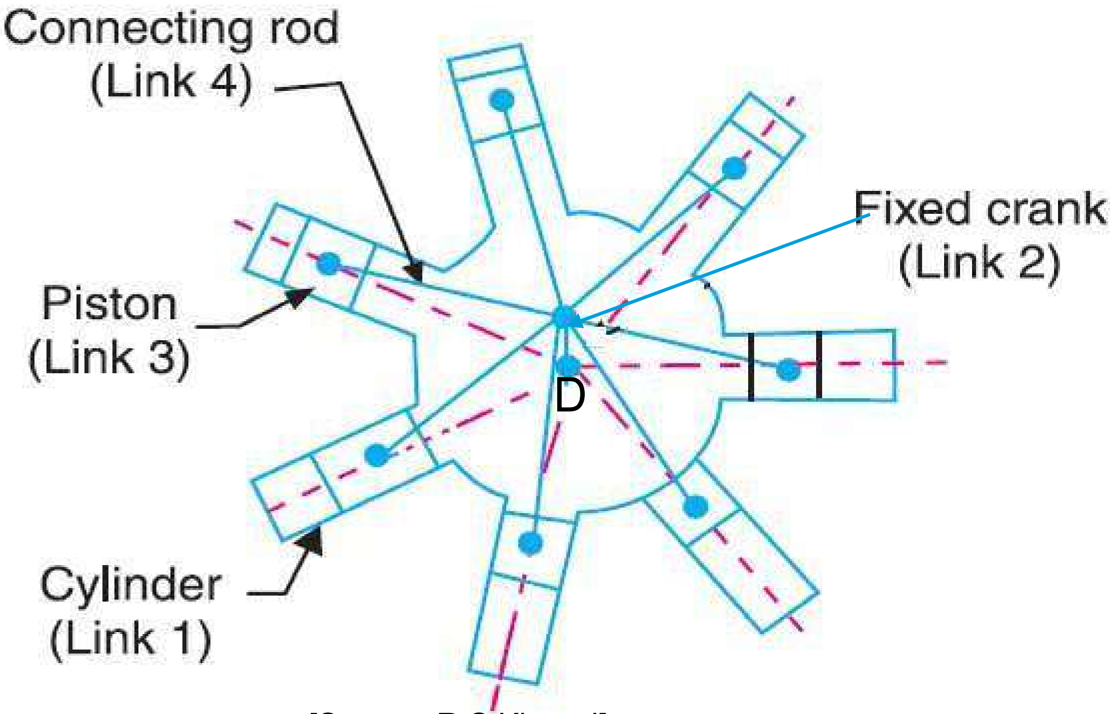
[Source: R S Khurmi]

- used to convert reciprocating motion into rotary motion
- the link 3 (Connecting Rod) forming the turning pair is fixed.

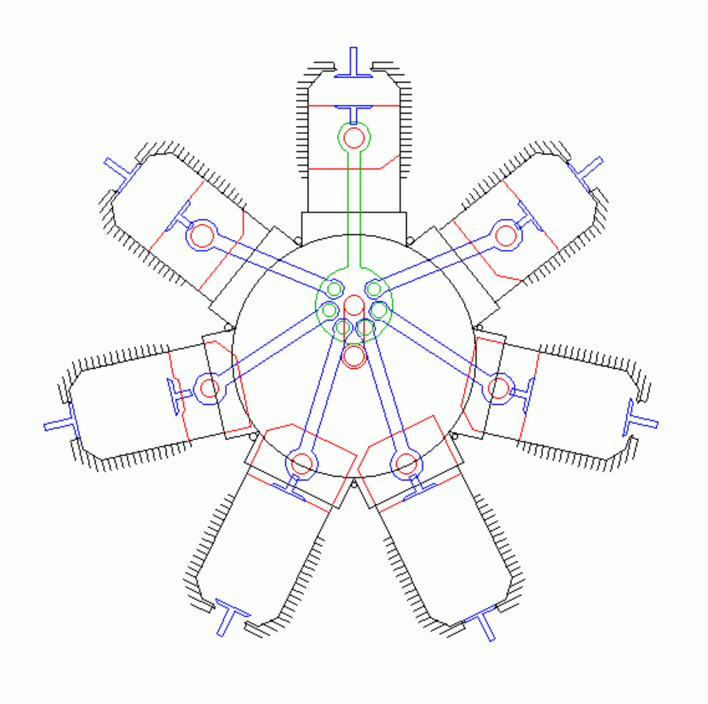
MULTI-CYLINDER RADIAL IC ENGINE



ROTARY INTERNAL COMBUSTION ENGINE (OR) GNOME ENGINE

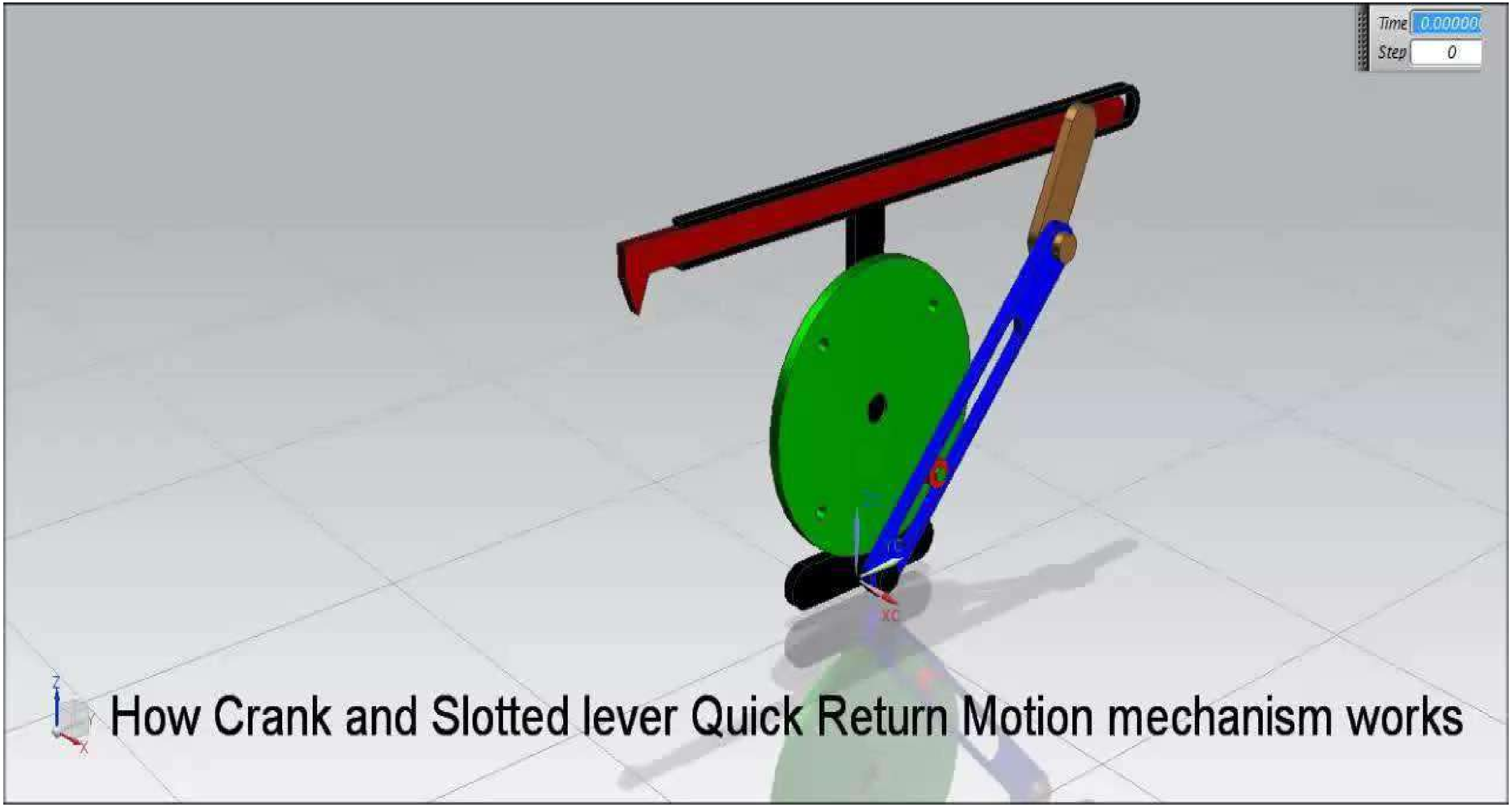


[Source: R S Khurmi]

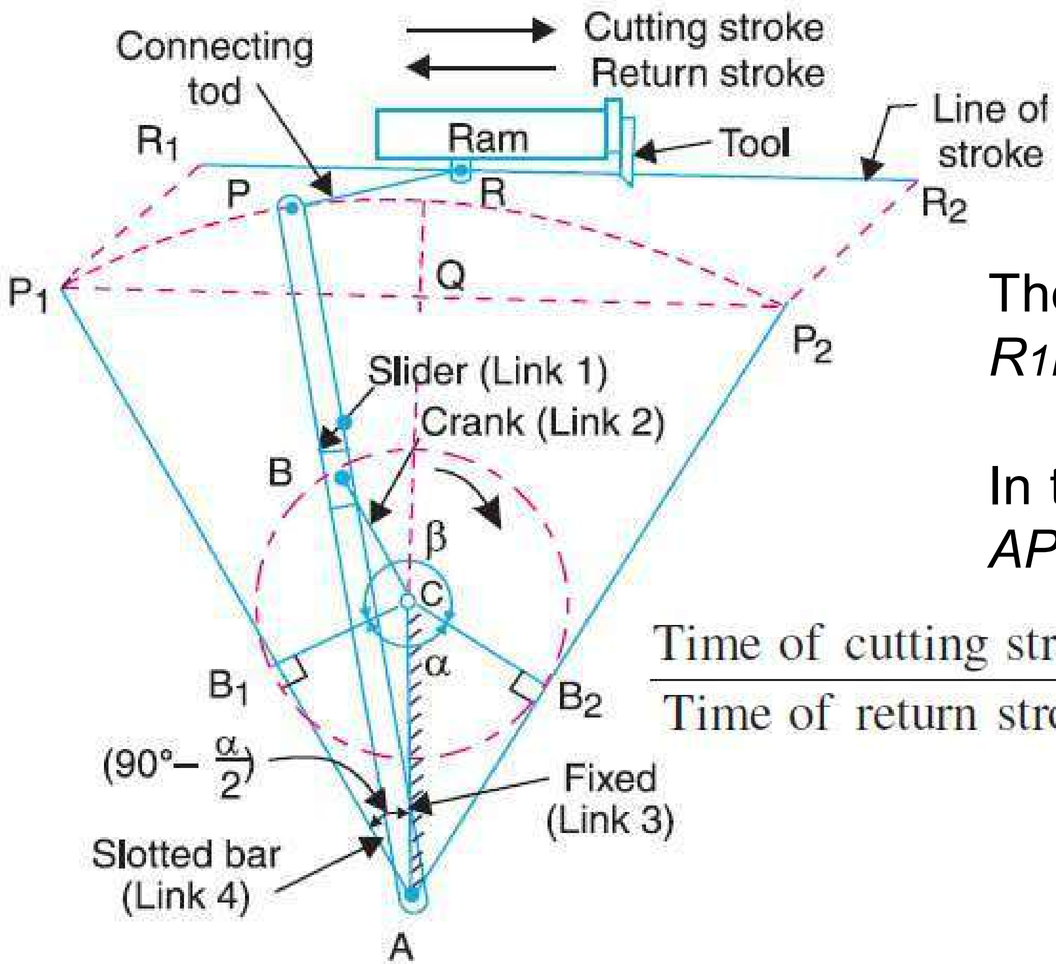


- Crank is fixed at center D
- Cylinder reciprocates
- Engine rotates in the same plane

CRANK AND SLOTTED LEVER QUICK RETURN MOTION MECHANISM



CRANK AND SLOTTED LEVER QUICK RETURN MOTION MECHANISM



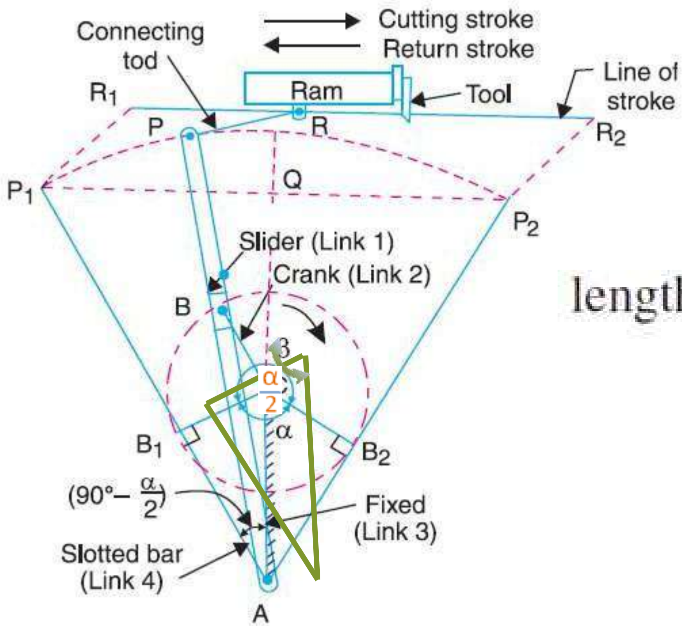
[Source: R S Khurmi]

The line of stroke of the ram (*i.e.* R_1R_2) is perpendicular to AC

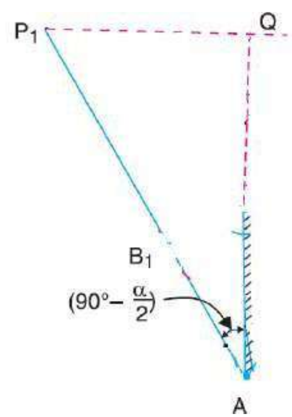
In the extreme positions, AP_1 and AP_2 are tangential to the circle

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta} \quad \text{or} \quad \frac{360^\circ - \alpha}{\alpha}$$

CRANK AND SLOTTED LEVER QUICK RETURN MOTION MECHANISM



[Source: R S Khurmi]



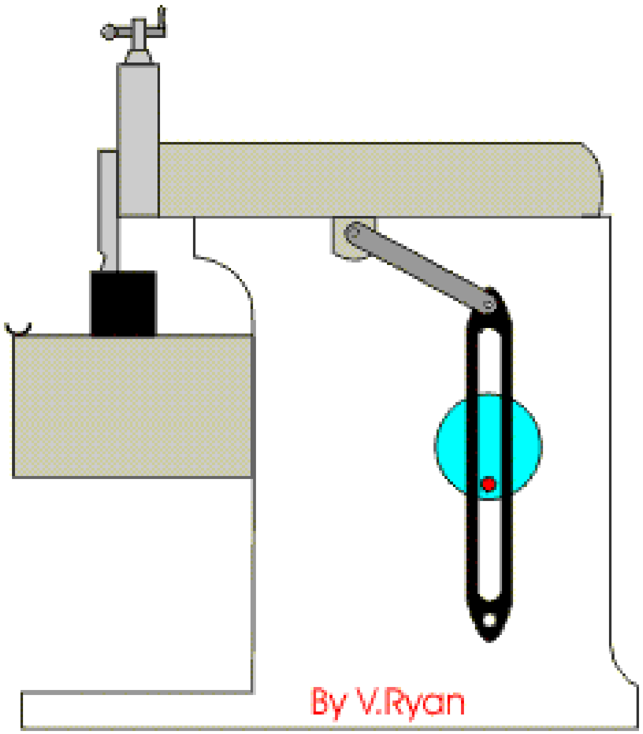
$$\text{length of stroke } = R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1 \sin \angle P_1AQ$$

$$= 2AP_1 \sin\left(90° - \frac{\alpha}{2}\right) = 2AP \cos\frac{\alpha}{2} \quad \text{.....} (\because AP_1 = AP)$$

$$= 2AP \times \frac{CB_1}{AC} \quad \dots \left(\because \cos\frac{\alpha}{2} = \frac{CB_1}{AC}\right)$$

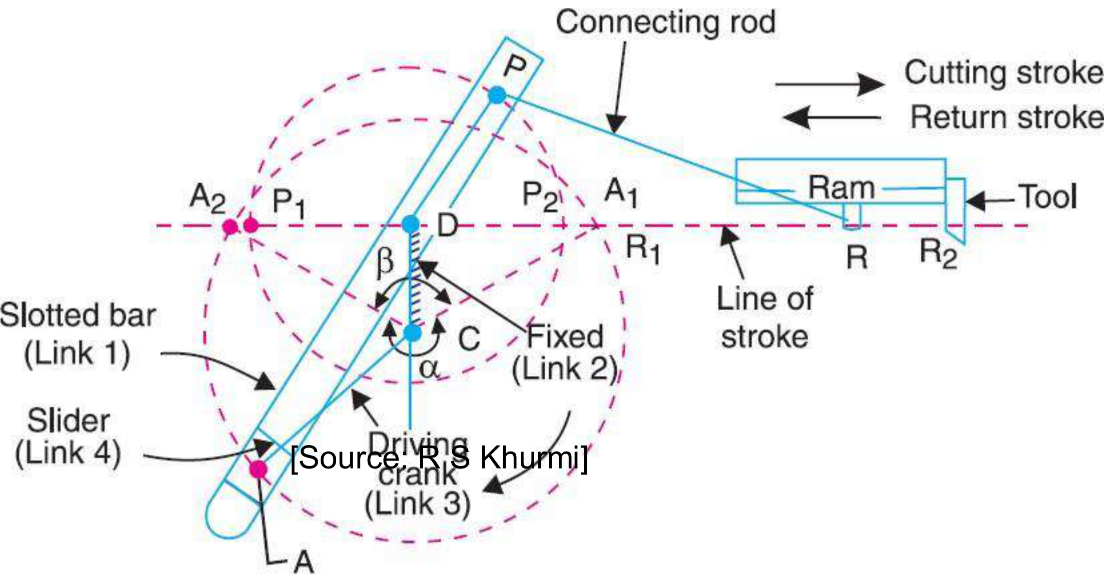
$$= 2AP \times \frac{CB}{AC} \quad \dots (\because CB_1 = CB)$$

Crank and slotted lever quick return mechanism is mostly used in shaping machines & slotting machines

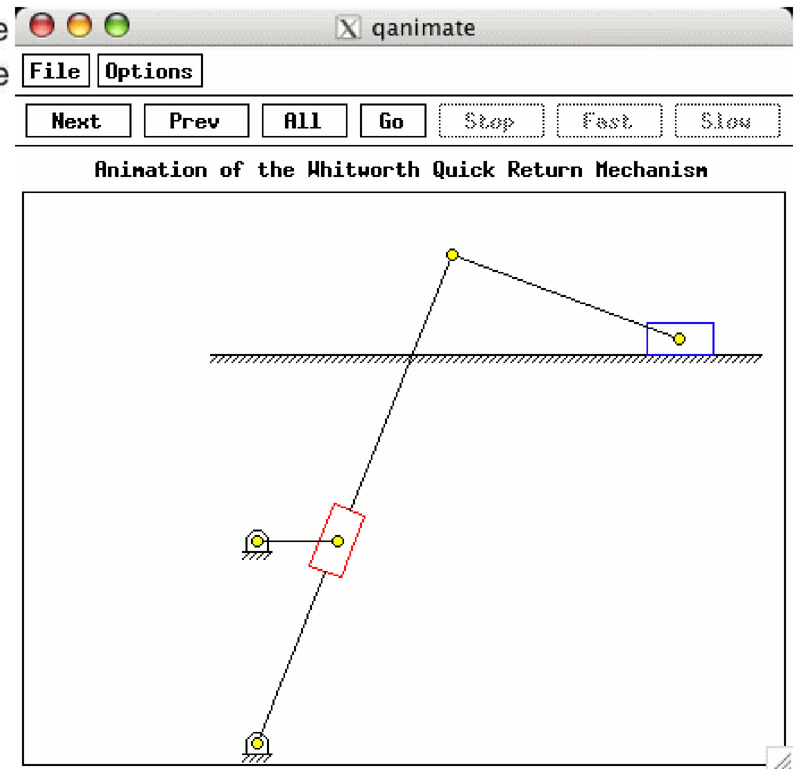


THE SHAPING MACHINE

WHITWORTH QUICK RETURN MOTION MECHANISM



[Source: R S Khurmi]



$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360° - \alpha} \quad \text{or} \quad \frac{360° - \beta}{\beta}$$

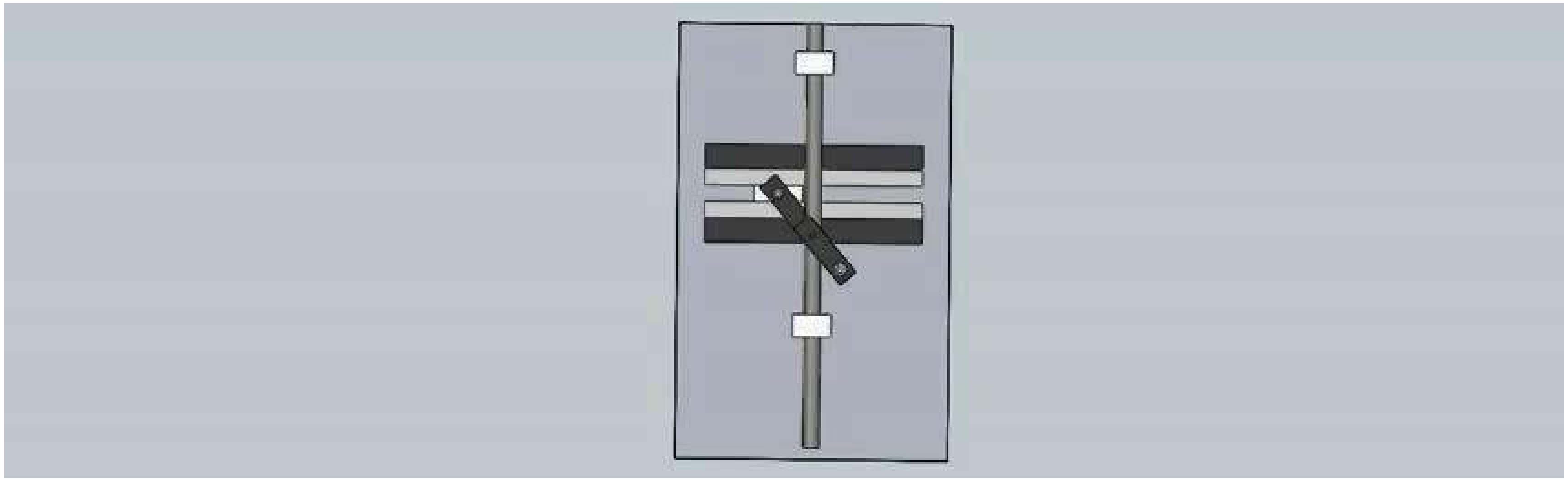
LECTURE 8

INVERSION OF DOUBLE SLIDER CRANK CHAINS

DEPARTMENT OF MECHANICAL ENGINEERING

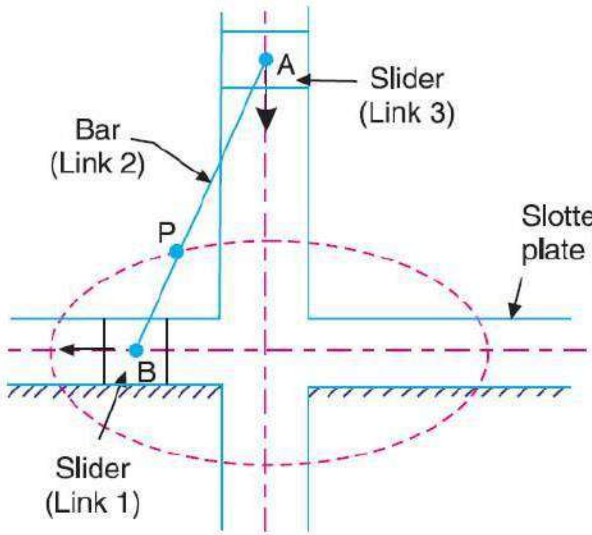
INVERSIONS OF DOUBLE SLIDER CRANK CHAIN

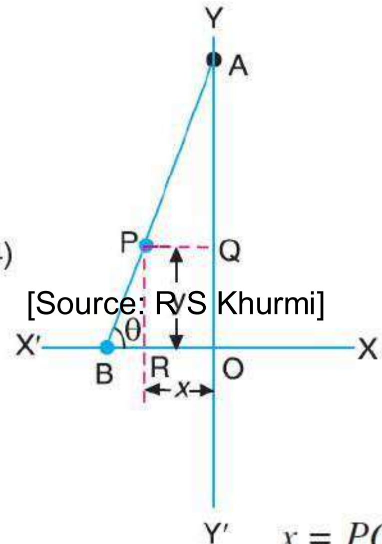
(1. ELLIPTICAL TRAMMELS)



INVERSIONS OF DOUBLE SLIDER CRANK CHAIN

(1. ELLIPTICAL TRAMMELS)





[Source: R/S Khurmi]

- used for drawing ellipses
- any point on the link 2 such as P traces out an ellipse on the surface of link 4
- AP - semi-major axis;
- BP - semi-minor axis

$$x = PQ = AP\cos\theta;\ \text{and}\ y = PR = BP\sin\theta$$

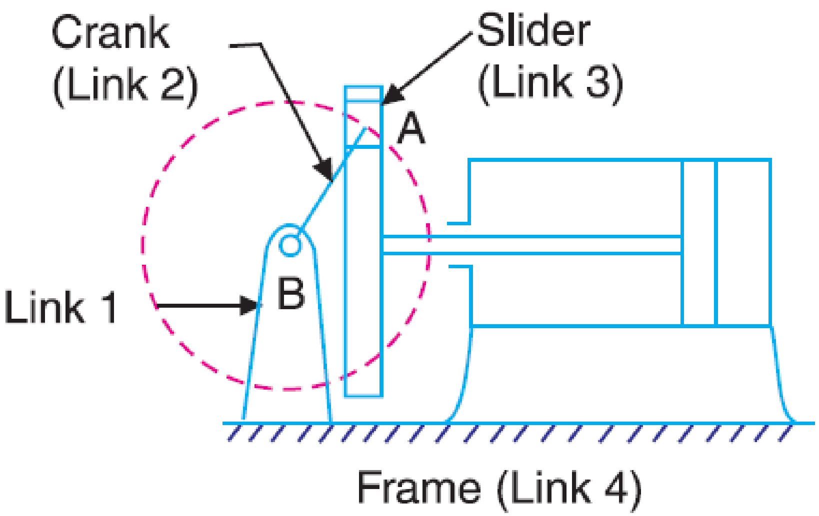
or

$$\frac{x}{AP} = \cos\theta;\ \text{and}\ \frac{y}{BP} = \sin\theta$$

Squaring and adding,

$$\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = \cos^2\theta + \sin^2\theta = 1$$

INVERSIONS OF DOUBLE SLIDER CRANK CHAIN
(2. SCOTCH YOKE MECHANISM)



Scotch yoke mechanism.

[Source: R S Khurmi]

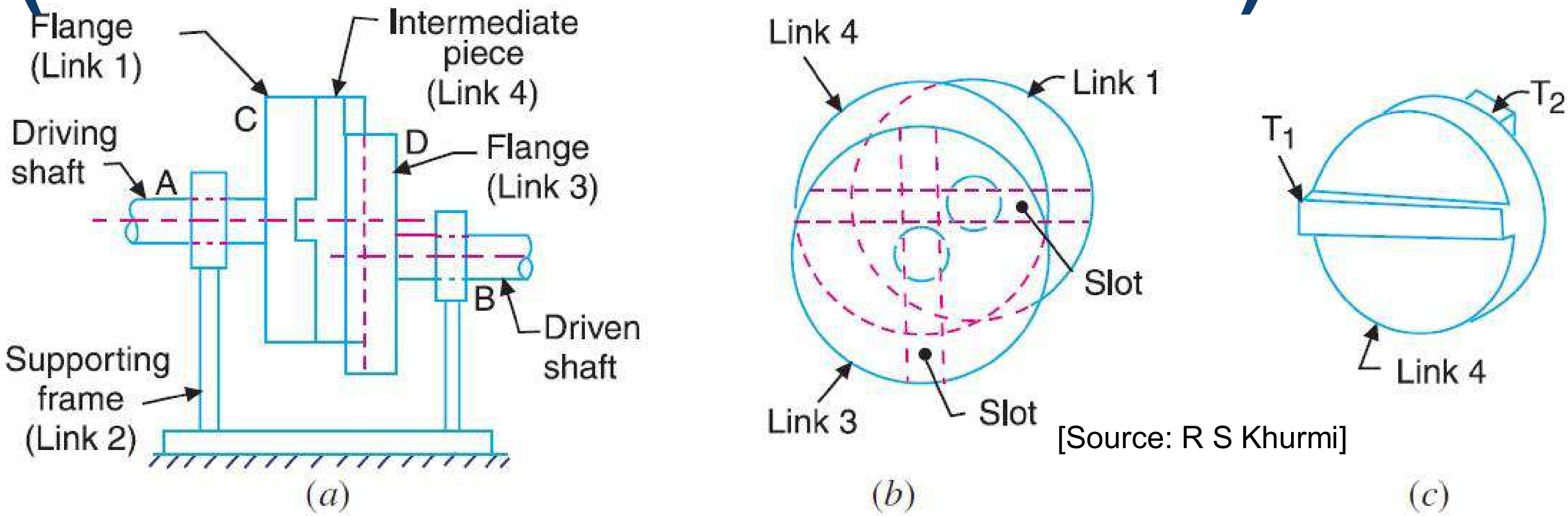
- This mechanism is used for converting rotary motion into a reciprocating motion.
- Link 1 is fixed.
- when the link 2 (crank) rotates about B as centre, reciprocation motion taking place.

INVERSIONS OF DOUBLE SLIDER CRANK CHAIN

(3. OLDHAM'S COUPLING)

INVERSIONS OF DOUBLE SLIDER CRANK CHAIN

(3. OLDHAM’S COUPLING)



[Source: R S Khurmi]

Oldham’s coupling.

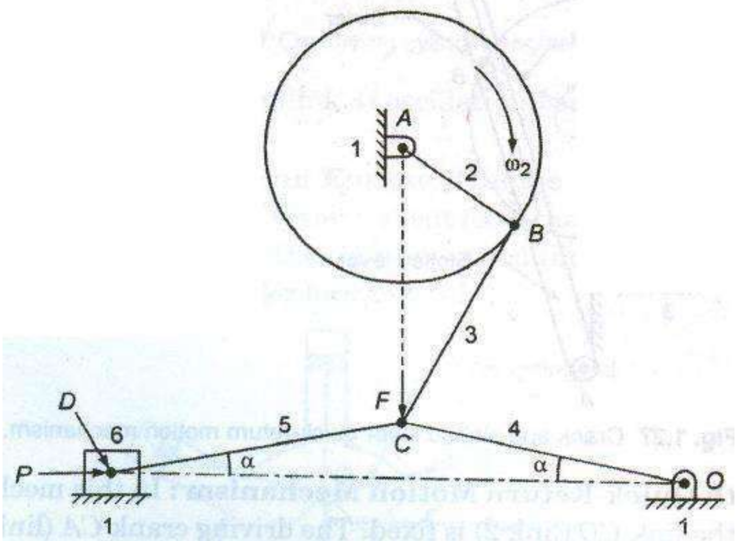
T_1 and T_2 two tongues (*i.e.* diametrical projections) on each face at right angles to each other

used for connecting two parallel shafts whose axes are at a small distance apart.

SOME COMMON MECHANISMS :TOGGLE MECHANISM

TOGGLE MECHANISM

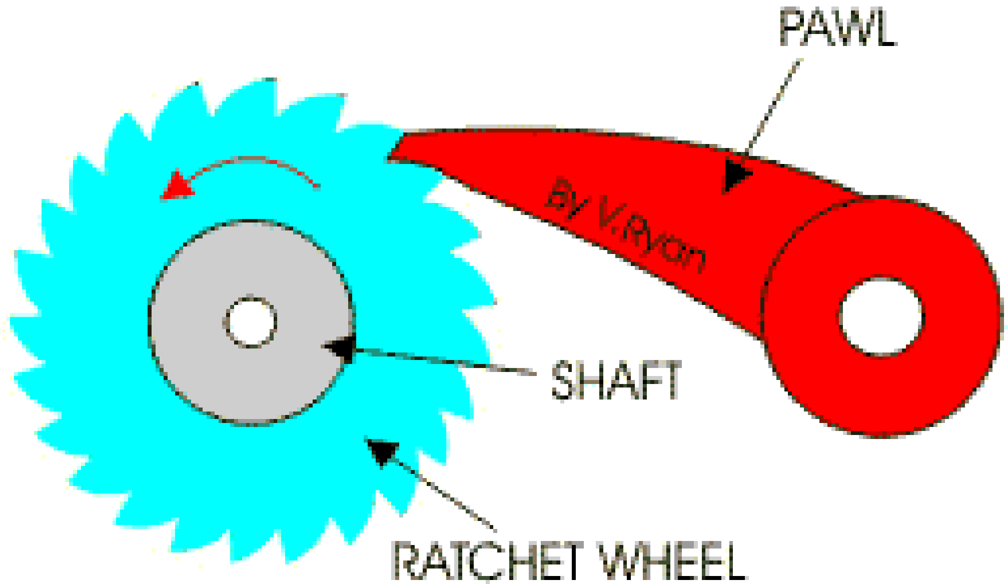
$$F = 2P \tan\alpha$$



- If α approaches to zero, for a given F, P approaches infinity.
- A stone crusher utilizes this mechanism to overcome a large resistance with a small force.
- It is used in numerous toggle clamping devices for holding work pieces.
- Other applications are: Clutches, Pneumatic riveters etc.,

INTERMEDIATE MOTION MECHANISM

RATCHET AND PAWL MECH



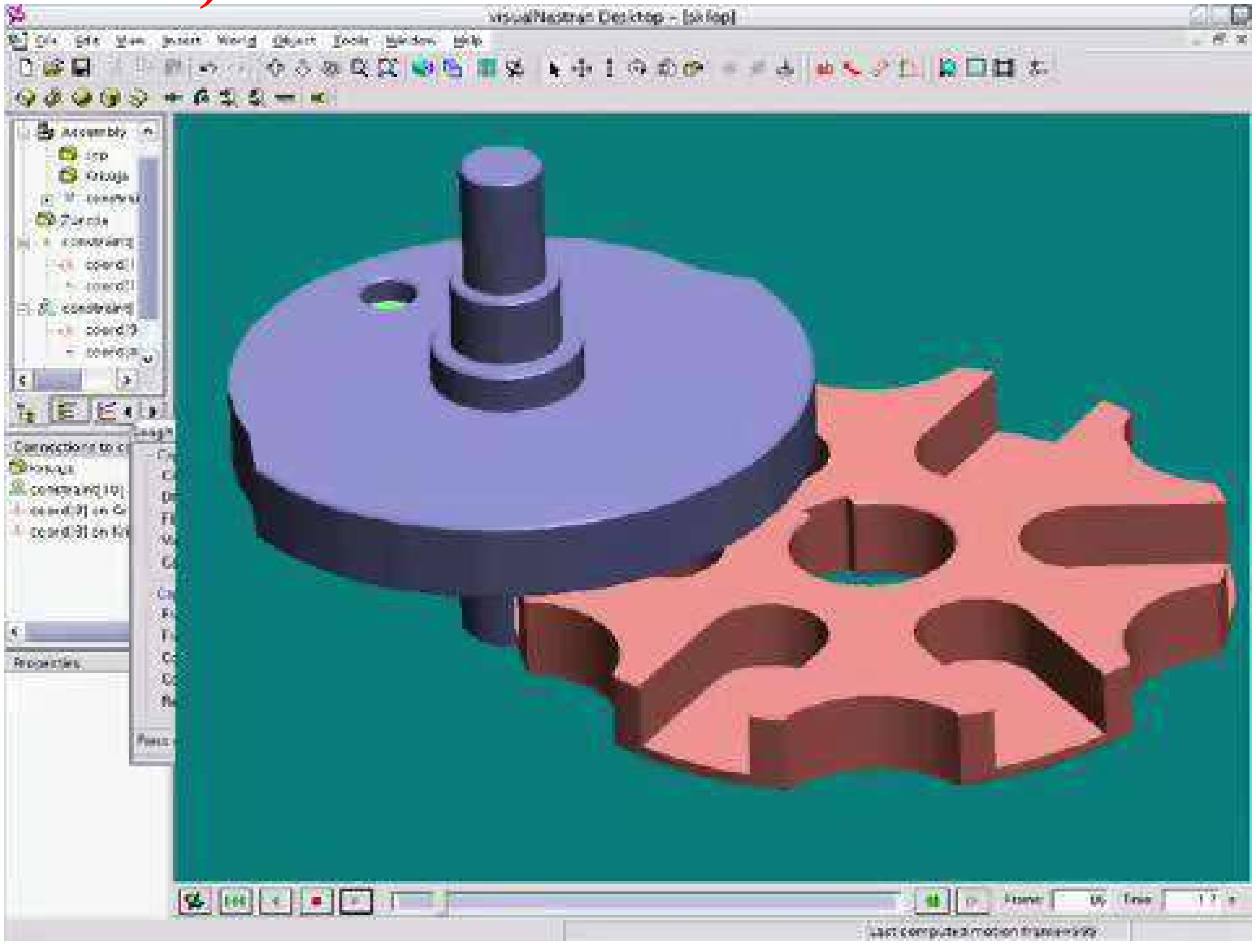
- There are many different forms of ratchets and escapements which are used in:
- locks, jacks, clockwork, and other applications requiring some form of intermittent motion.

APPLICATION OF RATCHET PAWL MECHANISM

Used in Hoisting Machines as safety measure

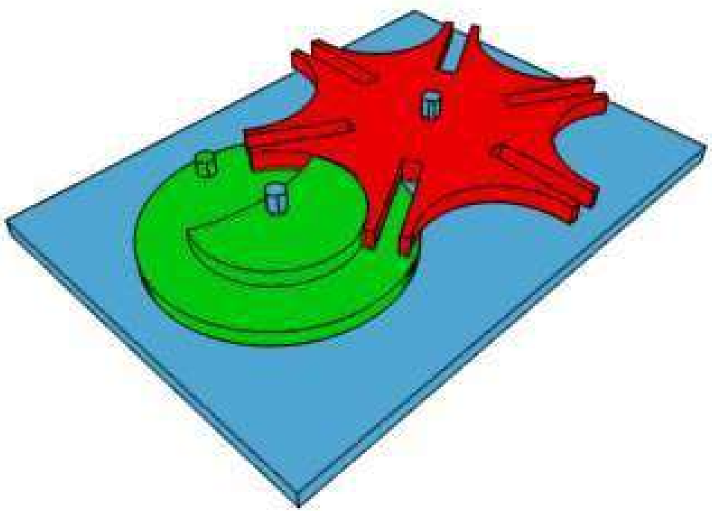
INTERMEDIATE MOTION MECHANISM

GENEVA MECHANISM (INDEXING MECHANISM)

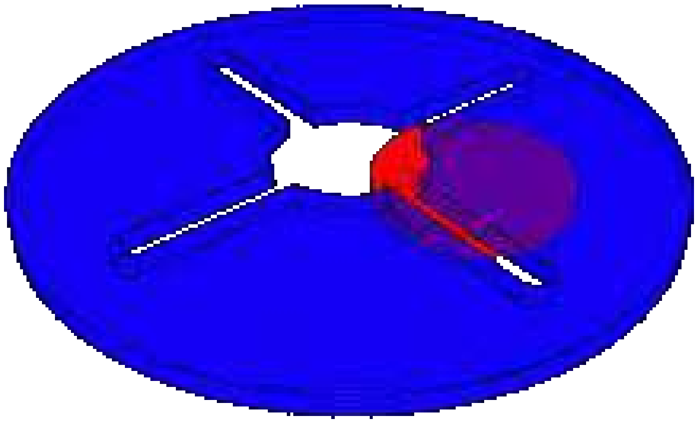


INTERMEDIATE MOTION MECHANISM

GENEVA MECHANISM

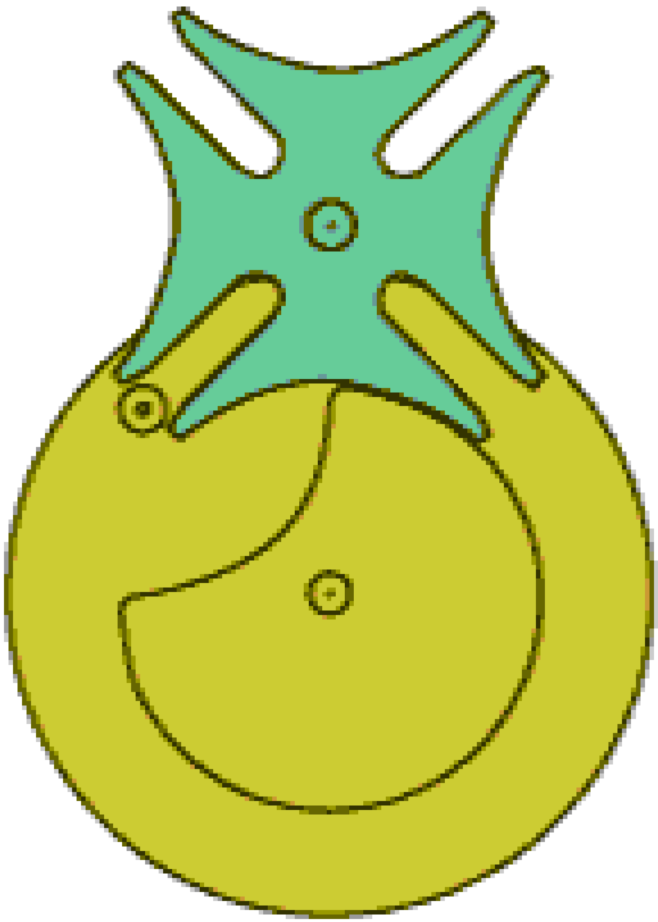


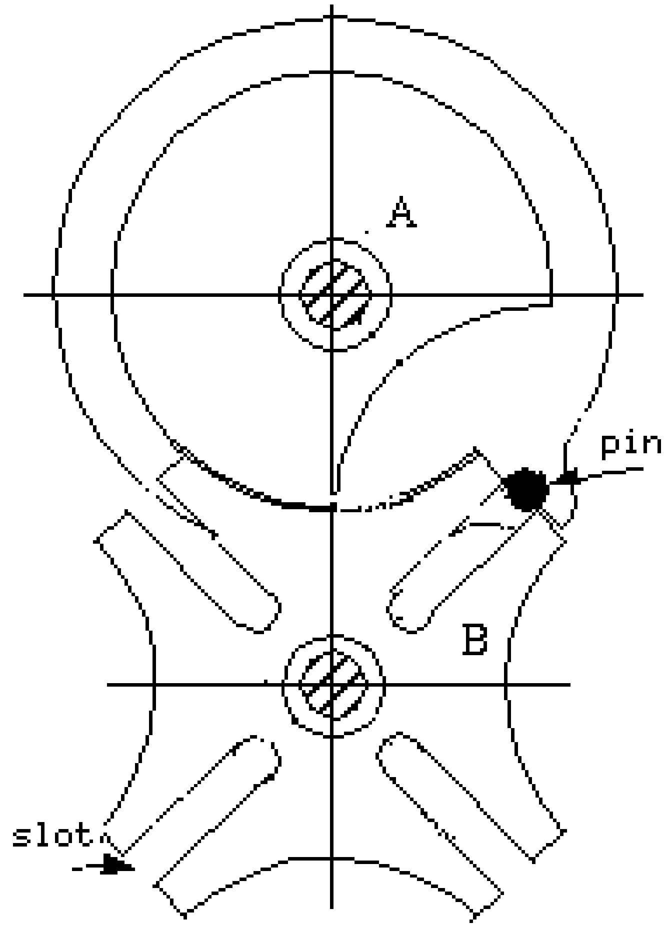
Animation showing a six-position external Geneva drive in operation



Animation showing an internal Geneva drive in operation.

INTERMITTENT MOTION MECHANISMS
GENEVA WHEEL MECHANISM





APPLICATIONS OF GENEVA MECHANISM

- Locating and locking mechanism
- Indexing system of a multi-spindle machine tool

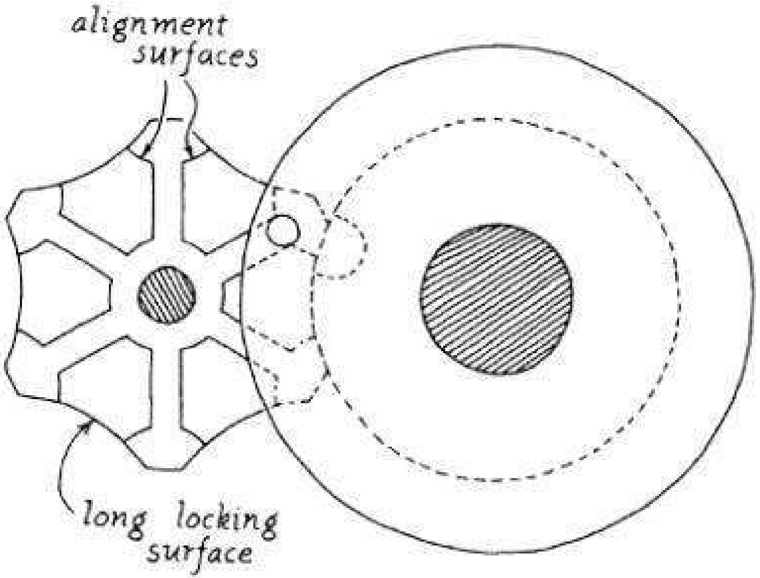
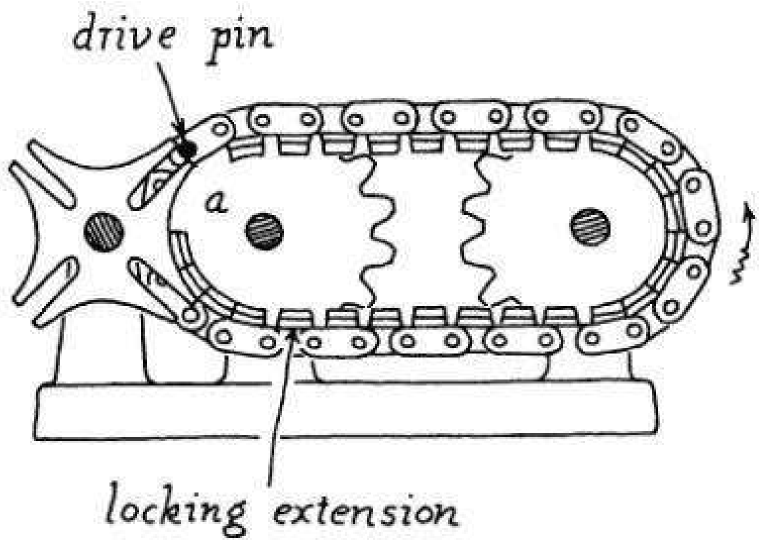


Fig. 9–15. Six-slot external Geneva used for light-duty instrument applications.



Drawing courtesy of PRODUCT ENGINEERING Magazine; June 8, 1964: pp. 67, 68

Fig. 9–18. Chain-mounted drive pins with blocks for locking during dwells.

Industry applications

1. Compliant mechanisms used in new age industries.
2. Mechanical Components form Specialized Motion-Control Systems
3. Mechanism for Planar Manipulation with Simplified Kinematics
4. Five linkages for straight-line motion
5. Seven linkages for transport mechanisms

Question Bank for Assignments

1. Explain inversions of a four bar chain in detail?
2. Explain the working of any two inversions of a single slider crank chain with neat sketches.
3. What is inversion of mechanism? Describe various inversions of double slider crank mechanism with sketches.
4. Explain with neat sketch the working of crank and slotted lever quick return motion mechanism. Deduce the expression for length of stroke in terms of link lengths.
5. State and explain Whitworth quick return mechanism. Also derive an equation for ratio of time taken for return strokes and forward strokes.
6. Define Kinematic pair and discuss various types of kinematic pairs with example.

Tutorial Questions

1. What is a machine? Giving example, differentiate between a machine and a structure.

2. Write notes on complete and incomplete constraints in lower and higher pairs, illustrating your answer with neat sketches.

3. Explain different kinds of kinematic pairs giving example for each one of them.

4. Explain the terms: 1. Lower pair, 2. Higher pair, 3. Kinematic chain, and 4. Inversion.

5. In what way a mechanism differ from a machine?

6. What is the significance of degrees of freedom of a kinematic chain when it functions as a mechanism? Give examples.

7. Explain Grubler's criterion for determining degree of freedom for mechanisms. Using Grubler's criterion for plane mechanism, prove that the minimum number of binary links in a constrained mechanism with simple hinges is four.

8. Sketch and explain the various inversions of a slider crank chain.

9. Sketch and describe the four bar chain mechanism. Why it is considered to be the basic chain?

10. Show that slider crank mechanism is a modification of the basic four bar mechanism.

11. Sketch slider crank chain and its various inversions, stating actual machines in which these are used in practice.

12. Sketch and describe the working of two different types of quick return mechanisms. Give examples of their applications. Derive an expression for the ratio of times taken in forward and return stroke for one of these mechanisms.

13. Sketch and explain any two inversions of a double slider crank chain.

UNIT 2

SPECIAL MECHANISMS

COURSE OBJECTIVE

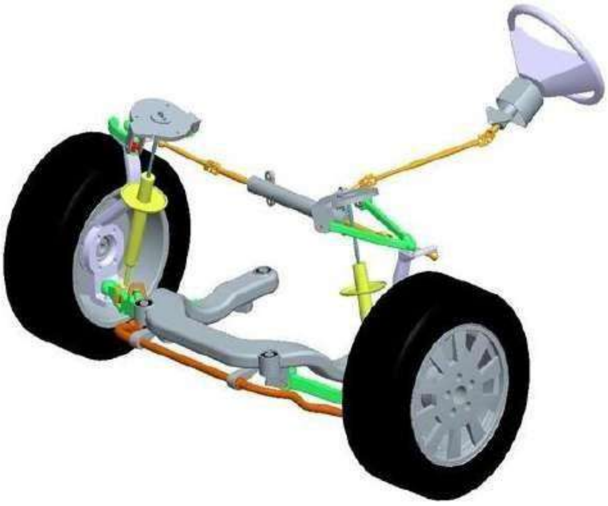
To Synthesize and analyze 4 bar mechanisms.

COURSE OUTCOME

LECTURE	LECTURE TOPIC	KEY ELEMENTS	LEARNING OBJECTIVES
1	**Straight Line Motion Mechanisms**	Definition of Straight line motion mechanism Classification of exact straight line motion mechanism	Understanding the inversion of mechanisms and its classifications (B2)
2	Approximate Straight Line Mechanism	Working of approximate straight-line mechanisms	Understanding the inversions of approximate straight line mechanism (B2) Analyse the approximate straight line mechanisms (B4)
3	Pantograph	Purpose of mechanisms Applications of Mechanism	Understanding the Pantograph mechanism (B2) List the applications of Pantograph (B4)
4	**Davi's Steering Gear Mechanism**	Definition of Davi's Steering Gear Mechanism Condition for correct steering condition	Understanding the working of Davi's Steering gear mechanism (B2) Apply the mechanism for correct steering condition (B3)
5	Ackerman's Steering Gear Mechanism	Definition of Ackerman's Steering Gear Mechanism Condition for correct steering condition	Understanding the working of Ackerman's Steering gear mechanism (B2) Apply the mechanism for correct steering condition (B3)
6	**Single and Double Hooke Joint**	Describe single and double Hooke's Joint List of applications using single and double Hooke's Joint	Remember the working single and double Hooke's Joint (B1) Apply single and double Hooke's joint for various applications (B3)
7	Ratio of Shaft Velocities	Derive the ratio of shaft velocities	Evaluate the velocity ratio for shafts (B5)

2

Special Mechanisms



Course Contents

2.1 Straight line mechanisms

2.2 Exact straight line mechanisms made up of turning pair

2.3 Peaucellier mechanism

2.4 Hart"s Mechanism

2.5 Exact straight line motion consisting of one sliding pair

Approximate straight line motion mechanisms

Steering gear mechanism

Devis steering gear

Ackerman steering gear

Universal or Hooke"s joint

Ratio of shaft velocities

Max. and Min. speed of driven shaft

Polar diagram

Double Hooke"s Joint

Examples

Straight Line Mechanisms

- It permits only relative motion of an oscillatory nature along a straight line. The mechanisms used for this purpose are called ***straight line mechanisms***.
  1. In which only turning pairs are used
  2. In which one sliding pair is used.
- These two types of mechanisms may produce exact straight line motion or approximate straight line motion.
- **Need of Straight Line:**
  1. Sewing Machine converts rotary motion to up/down motion.
  2. Want to constrain pistons to move only in a straight line.
  3. How do you create the first straight edge in the world? (Compass is easy)
  4. Windshield wipers, some flexible lamps made of solid pieces connected by flexible joints.

Exact Straight Line Motion Mechanisms Made Up Of Turning Pairs

- The principle adopted for a mathematically correct or exact straight line motion is described in Fig.2.1
- Let O be a point on the circumference of a circle of diameter OP. Let OA be any chord and B is a point on OA produced, suchthat

$$OA \times OB = \text{constant}$$

- The triangles OAP and OBQ are similar.

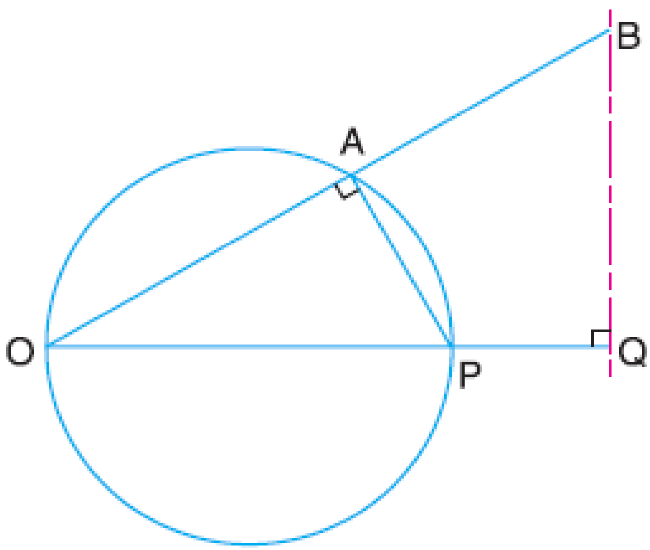


Fig. 2.1 Exact straight line motion mechanism

$$\frac{OA}{OP} = \frac{OQ}{OB}$$

$$OP \times OQ = OA \times OB$$

$$OQ = \frac{OA \times OB}{OP}$$

- But OP is constant as it is the diameter of a circle; therefore, if $OA \times OB$ is constant, then OQ will be constant.
- Hence

$$OA \times OB = constant$$

- So point B moves along the straight line.

Peaucellier Mechanism (Exact Straight Line)

- It consists of a fixed link OO1 and the other straight links O1A, OC, OD, AD, DB, BC and CA are connected by turning pairs at their intersections, as shown in Fig. 2.2
- The pin at A is constrained to move along the circumference of a circle with the fixed diameter OP, by means of the link O1A. In Fig. 2.2
- AC = CB = BD = DA
- OC = OD
- $OO_1 = O_1A$

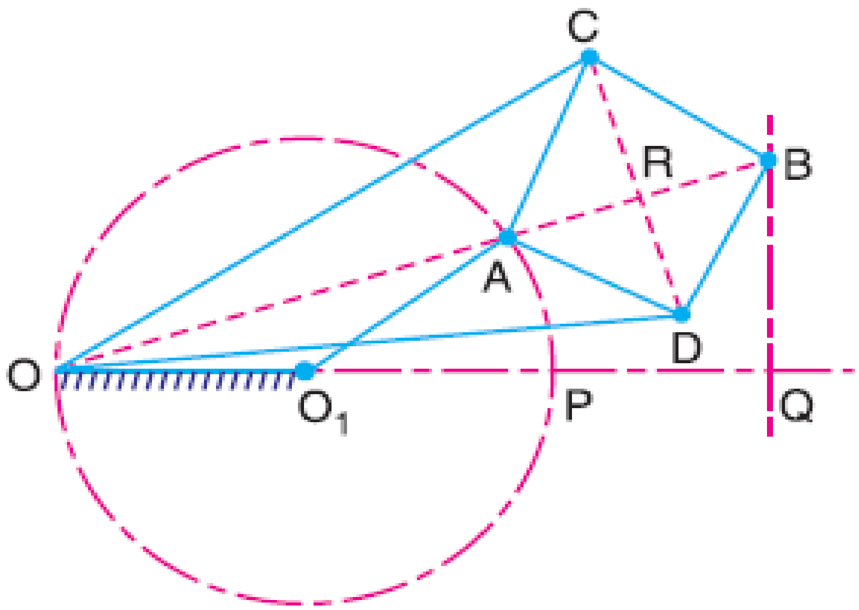


Fig. 2.2 Peaucellier Mechanism

- From right angled triangles ORC and BRC, we have

$$OC^2 = OR^2 + RC^2 \qquad (I)$$

$$BC^2 = RB^2 + RC^2 \qquad (ii)$$

- From (i) and (ii)

$$OC^2 - BC^2 = OR^2 - RB^2$$

$$= (OR - RB)(OR + RB)$$

$$= OB \times OA$$

- Since OC and BC are of constant length, therefore the product $OB \times OA$ remains constant.

Hart's Mechanism

- This mechanism requires only six links as compared with the eight links required by the Peaucellier mechanism.
- It consists of a fixed link OO1 and other straight links O1A, FC, CD, DE and EF are connected by turning pairs at their points of intersection, as shown in Fig. 2.3.
- The links FC and DE are equal in length and the lengths of the links CD and EF are also equal. The points O, A and B divide the links FC, CD and EF in the same ratio. A little consideration will show that BOCE is a trapezium and OA and OB are respectively parallel to FD and CE.

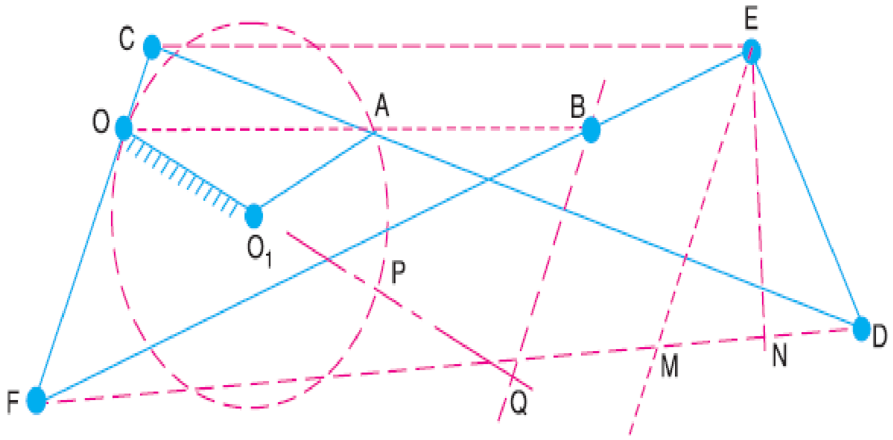


Fig. 2.3 Hart's Mechanism

- Here, FC = DE & CD = EF
- The point O, A and B divide the links FC, CD and EF in the sameratio.
- From similar triangles CFE and OFB,

$$\frac{CE}{FC} = \frac{OB}{OF} \text{ or } CB = \frac{CE \times OF}{FC} \text{ ... ... (i)}$$

- From similar triangle FCD and OCA

$$\frac{FD}{FC} = \frac{OA}{OC} \quad or\ OA = \frac{FD \times OC}{FC} \quad .... (ii)$$

- From above equations,

$$OA \times OB = \frac{FD \times OC}{FC} \times \frac{CE \times OF}{FC}$$

$$= FD \times CE \times \frac{OC \times OF}{FC^2}$$

- Since the lengths of OC, OF and FC are fixed, therefore

$$OA \times OB = FD \times CE \times cons. \ldots (iii)$$

- From point E, draw EM parallel to CF and EN perpendicular to FD.

$$FD \times CE = FD \times FM \quad (CE = FM)$$

$$= (FN + ND)(FN - MN)$$

$$= FN^2 - ND^2 \text{ (MN = ND)}$$

$$= (FE^2 - NE^2) - (ED^2 - NE^2) \text{ (From right angle triangles FEN and EDN)}$$

$$= E^2 - ED^2 = constant \quad (iv)$$

– From equation (iii) and (iv),

$$OA \times OB = constant$$

Exact Straight Line Motion consisting of one slidingpair-Scott Russell's Mechanism

– A is the middle point of PQ and OA = AP = AQ. The instantaneous center for the link PAQ lies at I in OA produced and is such that IP is perpendicular to OP.

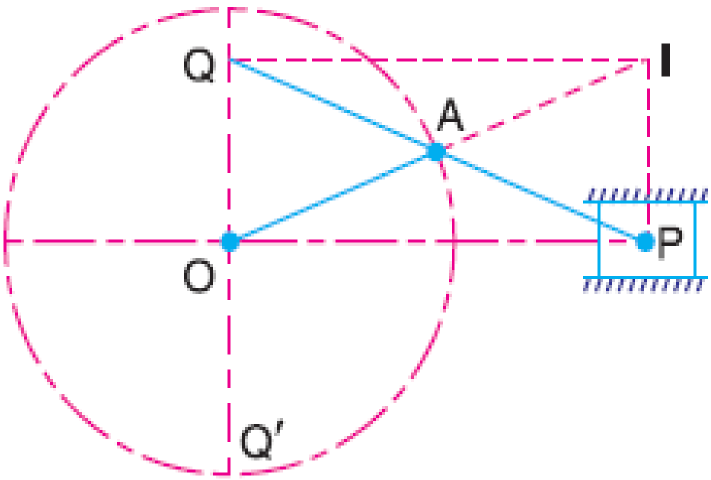


Fig. 2.4 Scott Russell's Mechanism

– Join IQ. Then Q moves along the perpendicular to IQ. Since OPIQ is a rectangle and IQ is perpendicular to OQ, therefore Q moves along the vertical line OQ for all positions of QP. Hence Q traces the straight line OQ′.

– If OA makes one complete revolution, then P will oscillate along the line OP through a distance 2 OA on each side of O and Q will oscillate along OQ′ through the same distance 2 OA above and below O. Thus, the locus of Q is a copy of the locus of P.

Approximate straight line motion mechanisms

Watt's Mechanism

– It has four links as shown in fig. OB, O1A, AB and OO1.

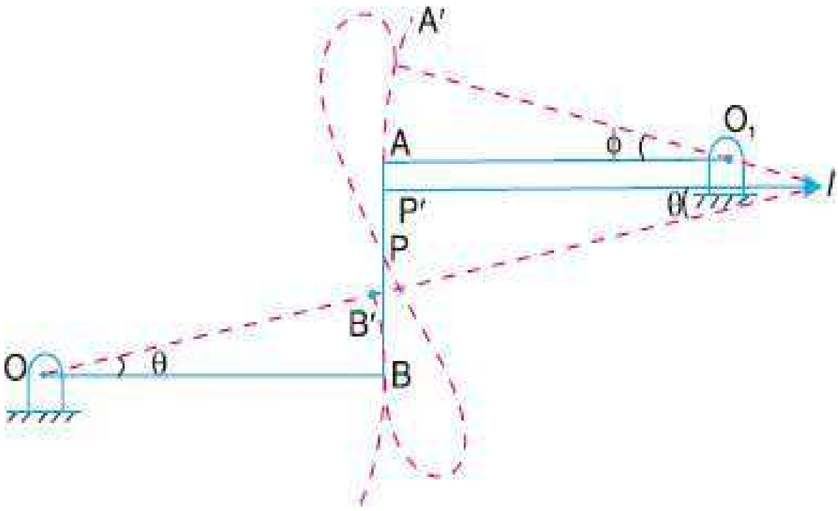


Fig. 2.5 watt's mechanism

– OB and O1A oscillates about centers O and O1 respectively. P is a point on AB such that,

$$\frac{O_1}{OB} = \frac{PB}{PA}$$

– As OB oscillates the point P will describe an approximate straight line.

Modified Scott-Russel Mechanism

– This is similar to Scott-Russel mechanism but in this case AP is not equal to AQ and the points P and Q are constrained to move in the horizontal and vertical directions.

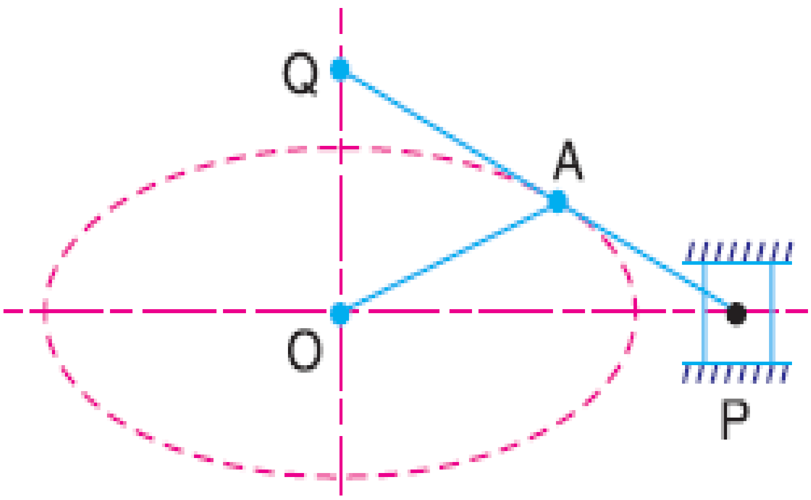


Fig. 2.6 Modified Scott-Russel Mechanisms

- A little consideration will show that it forms an elliptical trammel, so that any point *A* on *PQ* traces an ellipse with semi-major axis *AQ* and semi minor axis *AP*.
- If the point *A* moves in a circle, then for point *Q* to move along an approximate straight line, the length *OA* must be equal (*AP*)2 / *AQ*. This is limited to only small displacement of *P*.

Grasshopper Mechanism

- In this mechanism, the centers O and O1 are fixed. The link OA oscillates about O through an angle AOA1 which causes the pin P to move along a circular arc with O1 as center and O1P as radius.

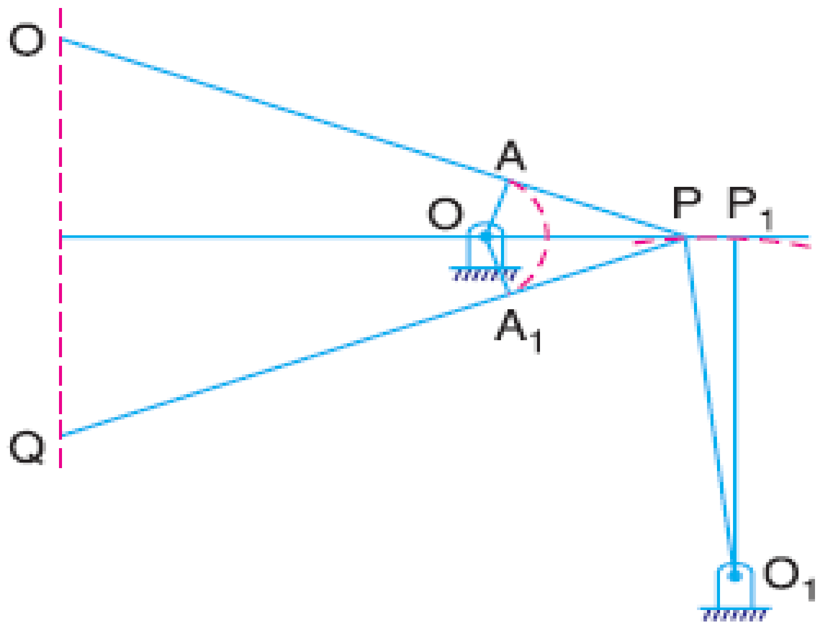


Fig. 2.7 Grasshopper Mechanism

- For small angular displacements of OP on each side of the horizontal, the point Q on the extension of the link PA traces out an approximately a straight path QQ′. if the lengths are such that

$$OA = \frac{AP^2}{AQ}$$

Tchebicheff's Mechanism

- It is a four bar mechanism in which the crossed links OA and O1B are of equal length, as shown in Fig. 2.8.
- The point P, which is the mid-point of AB, traces out an approximately straight line parallel to OO1.

– The proportions of the links are, usually, such that point P is exactly above O or O1 in the extreme positions of the mechanism i.e. when BA lies along OA or when BA lies along BO_1.

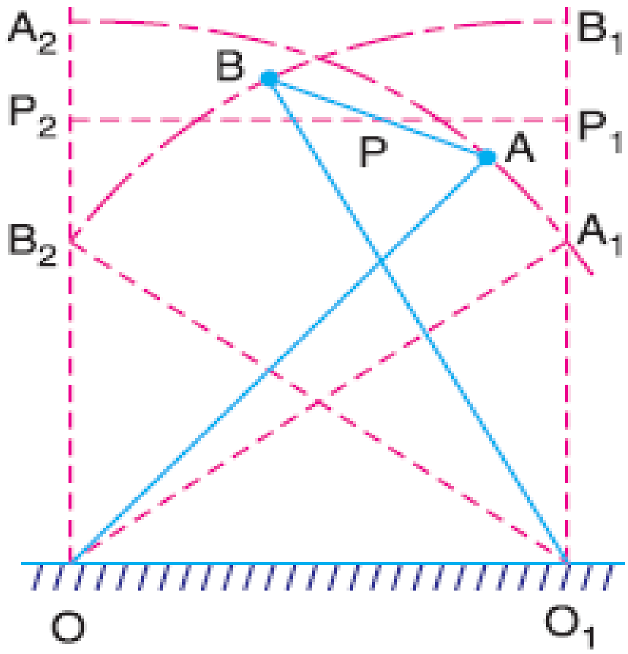


Fig. 2.8 Tchebicheff's mechanism

– It may be noted that the point *P* will lie on a straight line parallel to OO_1, in the two extreme positions and in the mid position, if the lengths of the links are in proportions

$$AB : OO_1 : OA = 1 : 2 : 4.5.$$

Roberts Mechanism

– It is also a four bar chain mechanism, which, in its mean position, has the form of a trapezium. The links OA and O1 B are of equal length and OO1 is fixed. A bar PQ is rigidly attached to the link AB at its middle point P.

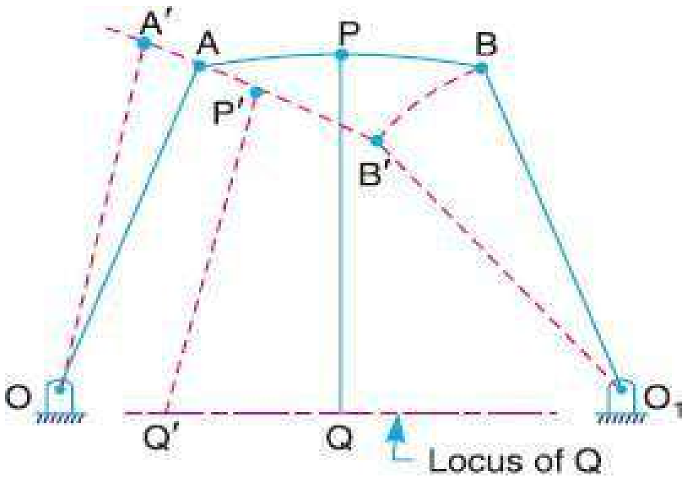


Fig. 2.9 Robert's Mechanism

– A little consideration will show that if the mechanism is displaced as shown by the dotted lines in Fig. the point Q will trace out an approximately straight line.

Steering gear mechanism

– The steering gear mechanism is used for changing the direction of two or more of the wheel axles with reference to the chassis, so as to move the automobile in any desired path.

– Usually the two back wheels have a common axis, which is fixed in direction with reference to the chassis and the steering is done by means of the front wheels.

– In automobiles, the front wheels are placed over the front axles, which are pivoted at the points A and B, as shown in Fig. 2.10.

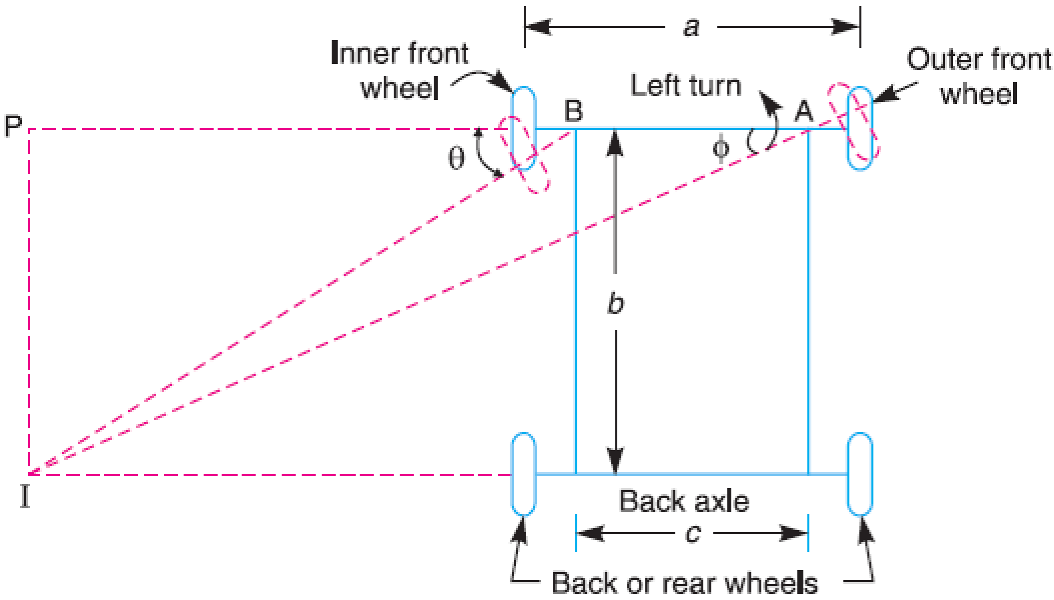


Fig. 2.10 steering gear mechanism

– These points are fixed to the chassis. The back wheels are placed over the back axle, at the two ends of the differential tube. When the vehicle takes a turn, the front wheels along with the respective axles turn about the respective pivoted points. The back wheels remain straight and do not turn. Therefore, the steering is done by means of front wheels only.

– In order to avoid skidding (i.e. slipping of the wheels sideways), the two front wheels must turn about the same instantaneous centre I which lies on the axis of the back wheels. If the instantaneous centre of the two front wheels do not coincide with the instantaneous Centre of the back wheels, the skidding on the front or back wheels will definitely take place, which will cause more wear and tear of the tyres

- Thus, the condition for correct steering is that all the four wheels must turn about the same instantaneous centre. The axis of the inner wheel makes a larger turning angle θ than the angle φ subtended by the axis of outer wheel.
- Let, a = wheel track

  b = wheel base

  c = Distance between the pivots A and B of the front axle.
- Now from triangle IBP,

$$\cot\theta = \frac{BP}{IP}$$

- And from triangle IAP,

$$\cot\emptyset = \frac{AP}{IP} = \frac{AB + BP}{IP} = \frac{c}{b} + \cot\theta$$

$$\boldsymbol{\cot\emptyset - \cot\theta = \frac{c}{b}}$$

- This is the fundamental equation for correct steering.

Devis Steering Mechanism

- The Davis steering gear is shown in Fig. 2.11. It is an exact steering gear mechanism. The slotted links AM and BH are attached to the front wheel axle, which turn on pivots A and B respectively.
- The rod CD is constrained to move in the direction of its length, by the sliding members at P and Q. These constraints are connected to the slotted link AM and BH by a sliding and a turning pair at each end. The steering is affected by moving CD to the right or left of its normal position. C 'D' shows the position of CD for turning to the left.

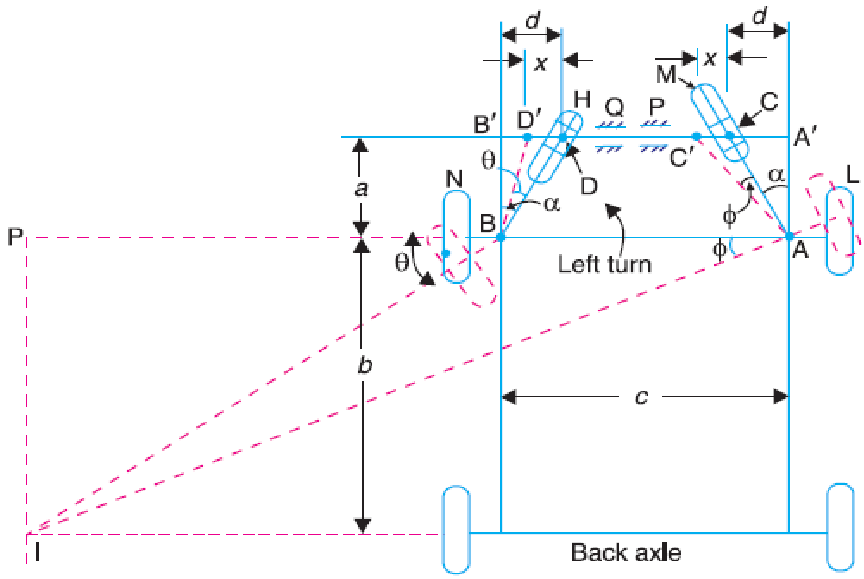


Fig. 2.11 Devis steering gear mechanism

- Let,

  a = Vertical distance between AB and CD,
  b = Wheel base,
  d = Horizontal distance between AC and BD,
  c = Distance between the pivots A and B of the front axle.
  x = Distance moved by AC to AC ′ = CC ′ = DD′, and
  α = Angle of inclination of the links AC and BD, to the vertical.

- From triangle AA‟C ′

$$\tan(\alpha+\emptyset) = \frac{A'C'}{A'A'} = \frac{d+x}{a} \quad \ldots\ldots (i)$$

- From triangle AA‟C

$$\tan\alpha = \frac{A'C}{A'A'} = \frac{d}{a} \ldots\ldots (ii)$$

- From triangle BB‟D‟

$$\tan(\alpha-\theta) = \frac{B'D'}{BB'} = \frac{d-x}{a} \ldots\ldots (iii)$$

- We know that,

$$\tan(\alpha+\emptyset) = \frac{\tan\alpha+\tan\emptyset}{1-\tan\alpha\times\tan\emptyset}$$

$$\frac{d+x}{a} = \frac{d/a+\tan\emptyset}{1-/_a\times\tan\emptyset} = \frac{d+(\ \times\ \tan\emptyset)}{a-(\ \times\ \tan\emptyset)}$$

$$d\cdot x\times(a-d\times\tan\emptyset) = a\times(d+a\times\tan\emptyset)$$

$$a\cdot d-d^2\times\tan\emptyset+a\cdot x-d\times x\times\tan\emptyset = a\cdot d+\ ^2\times\ \tan\emptyset$$

$$\tan\emptyset\times(a^2+d^2+d\cdot x) = a\cdot x$$

$$\tan\emptyset = \frac{a\cdot x}{(a^2+d^2+d\cdot x)} \ldots\ldots (iv)$$

- Similarly from $\tan(\alpha-\theta) = \frac{d-x}{a}$, we get

$$\tan\theta = \frac{a\cdot x}{(a^2+d^2-d\cdot x)} \ldots\ldots (v)$$

- We know that for correct steering,

$$\cot\emptyset-\cot\theta = \frac{c}{b}$$

$$\frac{(a^2+d^2+d\cdot x)}{a\cdot x} - \frac{(a^2+\ ^2-d\cdot x)}{a\cdot x} = \frac{c}{b}$$

$$\frac{2\,d}{a} = \frac{c}{b}$$

$$2\ \tan\alpha = \frac{c}{b}$$

$$\tan\alpha = \frac{c}{2\,b}$$

Ackerman steering Gear

- The Ackerman steering gear mechanism is much simpler than Davis gear. The difference between the Ackerman and Davis steering gears are :

1 The whole mechanism of the Ackerman steering gear is on back of the front wheels;
whereas in Davis steering gear, it is in front of the wheels.

2 The Ackerman steering gear consists of turning pairs, whereas Davis steering gear consists of sliding members.

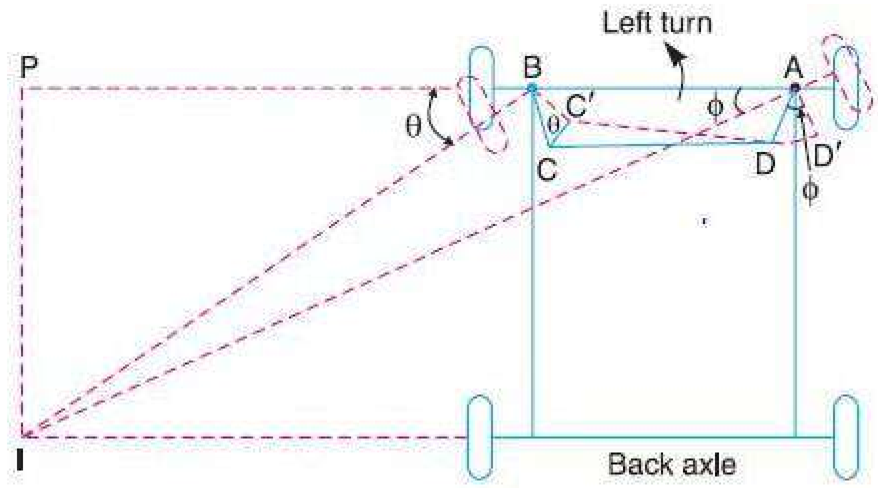


Fig. 2.12 Ackerman steering mechanism

- In Ackerman steering gear, the mechanism ABCD is a four bar crank chain, as shown in Fig. 2.12. The shorter links BC and A D are of equal length and are connected by hinge joints with front wheel axles. The longer links A B and CD are of unequal length.
- The following are the only three positions for correct steering.

1 When the vehicle moves along a straight path, the longer links A B and CD are parallel and the shorter links BC and AD are equally inclined to the longitudinal axis of the vehicle, as shown by firm lines in Fig. 2.12.

2 When the vehicle is steering to the left, the position of the gear is shown by dotted lines in Fig. 2.12. In this position, the lines of the front wheel axle intersect on the back wheel axle at I, for correct steering.

3 When the vehicle is steering to the right, the similar position may be obtained.

Universal or Hooke's Joint

- A Hooke‟s joint is used to connect two shafts, which are intersecting at a small angle, as shown in Fig.2.10. The end of each shaft is forked to U-type and each fork provides two bearings for the arms of a cross.
- The arms of the cross are perpendicular to each other. The motion is transmitted from the driving shaft to driven shaft through a cross.

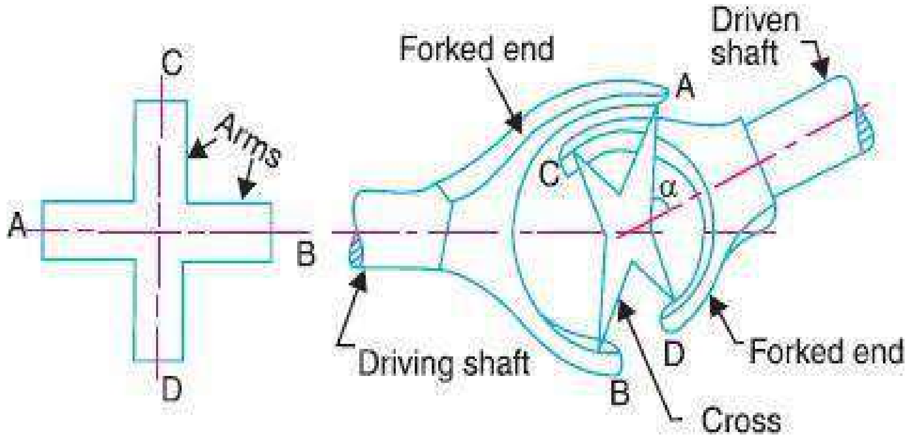


Fig. 2.13 Hooke's Joint

– The main application of the Universal or Hooke"s joint is found in the transmission from the gear box to the differential or back axle of the automobiles. It is also used for transmission of power to different spindles of multiple drilling machines.

Ratio of shaft velocities

– The top and front views connecting the two shafts by a universal joint are shown in Fig. 2.11. Let the initial position of the cross be such that both arms lie in the plane of the paper in front view, while the arm AB attached to the driving shaft lies in the plane containing the axes of the two shafts.

– Let the driving shaft rotates through an angle θ, so that the arm AB moves in a circle to a new position A1 B1 as shown in front view.

– A little consideration will show that the arm CD will also move in a circle of the same size. This circle when projected in the plane of paper appears to be an ellipse. Therefore the arm CD takes new position C1D1 on the ellipse, at an angle θ. But the true angle must be on the circular path.

– To find the true angle, project the point C1 horizontally to intersect the circle at C2. Therefore the angle COC2 (equal to φ) is the true angle turned by the driven shaft.

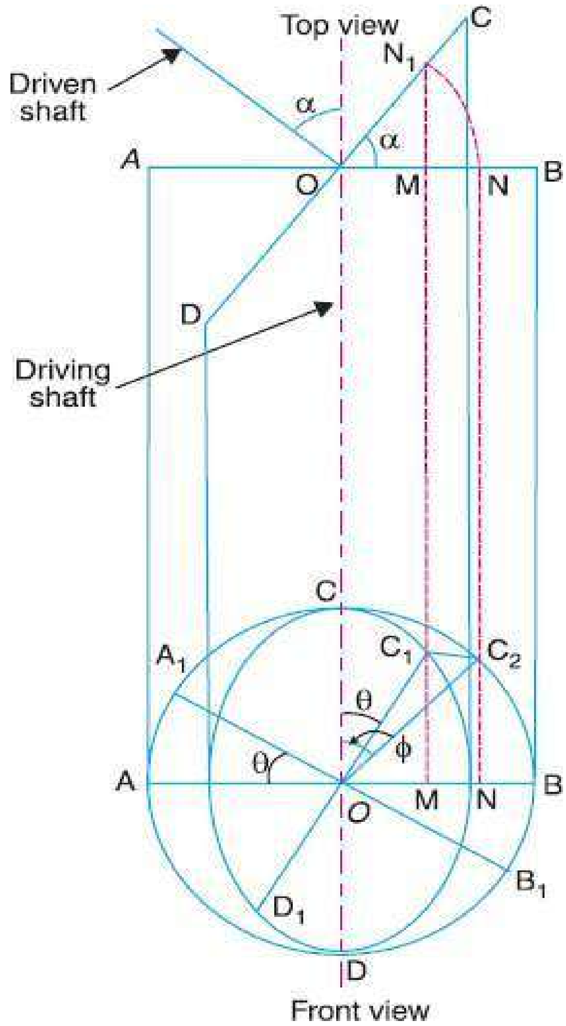


Fig. 2.14 ration of shaft velocities

- In triangle OC_1M, angle $OC_1M = \Theta$

$$\tan\theta = \frac{OM}{MC_1} \ldots\ldots\ldots\ldots..(i)$$

- In triangle OC_2N, angle $OC_2N = \emptyset$

$$\tan\emptyset = \frac{ON}{NC_2} = \frac{ON}{MC_1}\ldots\ldots..(ii) \quad (NC_2 = MC_1)$$

- Dividing eq. (i) by (ii)

$$\tan\emptyset = \frac{ON}{NC_2} = \frac{ON}{MC_1}$$

- But

  $OM = N_1 \cos\alpha = ON\cos\alpha$ (α = angle of inclination of driving and driven shaft)

$$\frac{\tan\theta}{\tan\emptyset} = \frac{ON\cos\alpha}{ON} = \cos\alpha$$

$$\tan\theta = \tan\emptyset \times \cos\alpha \ldots\ldots\ldots\ldots(iii)$$

- Let,

ω = angular velocity of driving shaft $= \frac{d\theta}{dt}$

ω_1 = angular velocity of driven shaft $= \frac{d\emptyset}{dt}$

- Differentiating both side of eq. (iii)

$$\sec^2\theta \times \frac{d\theta}{dt} = \cos\alpha \times \sec^2\emptyset \times \frac{\emptyset}{dt}$$

$$\sec^2\theta \times = \cos\alpha \times \sec^2\emptyset \times \omega_1$$

$$\frac{\omega_1}{\omega} = \frac{\sec^2\theta}{\cos\alpha \times \sec^2\emptyset}$$

$$= \frac{1}{\cos^2\theta \times \cos\alpha \times \sec^2\emptyset} \ldots\ldots.(iv)$$

- We know that,

$$\sec^2\emptyset = 1 + \tan^2\emptyset = 1 + \frac{\tan^2\theta}{\cos^2\alpha}$$

$$= 1 + \frac{\sin^2\theta}{\cos^2\theta \times \cos^2\alpha}$$

$$= \frac{\cos^2\theta \times \cos^2\alpha + \sin^2\theta}{\cos^2\theta \times \cos^2\alpha}$$

$$= \frac{\cos^2\theta \times (1 - \sin^2\alpha) + \sin^2\theta}{\cos^2\theta \times \cos^2\alpha}$$

$$= \frac{\cos^2\theta - \sin^2\alpha \times \cos^2\theta + \sin^2\theta}{\cos^2\theta \times \cos^2\alpha}$$

$$= \frac{1 - \sin^2\alpha \times \cos^2\theta}{\cos^2\theta \times \cos^2\alpha}$$

- Substituting this value in eq. (iv)

$$\frac{\omega_1}{\omega} = \frac{1}{\cos^2\theta \times \cos\alpha} \times \frac{\cos^2\theta \times \cos^2\alpha}{1 - \sin^2\alpha \times \cos^2\theta}$$

Maximum and Minimum speed of Driven Shaft

$$\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1 - \sin^2\alpha \times \cos^2\theta}$$

$$\omega_1 = \frac{\omega \times \cos\alpha}{1 - \sin^2\alpha \times \cos^2\theta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

- The value of ω_1 will be minimum for a given value of α, if the denominator of eq. (I) is minimum.

$\cos^2\theta = 1$, i.e. $\Theta = 0^o$, 180^o , 360^o etc.

- Maximum speed of the driven shaft,

$$\omega_{1(\max)} = \frac{\omega\cos\alpha}{1 - \sin^2\alpha} = \frac{\omega \times \cos\alpha}{\cos^2\alpha} = \frac{\omega}{\cos\alpha}$$

$$N_{1(\max)} = \frac{N}{\cos}$$

– Similarly, the value of ω_1 is minimum , if the denominator of eq. (i) is maximum, this will happen, when $(\sin^2 \alpha \times \cos^2 \theta)$ is maximum, or

$$\cos^2 \theta = 0, \text{ i.e. } \Theta = 90^\circ \ , 270^\circ \ \text{ etc.}$$

Polar diagram – salient features of driven shaftspeed

– For one complete revolution of the driven shaft, there are two points i.e. at 0° and 180° as shown by points 1 and 2 in Fig. Where the speed of the driven shaft is maximum and there are two points i.e. at 90° and 270° as shown by point 3 and 4 where the speed of the driven shaft is minimum.

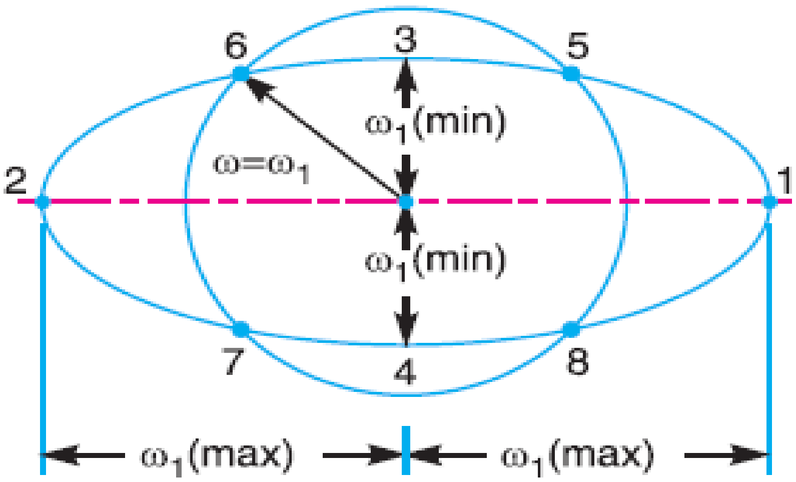


Fig. 2.15 polar diagram

– Since there are two maximum and two minimum speeds of the driven shaft, therefore there are four points when the speeds of the driven and driver shaft are same. This is shown by points, 5, 6, 7 and 8 in Fig.

– Since the angular velocity of the driving shaft is usually constant, therefore it is represented by a circle of radius ω. The driven shaft has a variation in angular velocity, the maximum value being ω/cos α and minimum value is ω cos α. Thus it is represented by an ellipse of semi-major axis ω/cos α and semi-minor axis ω cos α, as shown in Fig.2.15.

Double Hooke's Joint

– The velocity of the driven shaft is not constant, but varies from maximum to minimum values. In order to have a constant velocity ratio of the driving and driven shafts, an intermediate shaft with a Hooke‟s joint at each end as shown in Fig.2.16, is used. This type of joint is known as double Hooke‟s joint.

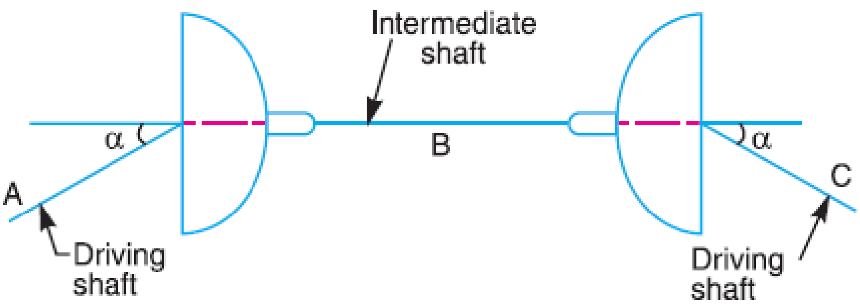


Fig. 2.16 double Hooke's joint

- For shaft A and B,

$$\tan\theta = \tan\emptyset \times \cos\alpha$$

- For shaft B and C,

$$\tan\gamma = \tan\emptyset \times \cos\alpha$$

- This shows that the speed of the driving and driven shaft is constant. In other words, this joint gives a velocity ratio equal to unity, if
  1. The axes of the driving and driven shafts are in the same plane, and
  2. The driving and driven shafts make equal angles with the intermediate shaft.

Examples:

1. In a Davis steering gear, the distance between the pivots of the front axle is 1.2 metres and the wheel base is 4.7 metres. Find the inclination of the track arm to the longitudinal axis of the car, when it is moving along a straight path .

- **Given:** c = 1.2 m ; b = 4.7 m
- Let, α = Inclination of the track arm to the longitudinal axis.
- We know that

$$\tan\alpha = \frac{C}{2\,b} = \frac{1.2}{2\times 4.7} = 0.222$$

$$= 14.5°$$

2. Two shafts with an included angle of 160° are connected by a Hooke's joint. The driving shaft runs at a uniform speed of 1500 r.p.m. The driven shaft carries a flywheel of mass 12 kg and 100 mm radius of gyration. Find the maximum angular acceleration of the driven shaft and the maximum torque required.

- **Given:** N = 1500 rpm ; m = 12 kg; k = 100 mm ; α = 20°
- We know that angular speed of driving shaft,

$$\omega = 2\,\pi\,\frac{1500}{60} = 157\,\frac{rad}{s}$$

- The mass moment of inertia of the driven shaft,

$$I = m \times K^2 = 12 \times 0.1^2 = 0.12\ kg.m$$

Max. angular acceleration of driven shaft,

$$\cos 2\theta = \frac{\sin^2 \alpha \times 2}{2 - \sin^2 \alpha} = \frac{\sin^2 20 \times 2}{2 - \sin^2 20} = 0.124$$

$$= 41.45°$$

$$\frac{d\omega_1}{dt} = \frac{\omega^2 \times \cos \alpha \times \sin 2\theta \times \sin^2 \alpha}{(1 - \sin^2 \alpha \times \cos^2 \theta)^2}$$

$$= \frac{157^2 \times \cos 20 \times \sin 84.9 \times \sin^2 20}{(1 - \sin^2 20 \times \cos^2 44.45)^2} = 3090\ \frac{rad}{s^2}$$

- Max torque req.

$$= I \times \frac{d\omega_1}{} = 0.12 \times 3090 = 371\ N.m$$

LECTURE 1

STRAIGHT LINE MOTION MECHANISMS

DEPARTMENT OF MECHANICAL ENGINEERING

STRAIGHT LINE MOTION MECHANISMS

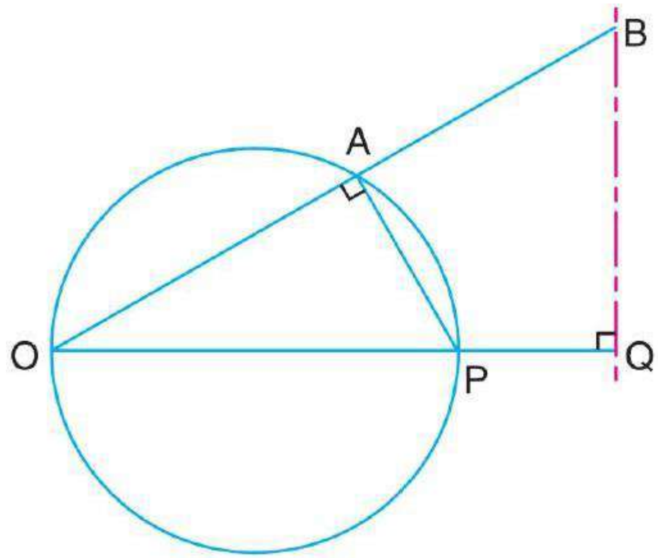
- One of the most common forms of the constraint mechanisms is that it permits only relative motion of an oscillatory nature along a straight line.
- The mechanisms used for this purpose are called straight line mechanisms. These mechanisms are of the following two types:
  - in which only turning pairs are used, an
  - in which one sliding pair is used.

These two types of mechanisms may produce **exact straight line motion or approximate straight line motion**, as discussed in the following articles.

EXACT STRAIGHT LINE MOTION MECHANISMS MADE UP OF TURNING PAIRS

- Let O be a point on the circumference of a circle of diameter OP.
- Let OA be any chord and B is a point on OA produced, such that,

$$OA \times OB = \text{constant}$$



Exact straight line motion mechanism

$$\frac{OA}{OP} = \frac{OQ}{OB}$$

$$OP \times OQ = OA \times OB$$

$$OQ = \frac{OA \times OB}{OP}$$

But OP is constant as it is the diameter of a circle, therefore, if OA×OB is constant, then OQ will be constant.
Hence the point B moves along the straight path BQ which is perpendicular to OP.

PEAUCELLIER MECHANISM

- It consists of a fixed link OO1 and the other straight links O1A , OC, OD, AD, DB, BC and CA are connected by turning pairs at their intersections, as shown in Fig.
- The pin at A is constrained to move along the circumference of a circle with the fixed diameter OP, by means of the link O1A .

$$AC = CB = BD = DA \; ; \; OC = OD \; ; \text{ and } OO_1 = O_1A$$

$$OC^2 = OR^2 + RC^2 \quad \ldots(i)$$

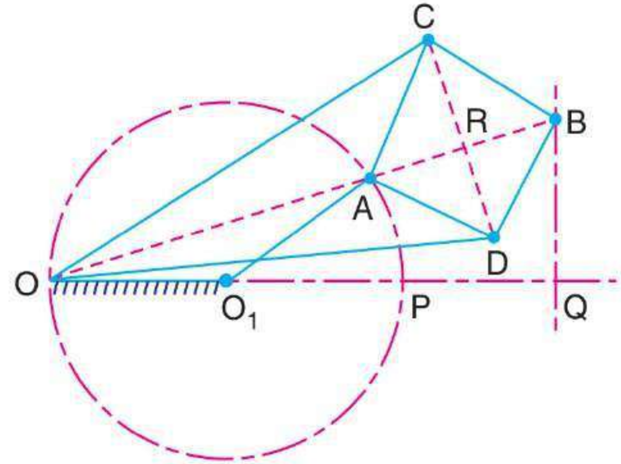
$$BC^2 = RB^2 + RC^2 \quad \ldots(ii)$$

Subtracting equation *(ii)* from *(i)*, we have

$$OC^2 - BC^2 = OR^2 - RB^2$$

$$= (OR + RB)\,(OR - RB)$$

$$= OB \times OA$$



HART'S MECHANISM

- This mechanism requires only six links as compared with the eight links required by the Peaucellier mechanism.
- It consists of a fixed link OO1 and other straight links O1A , FC, CD, DE and EF are connected by turning pairs at their points of intersection, as shown in Fig.
- The links FC and DE are equal in length and the lengths of the links CD and EF are also equal. The points O, A and B divide the links FC, CD and EF in the same ratio.
- A little consideration will show that BOCE is a trapezium and OA and OB are respectively parallel to *FD and CE.
- Hence OAB is a straight line. It may be proved now that the product OA× OB is constant.

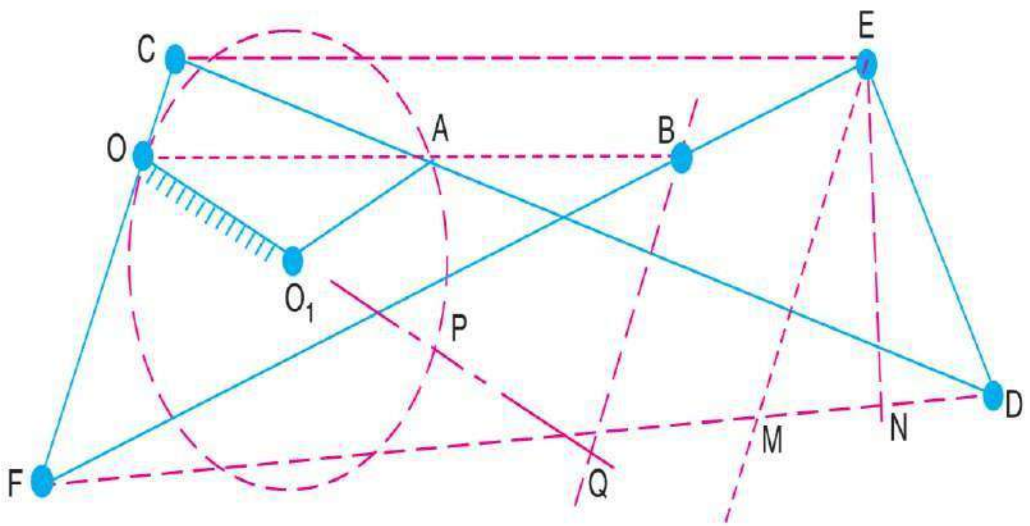
HART'S MECHANISM

From similar triangles CFE and OFB,

$$\frac{CE}{FC} = \frac{OB}{OF} \quad \text{or} \quad OB = \frac{CE \times OF}{FC}$$

and from similar triangles FCD and OCA

$$\frac{FD}{FC} = \frac{OA}{OC} \quad \text{or} \quad OA = \frac{FD \times OC}{FC}$$



It therefore follows that if the mechanism is pivoted about O as a fixed point and the point A is constrained to move on a circle with centre O1, then the point B will trace a straight line perpendicular to the diameter OP produced.

HART'S MECHANISM

Multiplying equations (*i*) and (*ii*), we have

$$OA \times OB = \frac{FD \times OC}{FC} \times \frac{CE \times OF}{FC} = FD \times CE \times \frac{OC \times OF}{FC^2}$$

Since the lengths of *OC*, *OF* and *FC* are fixed, therefore

$$OA \times OB = FD \times CE \times \text{constant} \quad \text{...(iii)}$$

$$\text{...}\left(\text{substituting } \frac{OC \times OF}{FC^2} = \text{constant}\right)$$

Now from point *E*, draw *EM* parallel to *CF* and *EN* perpendicular to *FD*. Therefore

$$FD \times CE = FD \times FM \quad \text{...}(\because CE = FM)$$

$$= (FN + ND)(FN - MN) = FN^2 - ND^2 \quad \text{...}(\because MN = ND)$$

$$= (FE^2 - NE^2) - (ED^2 - NE^2)$$

...(From right angled triangles *FEN* and *EDN*)

$$= FE^2 - ED^2 = \text{constant} \quad \text{...(iv)}$$

...($\because$ Length *FE* and *ED* are fixed)

From equations (*iii*) and (*iv*),

$$OA \times OB = \text{constant}$$

LECTURE 2

APPROXIMATE STRAIGHT LINE MECHANISM

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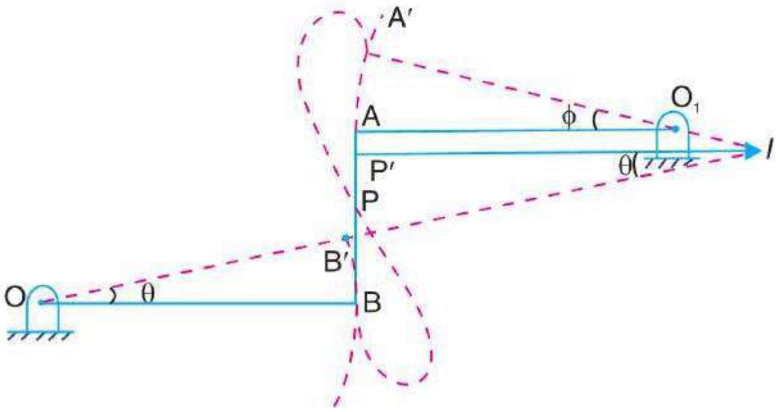
APPROXIMATE STRAIGHT LINE MOTION MECHANISMS

- The approximate straight line motion mechanisms are the modifications of the four-bar chain mechanisms. Following mechanisms to give approximate straight line motion, are important from the subject point of view:
- **Watt's mechanism:** It is a crossed four bar chain mechanism and was used by Watt for his early steam engines to guide the piston rod in a cylinder to have an approximate straight line motion.



$\text{arc } BB' = \text{arc } AA' \quad \text{or} \quad OB \times \theta = O_1A \times \phi$

$\therefore \quad OB / O_1A = \phi / \theta$

Also $\quad A'P' = IP' \times \phi, \text{ and } B'P' = IP' \times \theta$

$\therefore \quad A'P' / B'P' = \phi / \theta$

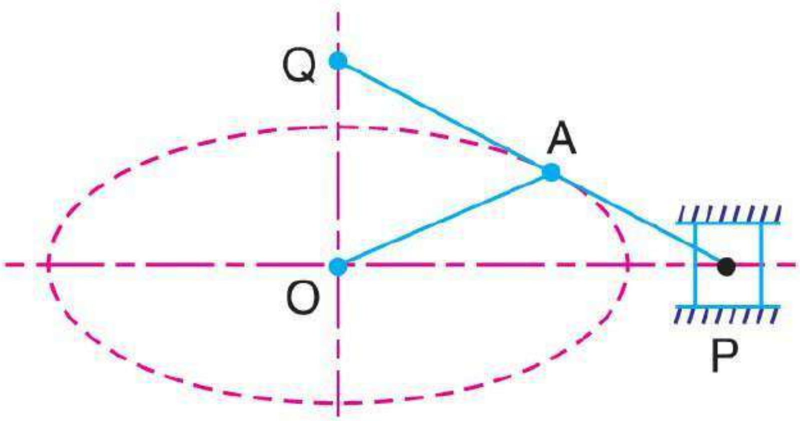
From equations (*i*) and (*ii*),

$$\frac{OB}{O_1A} = \frac{A'P'}{B'P'} = \frac{AP}{BP} \quad \text{or} \quad \frac{O_1A}{OB} = \frac{PB}{PA}$$

MODIFIED SCOTT-RUSSEL MECHANISM

- This mechanism is similar to Scott-Russel mechanism but in this case AP is not equal to AQ and the points P and Q are constrained to **move in the horizontal and vertical directions.**
- A little consideration will show that it forms an elliptical trammel, so that any point A on PQ traces an ellipse with semi-major axis AQ and semi-minor axis AP.

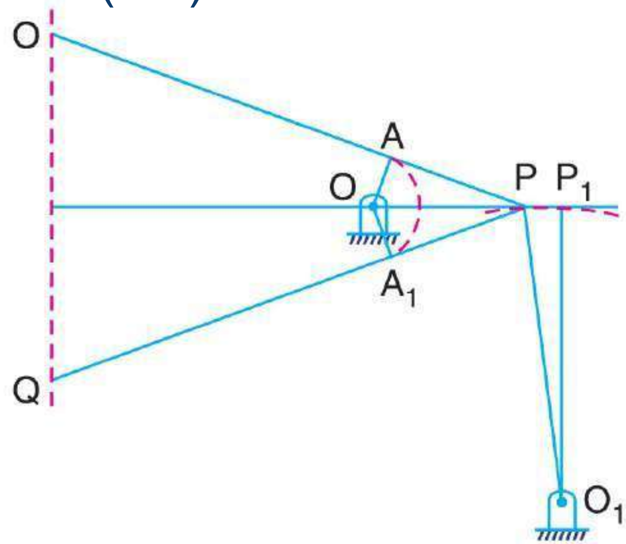
If the point A moves in a circle, then for point Q to move along an approximate straight line, the length OA must be equal (AP)2/AQ. This is limited to only small displacement of P.



Modified Scott-Russel mechanism

GRASSHOPPER MECHANISM

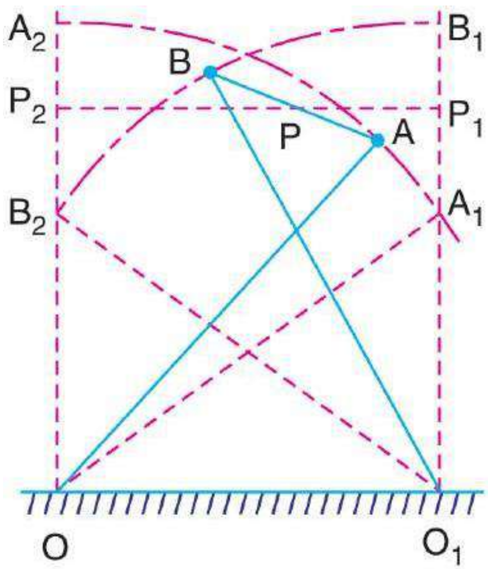
- This mechanism is a **modification of modified Scott-Russel's mechanism** with the difference that the point P does not slide along a straight line, but moves in a circular arc with centre O.
- It is a four bar mechanism and **all the pairs are turning pairs** as shown in Fig. In this mechanism, the centres O and O1 are fixed. The link OA oscillates about O through an angle AOA1 which causes the pin P to move along a circular arc with O1 as centre and O1P as radius. OA= (AP)2/ AQ.



TCHEBICHEFF'S MECHANISM

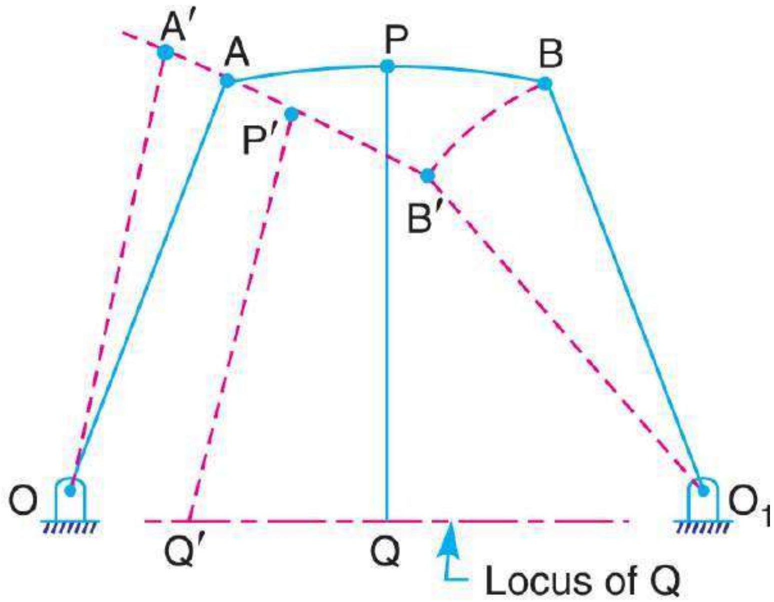
- It is a four bar mechanism in which the **crossed links OA and O1B are of equal length,** as shown in Fig. The point P, which is the mid-point of AB traces out an approximately straight line parallel to OO1.
- The proportions of the links are, usually, such that point P is exactly above O or O1 **in the extreme positions of the mechanism** i.e. when BA lies along OA or when BA lies along BO1.

It may be noted that the point P will lie on a straight line parallel to OO1, in the two extreme positions and in the mid position, if the **lengths of the links are in proportions AB: OO1: OA= 1 : 2 : 2.5.**



ROBERTS MECHANISM

- **It is also a four bar chain mechanism,** which, in its mean position, has the form of a trapezium.
- The links OA and O1 Bare of equal length and OO1 is fixed. A bar PQ is rigidly attached to the link AB at its middle point P.



LECTURE 3

PANTOGRAPH

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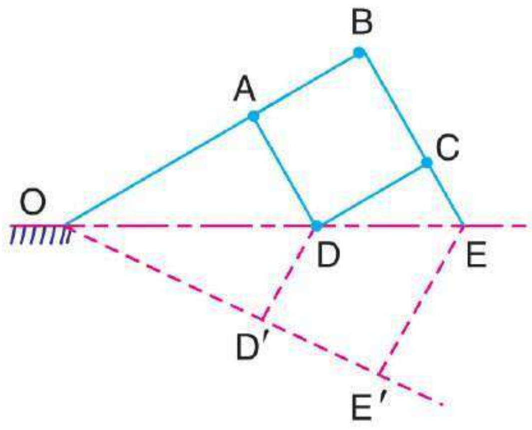
PANTOGRAPH

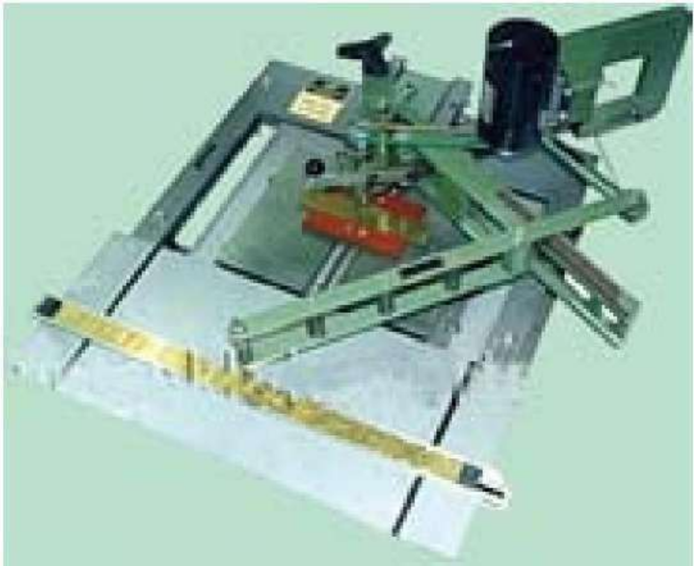
- A pantograph is an **instrument used to reproduce to an enlarged or a reduced scale** and as exactly as possible the path described by a given point.
- It consists of a jointed parallelogram ABCD as shown in Fig. It is made up of bars **connected by turning pairs.** The bars BA and BC are extended to O and E respectively, such that $OA/OB = AD/BE$
  - Thus, for all relative positions of the bars, the triangles OAD and OBE are similar and the points O, D and E are in one straight line.
  - It may be proved that point E traces out the same path as described by point D.



PANTOGRAPH

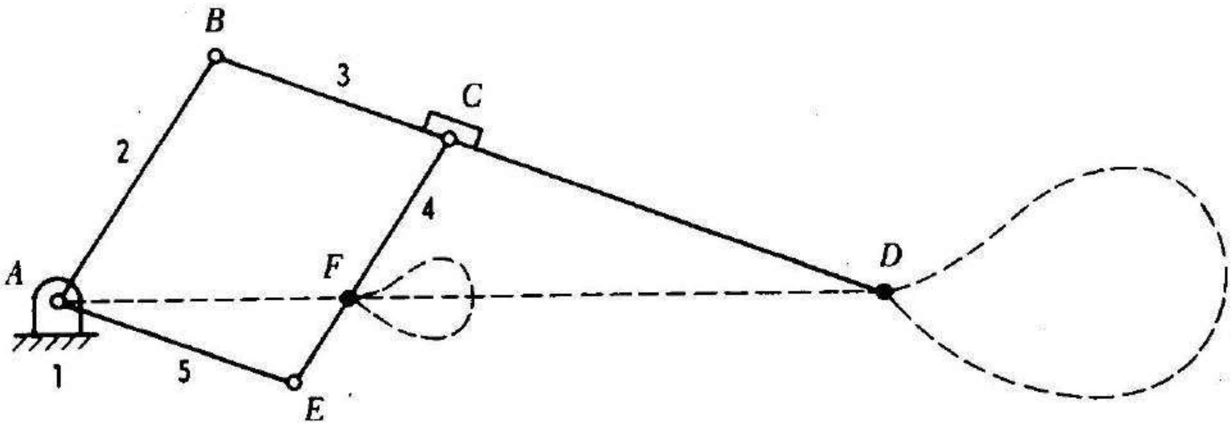
- From similar triangles OAD and OBE, we find that, $OD/OE = AD/BE$

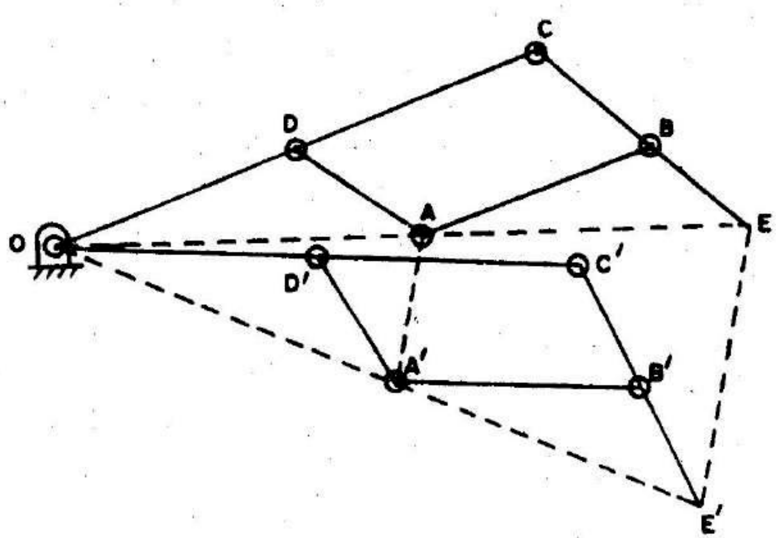
Let point O be fixed and the points D and E move to some new positions D′ and E′. Then $OD/OE = OD'/OE'$



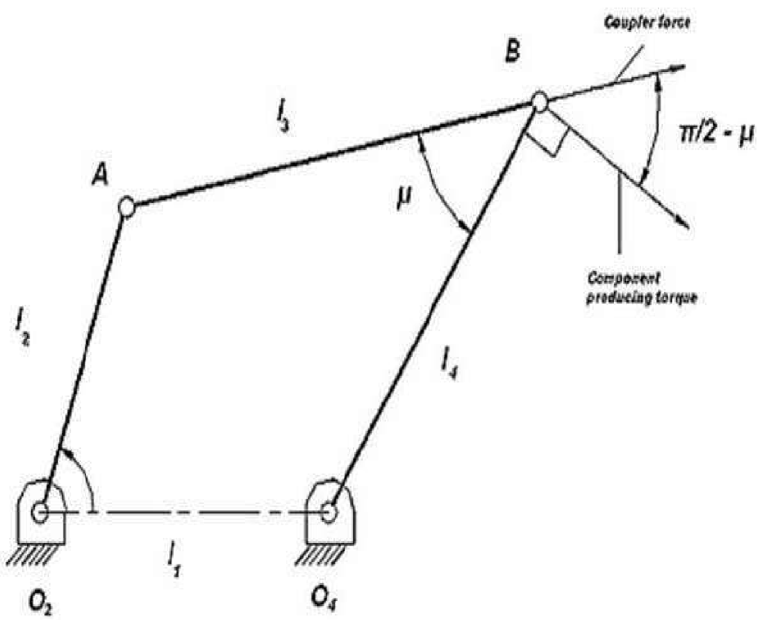
- A pantograph is mostly used for the reproduction of **plane areas and figures such as maps, plans etc., on enlarged or reduced scales.**
- It is, sometimes, used as an indicator rig in order to reproduce to a **small scale the displacement of the crosshead and therefore of the piston of a reciprocating steam engine.** It is also used to guide cutting tools.
- A modified form of pantograph is used to collect power at the top of an electric locomotive.

PANTOGRAPH





TRANSMISSION ANGLE



For a 4 R linkage, the transmission angle (μ) is defined as the acute angle between the coupler (AB) and the follower

For a given force in the coupler link, the torque transmitted to the output bar (about point O_4) is maximum when the angle μ between coupler bar AB and output bar BO_4 is π/2. Therefore, angle ABO_4 is called **transmission angle.**

LECTURE 4

DAVI'S STEERING GEAR MECHANISM

LIVE TO LEARN & LEARN TO SHARE

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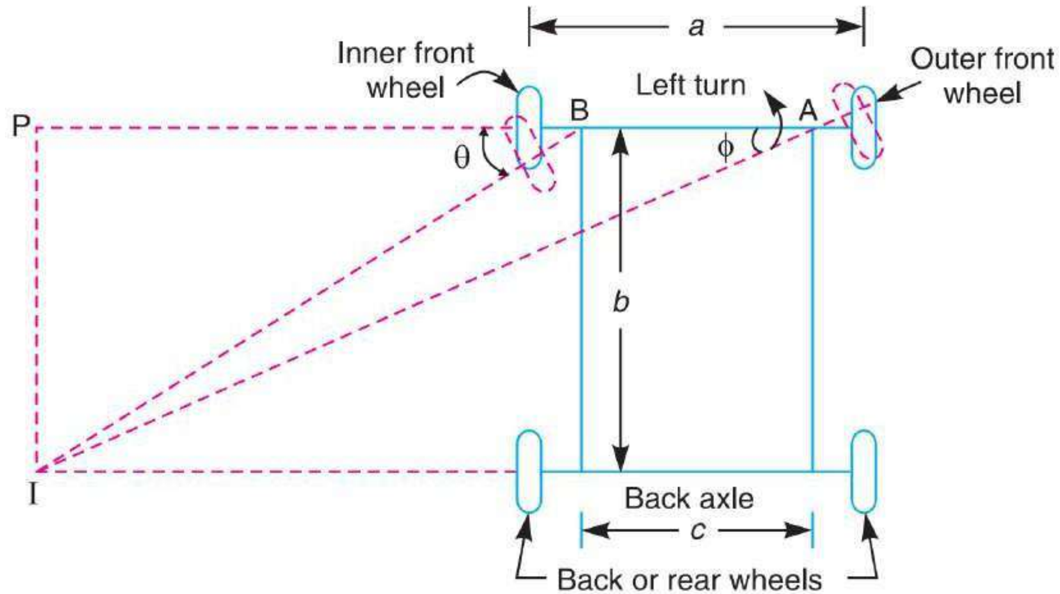
STEERING GEAR MECHANISM

- The steering gear mechanism is used for changing the direction of two or more of the **wheel axles with reference to the chassis, so as to move the automobile in any desired path.**
- Usually the **two back wheels have a common axis, which is fixed in direction with reference to the chassis** and the steering is done by means of the front wheels.



STEERING GEAR MECHANISM

Thus, the condition for correct steering is that all the four wheels **must turn about the same instantaneous centre.** The axis of the inner wheel makes a larger turning angle θ than the angle φ subtended by the axis of outer wheel.

Let a = Wheel track,

b = Wheel base, and

c = Distance between the pivots A and B of the front axle.

Now from triangle IBP,

$$\cot\theta = \frac{BP}{IP}$$

and from triangle IAP,

$$\cot\phi = \frac{AP}{IP} = \frac{AB + BP}{IP} = \frac{AB}{IP} + \frac{BP}{IP} = \frac{c}{b} + \cot\theta \qquad ...(\because IP = b)$$

$$\therefore \cot\phi - \cot\theta = c/b$$

This is the **fundamental equation for correct steering.** If this condition is satisfied, there will be no skidding of the wheels, when the vehicle takes a turn.

DAVIS STEERING GEAR

- It is an **exact steering gear mechanism**. The slotted links AM and BH are attached to the front wheel axle, which turn on pivots A and B respectively.
- The rod CD is constrained to move in the direction of its length, by the sliding members at P and Q. These **constraints are connected to the slotted link** AM and BH by a sliding and a turning pair at each end.

a = Vertical distance between AB and CD,

b = Wheel base,

d = Horizontal distance between AC and BD,

c = Distance between the pivots A and B of the front axle.

x = Distance moved by AC to $AC' = CC' = DD'$, and

α = Angle of inclination of the links AC and BD, to the vertical.

DAVIS STEERING GEAR

From triangle $A A' C'$,

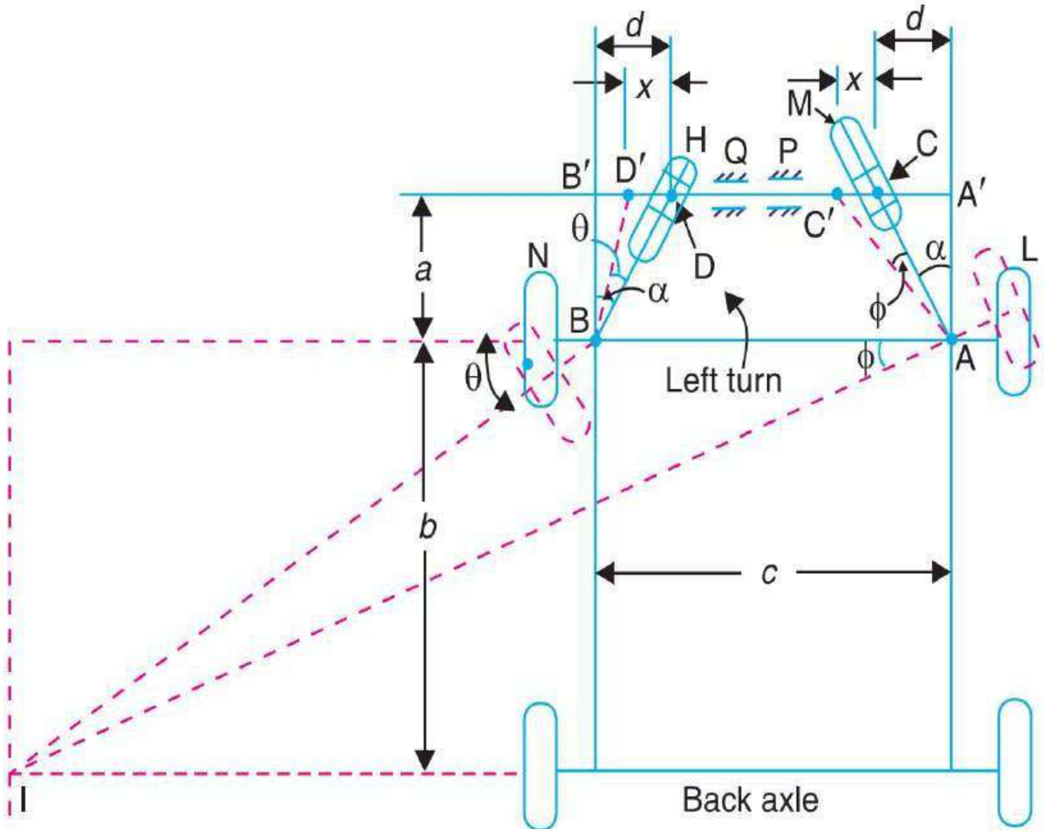
$$\tan(\alpha + \phi) = \frac{A'C'}{AA'} = \frac{d + x}{a}$$

From triangle $A A' C$,

$$\tan \alpha = \frac{A'C}{AA'} = \frac{d}{a}$$

From triangle $B B' D'$,

$$\tan(\alpha - \theta) = \frac{B'D'}{BB'} = \frac{d - x}{a}$$



We know that $\tan(\alpha + \phi) = \dfrac{\tan\alpha + \tan\phi}{1 - \tan\alpha . \tan\phi}$

or

$$\frac{d + x}{a} = \frac{d/a + \tan\phi}{1 - d/a \times \tan\phi} = \frac{d + a\tan\phi}{a - d\tan\phi}$$

...[From equations (*i*) and (*ii*)]

DAVIS STEERING GEAR

$$(d + x)\,(a - d\tan\phi) = a\,(d + a\tan\phi)$$

$$a.\,d - d^2\tan\phi + a.\,x - d.x\tan\phi = a.d + a^2\tan\phi$$

$$\tan\phi\,(a^2 + d^2 + d.x) = ax \quad \text{or} \quad \tan\phi = \frac{a.x}{a^2 + d^2 + d.x} \qquad \ldots(iv)$$

Similarly, from $\tan(\alpha - \theta) = \dfrac{d - x}{a}$, we get

$$\tan\theta = \frac{ax}{a^2 + d^2 - d.x} \qquad \ldots(v)$$

We know that for correct steering,

$$\cot\phi - \cot\theta = \frac{c}{b} \quad \text{or} \quad \frac{1}{\tan\phi} - \frac{1}{\tan\theta} = \frac{c}{b}$$

$$\frac{a^2 + d^2 + d.x}{a.x} - \frac{a^2 + d^2 - d.x}{a.x} = \frac{c}{b} \qquad \ldots\text{[From equations } (iv) \text{ and } (v)\text{]}$$

$$\frac{2d.x}{a.x} = \frac{c}{b} \quad \text{or} \quad \frac{2d}{a} = \frac{c}{b}$$

$$2\tan\alpha = \frac{c}{b} \quad \text{or} \quad \tan\alpha = \frac{c}{2b} \qquad \ldots(\because d/a = \tan\alpha)$$

LECTURE 5

ACKERMAN'S STEERING GEAR MECHANISM

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ACKERMAN'S STEERING GEAR MECHANISM

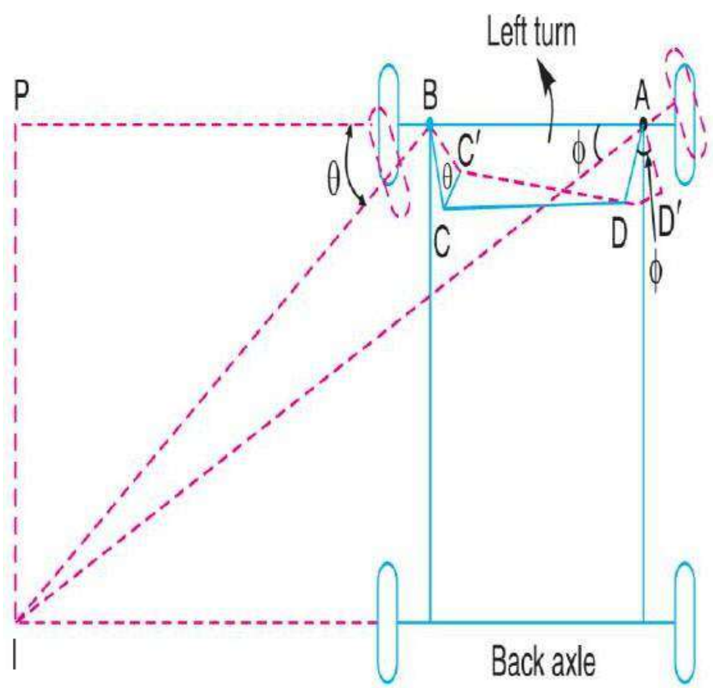
- The Ackerman steering gear **mechanism is much simpler than Davis gear.** The difference between the Ackerman and Davis steering gears are:
- The whole mechanism of the **Ackerman steering gear is on back of the front wheels; whereas in Davis steering gear, it is in front of the wheels.**
- The Ackerman **steering gear consists of turning pairs,** whereas Davis steering gear consists of sliding members.
- The **shorter links BC and AD are of equal length** and are connected by hinge joints with front wheel axles.
- The **longer links AB and CD are of unequal length.**

ACKERMAN’S STEERING GEAR MECHANISM

1. When the vehicle moves along a straight path, the **longer links AB and CD are parallel and the shorter links BC and AD are equally inclined** to the longitudinal axis of the vehicle, as shown by firm lines in Fig.

2. When the vehicle is steering to the left, the position of the gear is shown by dotted lines in Fig. In this position, the lines of the **front wheel axle intersect on the back wheel axle at I, for correct steering.**

3. When the vehicle is **steering to the right, the similar position may be obtained.**



LECTURE 6

SINGLE AND DOUBLE HOOKE JOINT

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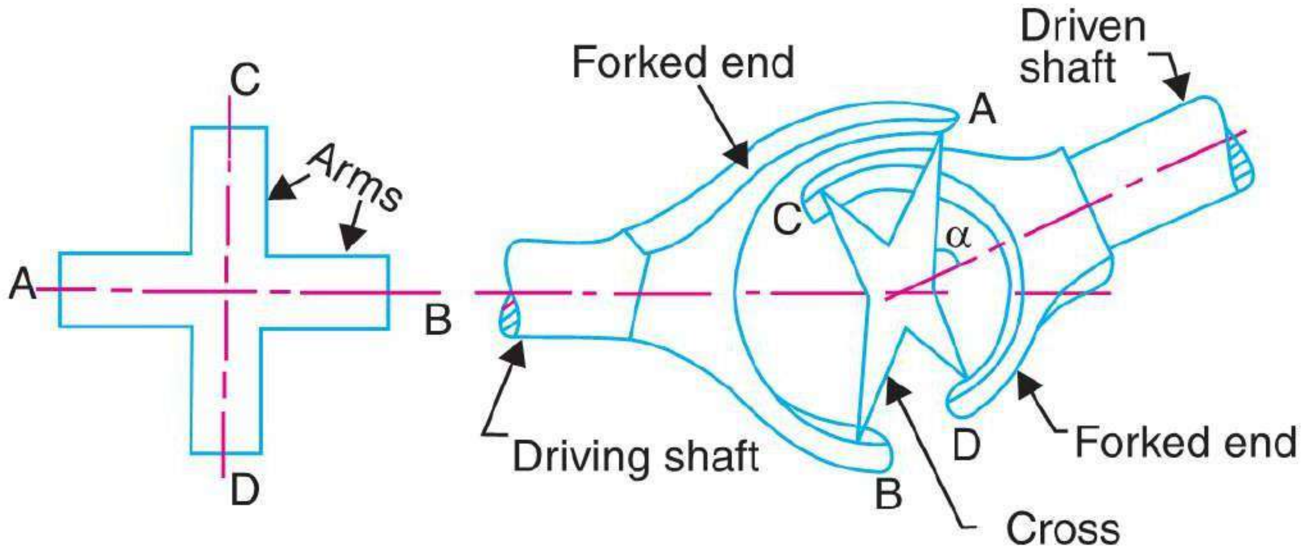
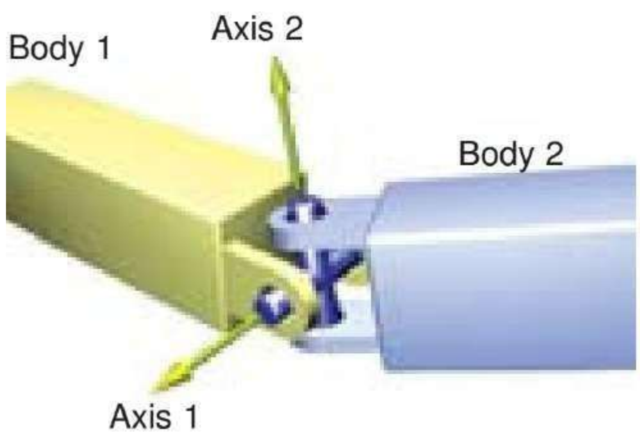
UNIVERSAL OR HOOKE'S JOINT

- A Hooke's joint is **used to connect two shafts**, which are intersecting at a small angle, as shown in Fig.
- The end of each shaft is **forked to U-type and each fork provides two bearings for the arms of a cross**. The arms of the cross are perpendicular to each other.
- The motion is transmitted from the driving shaft to driven shaft through a cross. **The inclination of the two shafts may be constant**, but in actual practice it varies, when the motion is transmitted.
- The main application of the **Universal or Hooke's joint is found in the transmission from the gear box to the differential or back axle** of the automobiles.
- It is also used for transmission of power to **different spindles of multiple drilling machine**. It is also used as a knee joint in milling machines.

UNIVERSAL OR HOOKE'S JOINT





UNIVERSAL OR HOOKE'S JOINT

- The arms of the cross are perpendicular to each other. The motion is **transmitted from the driving shaft to driven shaft** through a cross. The **inclination of the two shafts may be constant,** but in actual practice it varies, when the motion is transmitted.
- The main application of the Universal or Hooke's joint is found in the **transmission from the gear box to the differential or back axle of the automobiles.**
- It is also used for **transmission of power to different spindles of multiple drilling machine.** It is also used as a knee joint in milling machines.

LECTURE 7

RATIO OF SHAFT VELOCITIES

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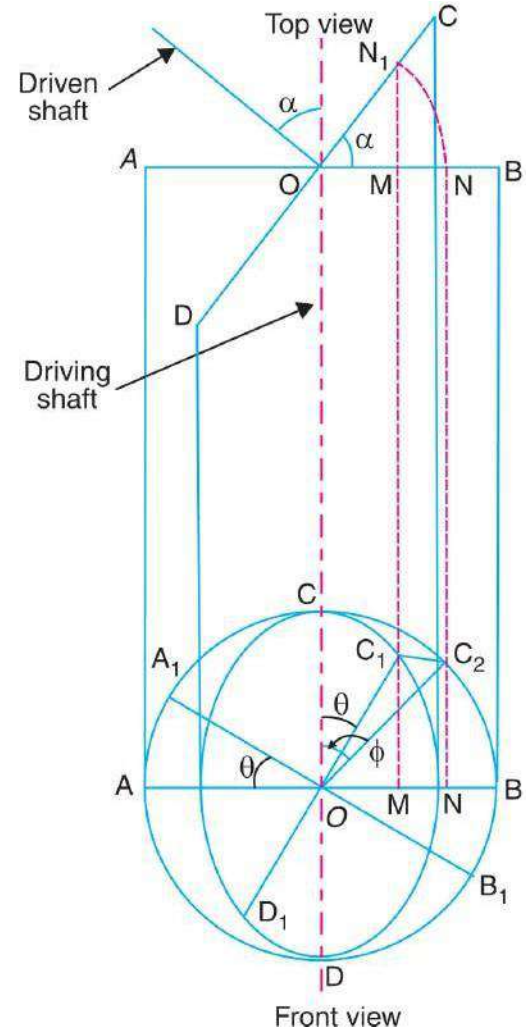
RATIO OF SHAFT VELOCITIES

- The top and front views connecting the two shafts by a universal joint are shown in Fig. Let the initial position of the cross be such that both arms lie in the plane of the paper in front view, while the arm AB attached to the driving shaft lies in the plane containing the axes of the two shafts.
- Let the driving shaft rotates through an angle θ, so that the arm AB moves in a circle to a new position A1B1 as shown in front view.
- A little consideration will show that the arm CD will also move in a circle of the same size. This circle when projected in the plane of paper appears to be an ellipse.



RATIO OF SHAFT VELOCITIES

- Therefore the arm CD takes new position C1D1 on the ellipse, at an angle θ. But the true angle must be on the circular path.
- To find the true angle, project the point C1 horizontally to intersect the circle at C2.
- Thus when the driving shaft turns through an angle θ, the driven shaft turns through an angle φ.

In triangle OC_1M, $\angle OC_1M = \theta$

$$\therefore \quad \tan\theta = \frac{OM}{MC_1} \quad \ldots(i)$$

and in triangle OC_2N, $\angle OC_2N = \phi$

$$\therefore \quad \tan\phi = \frac{ON}{NC_2} = \frac{ON}{MC_1}$$

Dividing equation (*i*) by (*ii*),

$$\frac{\tan\theta}{\tan\phi} = \frac{OM}{MC_1} \times \frac{MC_1}{ON} = \frac{OM}{ON}$$

But $\quad OM = ON_1 \cos\alpha = ON \cos\alpha$

RATIO OF SHAFT VELOCITIES

$$\therefore \quad \frac{\tan\theta}{\tan\phi} = \frac{ON\cos\alpha}{ON} = \cos\alpha$$

$$\tan\theta = \tan\phi \,.\, \cos\alpha$$

Let ω = Angular velocity of the driving shaft = $d\theta / dt$

ω_1 = Angular velocity of the driven shaft = $d\phi / dt$

Differentiating both sides of equation *(iii)*,

$$\sec^2\theta \times d\theta / dt = \cos\alpha \,.\, \sec^2\phi \times d\phi / dt$$

$$\sec^2\theta \times \omega = \cos\alpha \,.\, \sec^2\phi \times \omega_1$$

$$\therefore \quad \frac{\omega_1}{\omega} = \frac{\sec^2\theta}{\cos\alpha.\sec^2\phi} = \frac{1}{\cos^2\theta.\cos\alpha.\sec^2\phi}$$

RATIO OF SHAFT VELOCITIES

We know that $\sec^2\phi = 1 + \tan^2\phi = 1 + \dfrac{\tan^2\theta}{\cos^2\alpha}$ ...[From equation (*iii*)]

$$= 1 + \frac{\sin^2\theta}{\cos^2\theta.\cos^2\alpha} = \frac{\cos^2\theta.\cos^2\alpha + \sin^2\theta}{\cos^2\theta.\cos^2\alpha}$$

$$= \frac{\cos^2\theta(1-\sin^2\alpha) + \sin^2\theta}{\cos^2\theta.\cos^2\alpha} = \frac{\cos^2\theta - \cos^2\theta.\sin^2\alpha + \sin^2\theta}{\cos^2\theta.\cos^2\alpha}$$

$$= \frac{1 - \cos^2\theta.\sin^2\alpha}{\cos^2\theta.\cos^2\alpha} \quad \text{...}(\because \cos^2\theta + \sin^2\theta = 1)$$

Substituting this value of $\sec^2\phi$ in equation (*iv*), we have veloity ratio,

$$\frac{\omega_1}{\omega} = \frac{1}{\cos^2\theta.\cos\alpha} \times \frac{\cos^2\theta.\cos^2\alpha}{1 - \cos^2\theta.\sin^2\alpha} = \frac{\cos\alpha}{1 - \cos^2\theta.\sin^2\alpha} \quad \text{...}(v)$$

e: If N = Speed of the driving shaft in r.p.m., and

N_1 = Speed of the driven shaft in r.p.m.

Then the equation (*v*) may also be written as

$$\frac{N_1}{N} = \frac{\cos\alpha}{1 - \cos^2\theta.\sin^2\alpha}.$$

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Industry applications

1. An Evans' linkage has an oscillating drive arm that should have a maximum operating angle of about 40. For a relatively short guideway, the reciprocating output stroke is large. Output motion is on a true straight line in true harmonic motion. If an exact straight-line motion is not required, a link can replace the slide. The longer this link, the closer the output motion approaches that of a true straight line. If the link-length equals the output stroke, deviation from straight-line motion is only 0.03% of the output stroke.
2. A simplified Watt's linkage generates an approximate straight-line motion. If the two arms are of equal length, the tracing point describes a symmetrical figure 8 with an almost straight line throughout the stroke length. The straightest and longest stroke occurs when the connecting-link length is about two thirds of the stroke, and arm length is 1.5 times the stroke length.
3. Four-bar linkage produces an approximately straight-line motion. This arrangement provides motion for the stylus on self-registering measuring instruments. A comparatively small drive displacement results in a long, almost-straight line.
4. A D-drive is the result when linkage arms are arranged as shown here. The output link point describes a path that resembles the letter D, so there is a straight part of its cycle. This motion is ideal for quick engagement and disengagement before and after a straight driving stroke.

Question Bank for Assignments

1. Draw a neat sketch and explain Peaucellier's exact straight line mechanism.
2. Explain Hart's straight line mechanism in detail.
3. Draw a neat sketch and explain any three approximate straight line generating mechanisms.
4. With a neat sketch, explain the Ackermann steering gear of an automobile.
5. With a neat sketch, explain the Davis steering gear mechanism in detail.
6. Two shafts are connected by universal Hooke's joint. The driving shaft rotates at uniform speed of 1200 rpm. Determine the greatest permissible angle between the shaft axis so that the total fluctuation of speed does not exceed 100 rpm also calculate the maximum and minimum speeds of driven shaft.
7. Derive an expression for the ratio of shafts velocities for Hooke's joint and draw the polar diagram.

Tutorial Questions

1. Sketch a pantograph, explain its working and show that it can be used to reproduce to an enlarged scale a given figure.

2. A circle has OR as its diameter and a point Q lies on its circumference. Another point P lies on the line OQ produced. If OQ turns about O as centre and the product OQ× OP remains constant, show that the point P moves along a straight line perpendicular to the diameter OR.

3. What are straight line mechanisms? Describe one type of exact straight line motion mechanism with the help of a sketch.

4. Describe the Watt's parallel mechanism for straight line motion and derive the condition under which the straight line is traced.

5. Sketch an intermittent motion mechanism and explain its practical applications.

6. Give a neat sketch of the straight line motion 'Hart mechanism' Prove that it produces an exact straight line motion.

10. What is the condition for correct steering? Sketch and show the two main types of steering gears and discuss their relative advantages.

11. Explain why two Hooke's joints are used to transmit motion from the engine to the differential of an automobile.

UNIT 3

VELOCITY AND ACCELERATION

COURSE OBJECTIVE

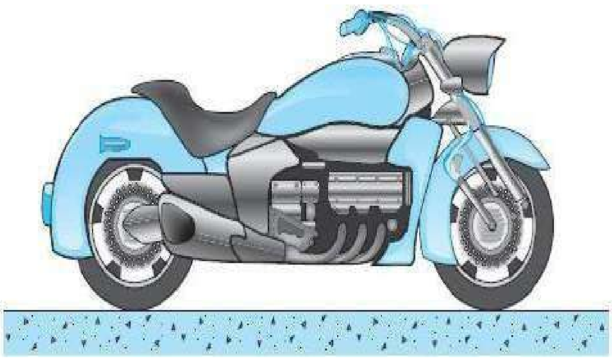
To impart skills to analyze the position, velocity and acceleration of mechanisms and synthesis of mechanism by analytical and graphical method.

COURSE OUTCOME

LECTURE	LECTURE TOPIC	KEY ELEMENTS	LEARNING OBJECTIVES
1	**Motion of link in machine**	Introduction to motion of link Methods to determine the motion of the link	Analyse the motion of the link (B4) Remember the types of motion (B1)
2	Velocity and acceleration diagrams	Relative velocity method Instantaneous Center Method	Remember the expressions for velocity and accelerations (B1) Calculate the Instantaneous centres (B4)
3	Graphical method	Velocity and acceleration on a Link by Relative Velocity Method Rubbing Velocity at a Pin Joint	Understanding different types of graphical method for velocity and acceleration calculation (B2) Apply graphical method for various methods (B3)
4	Relative velocity method four bar chain	Numerical examples to estimate the velocity and acceleration using relative velocity method	Apply relative velocity method to estimate the velocity and acceleration for four bar mechanisms (B3)
5	Instantaneous centre of rotation	Definition of instantaneous centre of rotation Types of instantaneous centre of rotation	Understanding the Instantaneous axis (B2) Compare the two components of acceleration (B1)
6	Three centers in line theorem	Aronhold Kennedy Theorem	Understanding the Three centers in line theorem (B2) Locate the instantaneous centres by Aronhold Kennedy's theorem (B5)
7	Graphical determination of instantaneous center	Number of Instantaneous Centres in a Mechanism Numerical Examples using instantaneous centre of rotation	Evaluate instantaneous centers of the slider crank mechanism (B5) Apply graphical method for Instantaneous Centres (B3)

3

Velocity and Acceleration Analysis



Course Contents

Introduction

- There are many methods for determining the velocity of any point on a link in a mechanism whose direction of motion (i.e. path) and velocity of some other point on the same link is known in magnitude and direction, yet the following two methods :
    1. Instantaneous centre method
    2. Relative velocity method
- The instantaneous centre method is convenient and easy to apply in simple mechanisms, whereas the relative velocity method may be used to any configuration diagram.

Velocity of Two Bodies Moving In Straight Lines

- Here we shall discuss the application of vectors for the relative velocity of two bodies moving along parallel lines and inclined lines, as shown in Fig. 3.1 (a) and 3.2 (a) respectively.
- Consider two bodies A and B moving along parallel lines in the same direction with absolute velocities v_A and v_B such that $v_A > v_B$, as shown in Fig. 3.1 (a). The relative velocity of A with respect to B,

$$v_{AB} = \text{vector difference of } v_A \text{ and } v_B = \vec{v_A} - \vec{v_B}$$

- From Fig. 3.1 (b), the relative velocity of A with respect to B (i.e. v_{AB}) may be written in the vector form as follows :

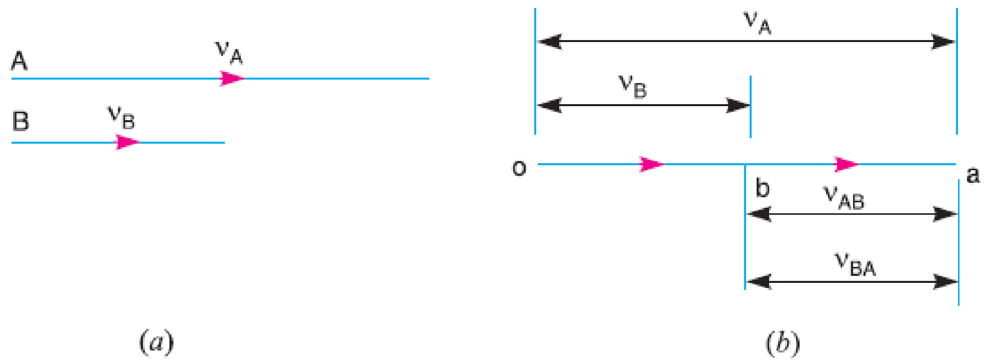


Fig. 3.1 relative velocity of two bodies moving along parallel line

- Similarly, the relative velocity of B with respect to A,

$$v_{AB} = \text{vector difference of } v_A \text{ and } v_B$$

- Now consider the body B moving in an inclined direction as shown in Fig. 3.2 (a). The relative velocity of A with respect to B may be obtained by the law of parallelogram of velocities or triangle law of velocities. Take any fixed point o and draw vector oa to represent v_A in magnitude and direction to some suitable scale. Similarly, draw vector ob to represent v_B in magnitude and direction to the same scale. Then vector ba represents the relative velocity of A with respect to B as shown in Fig. 3.2 (b). In the similar way as discussed above, the relative velocity of A with respect to B,

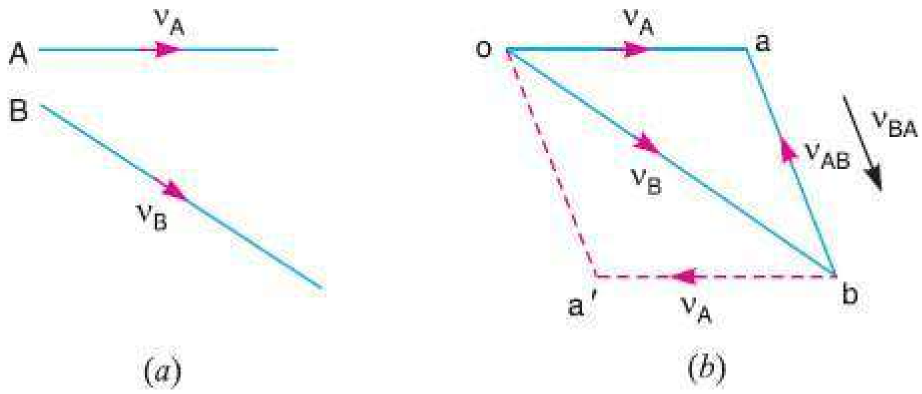


Fig. 3.2 relative velocity of two bodies moving along inclined line

$$v_{AB} = vector\ differece\ of\ v_A\ and\ v_B$$

- Similarly, the relative velocity of B with respect to A

$$v_{BA} = vector\ differece\ of\ v_B\ and\ v_A$$

- From above, we conclude that the relative velocity of a point A with respect to B (v_{AB}) and the relative velocity of point B with respect to A (v_{BA}) are equal in magnitude but opposite in direction

$$v_{AB} = -v_{BA}$$

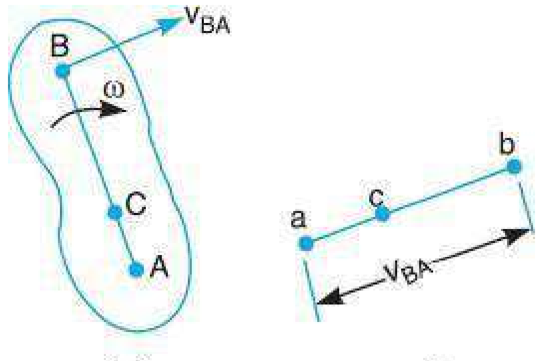
Motion Of A Link

- Consider two points A and B on a rigid link A B, as shown in Fig. 3.3 (a). Let one of the extremities (B) of the link move relative to A, in a clockwise direction. Since the distance from A to B remains the same, therefore there can be no relative motion between A and B, along the line AB. It is thus obvious, that the relative motion of B with respect to A must be perpendicular to AB.
- Hence velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.
- The relative velocity of B with respect to A (i.e. v_{BA}) is represented by the vector ab and is perpendicular to the line A B as shown in Fig. 3.3 (b).
- We know that the velocity of the point B with respect to A

$$v_{BA} = \omega \times AB \ldots\ldots \ldots \ldots \ldots\ldots \ldots \ldots .. (i)$$

- Similarly the velocity of the point C on AB with respect to A

$$v_{CA} = \omega \times AC \ldots \ldots \ldots \ldots \ldots .. \ldots \ldots \ldots (ii)$$



(a) (b)

Fig. 3.3 Motion of a Link

– Form equation (i) and (ii),

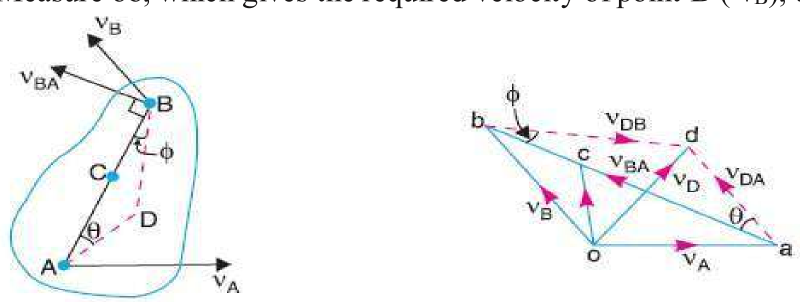
$$\frac{v_{CA}}{v_{BA}} = \frac{\omega \times AC}{\omega \times AB} = \frac{AC}{AB} \ldots \ldots \ldots \ldots \ldots \ldots \ldots (iii)$$

– Thus, we see from equation (iii), that the point c on the vector ab divides it in the same ratio as C divides the link AB.

Velocity of A Point On A Link By Relative Velocity Method

– Consider two points A and B on a link as shown in Fig. 3.4 (a). Let the absolute velocity of the point A i.e. v_A is known in magnitude and direction and the absolute velocity of the point B i.e. v_B is known in direction only. Then the velocity of B may be determined by drawing the velocity diagram as shown in Fig. 3.4 (b). The velocity diagram is drawn as follows :

1. Take some convenient point o, known as the pole.
2. Through o, draw oa parallel and equal to v_A, to some suitable scale.
3. Through a, draw a line perpendicular to AB of Fig. 3.4 (a). This line will represent the velocity of B with respect to A, i.e. v_{BA}.
4. Through o, draw a line parallel to v_B intersecting the line of v_{BA} at b
5. Measure ob, which gives the required velocity of point B (v_B), to the scale



(a) Motion of points on a link. (b) Velocity diagram.

Fig. 3.4

Velocities In Slider Crank Mechanism

- In the previous article, we have discussed the relative velocity method for the velocity of any point on a link, whose direction of motion and velocity of some other point on the same link is known. The same method may also be applied for the velocities in a slider crank mechanism.
- A slider crank mechanism is shown in Fig. 3.5 (a). The slider A is attached to the connecting rod AB. Let the radius of crank OB be r and let it rotates in a clockwise direction, about the point O with uniform angular velocity ω rad/s. Therefore, the velocity of B i.e. v_B is known in magnitude and direction. The slider reciprocates along the line of stroke AO.

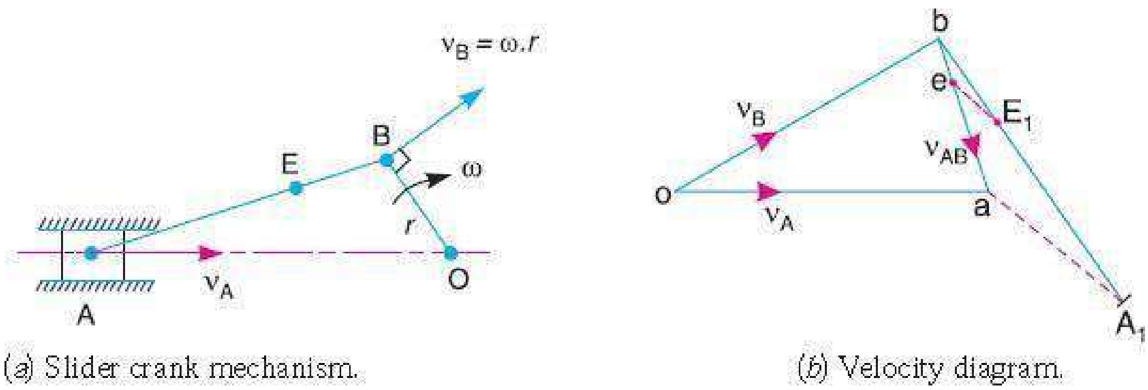


Fig. 3.5

- The velocity of the slider A (i.e. v_A) may be determined by relative velocity method as discussed below :
  1. From any point o, draw vector ob parallel to the direction of v_B (or perpendicular to OB) such that ob = v_B = ω.r, to some suitable scale, as shown in Fig. 3.5 (b).
  2. Since AB is a rigid link, therefore the velocity of A relative to B is perpendicular to AB. Now draw vector ba perpendicular to AB to represent the velocity of A with respect to B i.e. v_{AB}.
  3. From point o, draw vector oa parallel to the path of motion of the slider A (which is along AO only). The vectors ba and oa intersect at a. Now oa represents the velocity of the slider I.e. v_A, to the scale.
- The angular velocity of the connecting rod A B (ω_{AB}) may be determined as follows:

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB}$$

Rubbing Velocity at A Pin Joint

- The links in a mechanism are mostly connected by means of pin joints. The rubbing velocity is defined as the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of the pin.
- Consider two links OA and OB connected by a pin joint at O as shown in fig.

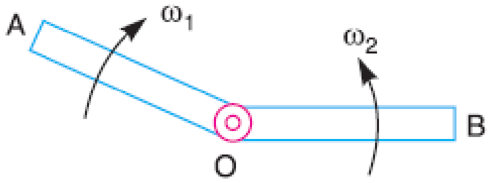


Fig. 3.6 Links connected by pin joints

- Let,
  $\omega 1$ = angular velocity of link OA
  $\omega 2$ = angular velocity of link OB
- According to the definition,
- Rubbing velocity at the pin joint O

$$= (\omega_1 - \omega_1) \times r \text{ if the links move in the same direction}$$
$$= (\omega_1 + \omega_1) \times r \text{ if the links move in the same direction}$$

Examples Based On Velocity

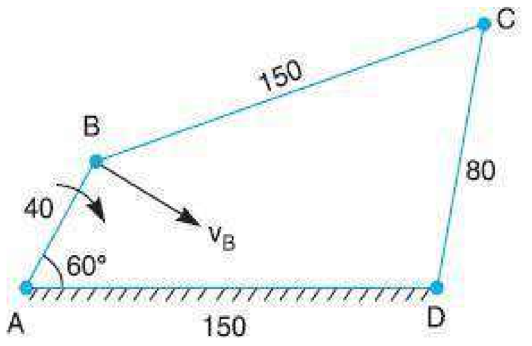
In a four bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40 mm long and rotates at 120 r.p.m. clockwise, while the link CD = 80 mm oscillates about D. BC and AD are of equal length. Find the angular velocity of link CD when angle BAD = 60°.

- **Given :** N_{BA} = 120 r.p.m. or $\omega_{BA} = 2\pi \times 120/60 = 12.568$ rad/s
- Since the length of crank A B = 40 mm = 0.04 m, therefore velocity of B with respect to A or velocity of B, (because A is a fixed point),
- Since the length of crank A B = 40 mm = 0.04 m, therefore velocity of B with respect to A or velocity of B, (because A is a fixed point),

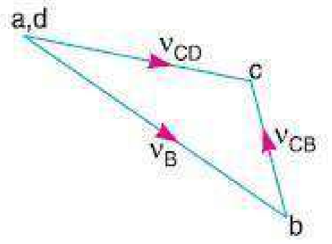
$$v_{BA} = v_B = \omega_{BA} \times AB = 12.568 \times 0.04 = 0.503 \text{ m/s}$$

- Since the link AD is fixed, therefore points a and d are taken as one point in the velocity diagram. Draw vector ab perpendicular to B A, to some suitable scale, to represent the velocity of B with respect to A or simply velocity of B (i.e. v_{BA} or v_B) such that

$$\text{Vector } ab = v_{BA} = v_B = 0.503 \text{ m/s}$$



(a) Space diagram (All dimensions in mm).



(b) Velocity diagram.

Fig. 3.7

- Now from point b, draw vector bc perpendicular to CB to represent the velocity of C with respect to B (i.e. v_{CB}) and from point d, draw vector dc perpendicular to CD to represent the velocity of C with respect to D or simply velocity of C (i.e. v_{CD} or v_C). The vectors bc and dc intersect at c.

  By measurement, we find that

$$V_{CD} = v_C = \text{vector dc} = 0.385 \text{ m/s}$$

- Angular velocity of link CD,

$$\omega_{CD} = \frac{v_{CD}}{CD} = \frac{0.385}{0.08} = 4.8 \ rad/s$$

The crank and connecting rod of a theoretical steam engine are 0.5 m and 2 m long respectively. The crank makes 180 r.p.m. in the clockwise direction. When it has turned 45° from the inner dead centre position, determine:

1. Velocity of piston, 2. Angular velocity of connecting rod, 3. Velocity of point E on the connecting rod 1.5 m from the gudgeon pin, 4. velocities of rubbing at the pins of the crank shaft, crank and crosshead when the diameters of their pins are 50 mm, 60 mm and 30 mm respectively, 5. Position and linear velocity of any point G on the connecting rod which has the least velocity relative to crank shaft.

- **Given:**
- N_{BO} = 180 r.p.m. or $\omega_{BO} = 2\ \pi \times 180/60 = 18.852$ rad/s
- Since the crank length *OB* = 0.5 m, therefore linear velocity of *B* with respect to *O* or velocity of *B* (because *O* is a fixed point),

$$v_{BO} = v_B = \omega_{BO} \times OB = 18.852 \times 0.5 = 9.426 \text{ m/s}$$

- First of all draw the space diagram and then draw the velocity diagram as shown in fig.

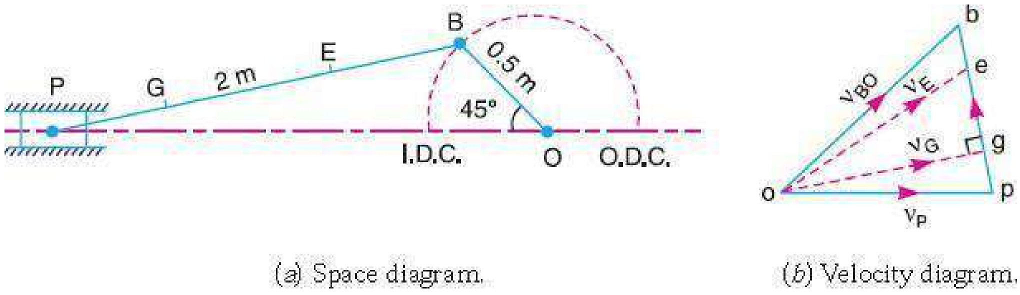


Fig. 3.8

- By measurement, we find that velocity of piston P,

$$v_P = vector\ op = 8.15\ m/s$$

- From the velocity diagram, we find that the velocity of P with respect to B

$$v_{PB} = \text{vector } bp = 6.8 \text{ m/s}$$

- Since the length of connecting rod PB is 2 m, therefore angular velocity of the connecting rod,

$$\omega_{PB} = \frac{v_{PB}}{PB} = \frac{6.8}{2} = 3.4\ rad/s$$

$$v_E = vector\ oe = 8.5\ m/s$$

- We know that velocity of rubbing at the pin of crank-shaft

$$= \frac{d_0}{2} \times \omega_{BO} = 0.47\ m/s$$

- Velocity of rubbing at the pin of crank

$$= \frac{d_B}{2}(\omega_{BO} + \omega_{PB}) = 0.6675\ m/s$$

- Velocity of rubbing at the pin of crank

$$= \frac{d_c}{2} \times \omega_{PB} = 0.051\ m/s$$

- By measurement we find that

$$vector\ bg = 5\ m/s$$

- By measurement we find linear velocity of point G

$$v_G = vector\ og = 8\ m/s$$

In Fig. , the angular velocity of the crank OA is 600 r.p.m. Determine the linear velocity of the slider D and the angular velocity of the link BD, when the crank is inclined at an angle of 75° to the vertical. The dimensions of various links are: OA = 28 mm; AB = 44 mm; BC 49 mm; and BD = 46 mm. The centre distance between the canters of rotation O and C is 65 mm. The path of travel of the slider is 11 mm below the fixed point C. The slider moves along a horizontal path and OC is vertical.

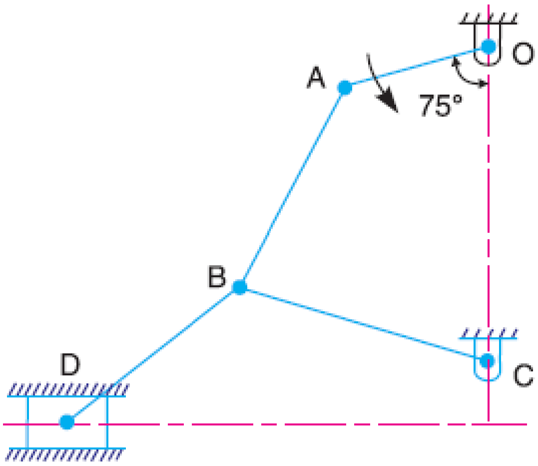


Fig. 3.9

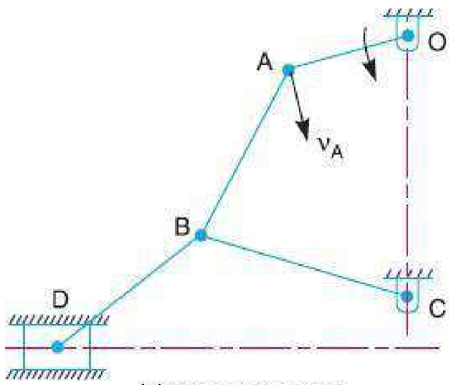
- **Given :**
- N_{AO} = 180 r.p.m. or $\omega_{BO} = 2\pi \times 180/60 = 18.852$ rad/s
- OA = 28 mm

$$v_{OA} = v_A = \omega_{AO} \times AO = 1.76\ m/s$$

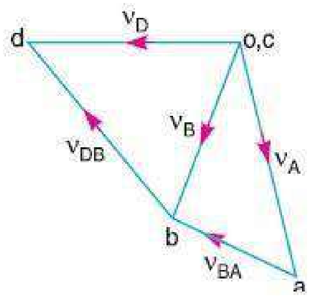
- Since the points O and C are fixed, therefore these points are marked as one point, in the velocity diagram. Now from point o, draw vector oa perpendicular to O A, to some suitable scale, to represent the velocity of A with respect to O or simply velocity of A such that

$$vector\ oa = v_{OA} = v_A = 1.76\ m/s$$

- From point a, draw vector ab perpendicular to A B to represent the velocity of B with respect A (i.e. v_{BA}) and from point c, draw vector cb perpendicular to CB to represent the velocity of B with respect to C or simply velocity of B (i.e. v_{BC} or v_B). The vectors ab and cb intersect at b.
- From point b, draw vector bd perpendicular to BD to represent the velocity of D with respect to B (i.e. v_{DB}) and from point o, draw vector od parallel to the path of motion of the slider D which is horizontal, to represent the velocity of D (i.e. v_D). The vectors bd and od intersect at d.



(a) Space diagram.



(b) Velocity diagram.

Fig.3.10

- By measurement, we find that velocity of slider D,

$$v_D = vector\ od = 1.6\ m/s$$

- By measurement from velocity diagram, we find that velocity of D with respect to B,

$$v_{DB} = vector\ bd = 1.7\ m/s$$

- Therefore angular velocity of link BD

$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{1.7}{0.046} = 36.96\ rad/s$$

The mechanism, as shown in Fig. 3.11, has the dimensions of various links as follows: AB = DE = 150 mm; BC = CD = 450 mm; EF = 375 mm. The crank AB makes an angle of 45° with the horizontal and rotates about A in the clockwise direction at a uniform speed of 120 r.p.m. The lever DC oscillates about the fixed point D, which is connected to AB by the coupler BC. The block F moves in the horizontal guides, being driven by the link EF. Determine: 1. velocity of the block F, 2. angular velocity of DC, and 3. rubbing speed at the pin C which is 50 mm in diameter.

- Given :
- N_{BA} = 120 r.p.m. or $\omega_{BA} = 2\pi \times 120/60 = 4\pi$ rad/s
- Since the crank length A B = 150 mm = 0.15 m, therefore velocity of B with respect to A or simply velocity of B (because A is a fixed point),

$$v_{BA} = v_B = \omega_{BA} \times AB = 4\pi \times 0.15 = 1.885\ m/s$$

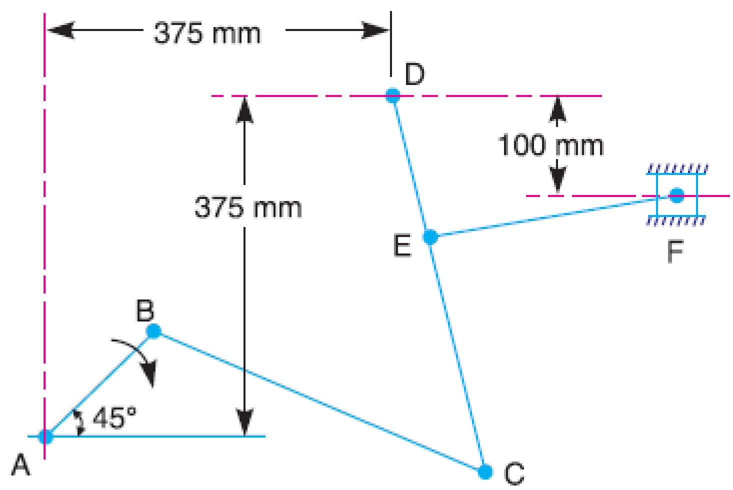
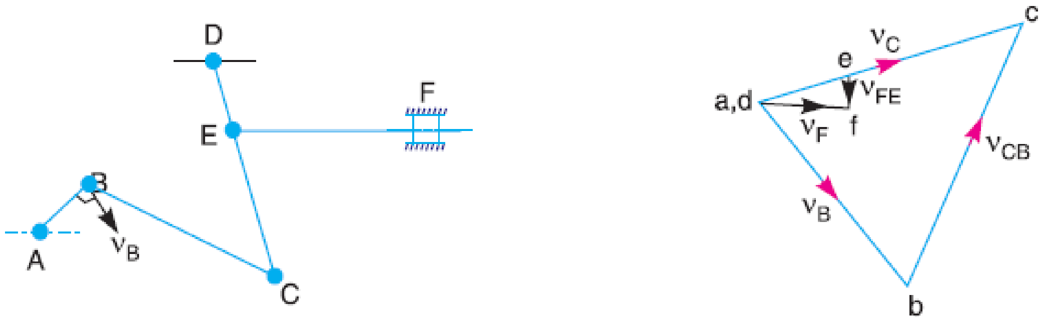


Fig.3.11

- Since the points A and D are fixed, therefore these points are marked as one point as shown in Fig. (b). Now from point a, draw vector ab perpendicular to A B, to some suitable scale, to represent the velocity of B with respect to A or simply velocity of B, such that

Vector ab = $v_{BA} = v_B$ = 1.885 m/s

- The point C moves relative to B and D, therefore draw vector bc perpendicular to BC to represent the velocity of C with respect to B (i.e. v_{CB}), and from point d, draw vector dc perpendicular to DC to represent the velocity of C with respect to D or simply velocity of C (i.e. v_{CD} or v_C). The vectors bc and dc intersect at c.



(a) Space diagram. (b) Velocity diagram.

Fig. 3.12

- Since the point E lies on DC, therefore divide vector dc in e in the same ratio as E divides CD in Fig. (a). In other words

ce/cd = CE/CD

- From point e, draw vector ef perpendicular to EF to represent the velocity of F with respect to E (i.e. v_{FE}) and from point d draw vector df parallel to the path of motion of F, which is horizontal, to represent the velocity of F i.e. v_F. The vectors ef and df intersect at f.

v_F = vector df = 0.7 m/s

- By measurement from velocity diagram, we find that velocity of C with respect to D,

v_{CD} = vector *dc* = 2.25 m/s

$$\omega_{DC} = \frac{v_{CD}}{DC} = 5\ \frac{rad}{s}$$

- From velocity diagram, we find that velocity of C with respect to B,

v_{CB} = vector *bc* = 2.25 m/s

- Angular velocity of BC,

$$\omega_{CD} = \frac{v_{CD}}{BC} = \frac{2.25}{0.45} = 5\ rad/s$$

Velocity Of A Point On A Link By Instantaneous Centre Method

- The instantaneous centre method of analyzing the motion in a mechanism is based upon the concept that any displacement of a body (or a rigid link) having motion in one plane, can be considered as a pure rotational motion of a rigid link as a whole about some centre, known as instantaneous centre or virtual centre of rotation.

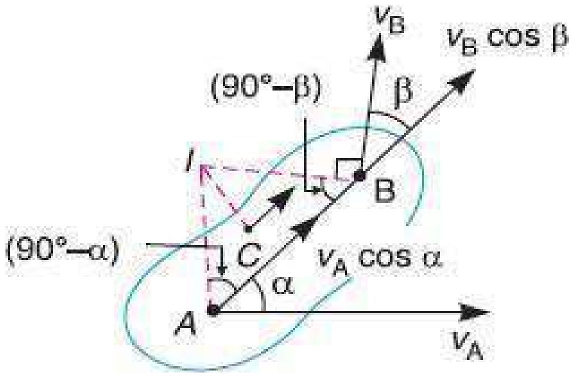


Fig. 3.13 velocity of a point on a link

- The velocities of points A and B, whose directions are given a link.by angles α and β as shown in Fig. If v_A is known in magnitude and direction and v_B in direction only, then the magnitude of v_B may be determined by the instantaneous centre method as discussed below :
- Draw AI and BI perpendiculars to the directions v_A and v_B respectively. Let these lines intersect at I, which is known as instantaneous centre or virtual centre of the link. The complete rigid link is to rotate or turn about the centre I.
- Since A and B are the points on a rigid link, therefore there cannot be any relative motion between them along line AB.
- Now resolving the velocities along AB,

$$v_A \times \cos \alpha = v_B \times \cos \beta$$

$$\frac{v_A}{v_B} = \frac{\cos \beta}{\cos \alpha} = \frac{\sin(90-\beta)}{\sin(90-\alpha)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (i)$$

- Applying Lami"s theorem to triangle ABI,

$$\frac{AI}{\sin(90-\beta)} = \frac{BI}{\sin(90-\alpha)}$$

$$\frac{AI}{BI} = \frac{\sin(90-\beta)}{\sin(90-\alpha)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

- Hence,

$$\frac{v_A}{v_B} = \frac{AI}{BI}$$

$$\frac{v_A}{AI} = \frac{v_B}{BI} = \omega \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (iii)$$

- If C is any other point on link, then

$$\frac{v_A}{AI} = \frac{v_B}{BI} = \frac{v_C}{CI} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (iv)$$

Properties of Instantaneous Method

- The following properties of instantaneous centre are important :
  1. A rigid link rotates instantaneously relative to another link at the instantaneous centre for the configuration of the mechanism considered.
  2. The two rigid links have no linear velocity relative to each other at the instantaneous centre. At this point (i.e. instantaneous centre), the two rigid links have the same linear velocity relative to the third rigid link. In other words, the velocity of the instantaneous centre relative to any third rigid link will be same whether the instantaneous centre is regarded as a point on the first rigid link or on the second rigid link

Number of Instantaneous Centre in A Mechanism:

- The number of instantaneous centres in a constrained kinematic chain is equal to the number of possible combinations of two links. The number of pairs of links or the number 3 of instantaneous centres is the number of combinations of n links taken two at a time. Mathematically, number of instantaneous centres

$$N = \frac{n\,(n-1)}{2}, \text{where } n = \text{Number of Link}$$

Location of Instantaneous centres:

- The following rules may be used in locating the instantaneous centres in a mechanism :
  1. When the two links are connected by a pin joint (or pivot joint), the instantaneous centre lies on the centre of the pin as shown in Fig. (a). such an instantaneous centre is of permanent nature, but if one of the links is fixed, the instantaneous centre will be of fixed type.
  2. When the two links have a pure rolling contact (i.e. link 2 rolls without slipping upon the fixed link 1 which may be straight or curved), the instantaneous centre lies on their point of contact, as shown in Fig.(b). The velocity of any point A on the link 2 relative to fixed link 1 will be perpendicular to I_{12} A and is proportional to I_{12} A.
  3. When the two links have a sliding contact, the instantaneous centre lies on the common normal at the point of contact. We shall consider the following three cases :
  a. When the link 2 (slider) moves on fixed link 1 having straight surface as shown in Fig.(c), the instantaneous centre lies at infinity and each point on the slider have the same velocity.

b. When the link 2 (slider) moves on fixed link 1 having curved surface as shown in Fig.(d),the instantaneous centre lies on the centre of curvature of the curvilinear path in the configuration at that instant.

c. When the link 2 (slider) moves on fixed link 1 having constant radius of curvature as shown in Fig. 3.14 (e), the instantaneous centre lies at the centre of curvature i.e. the centre of the circle, for all configuration of the links.

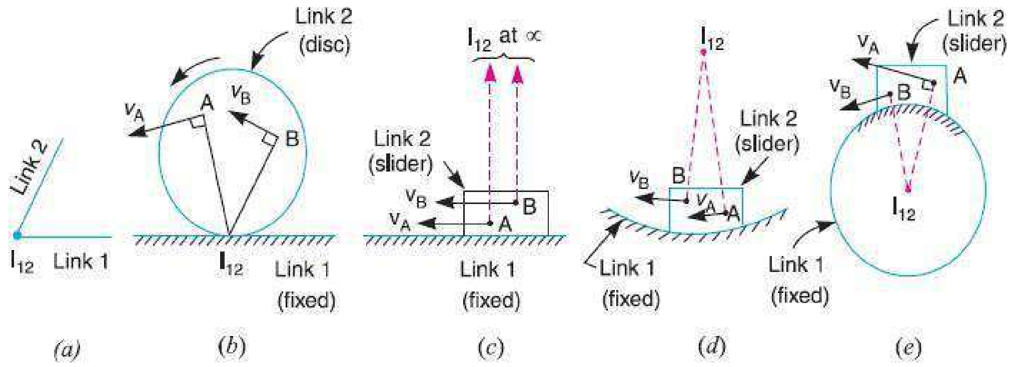


Fig. 3.14 Location of Instantaneous centres

Kennedy's Theorem

– The Aronhold Kennedy"s theorem states that "if three bodies move relatively to each other, they have three instantaneous centres and lie on a straight line."

– Consider three kinematic links A, B and C having relative plane motion. The number of instantaneous centres (N) is given by

$$N = \frac{n\,(n-1)}{2} = \frac{3\,(3-1)}{2} = 3$$

– The two instantaneous centres at the pin joints of B with A , and C with A (i.e. I_{ab} and I_{ac}) are the permanent instantaneous centre According to Aronhold Kennedy"s theorem, the third instantaneous centre I_{bc} must lie on the line joining I_{ab} and I_{ac}. In order to prove this let us consider that the instantaneous centre I_{bc} lies outside the line joining I_{ab} and I_{ac} as shown in Fig. The point I_{bc} belongs to both the links B and C. Let us consider the point I_{bc} on the link B. Its velocity v_{BC} must be perpendicular to the line joining I_{ab} and I_{bc}. Now consider the point I_{bc} on the link C. Its velocity v_{BC} must be perpendicular to the line joining I_{ac} and I_{bc}.

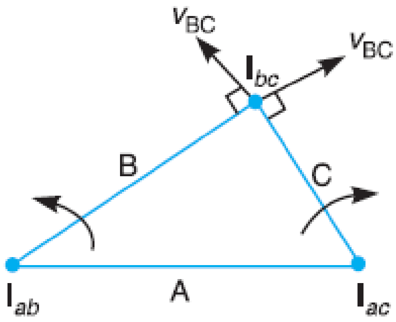


Fig. 3.15 Aronhold Kennedy's theorem

– We have already discussed that the velocity of the instantaneous centre is same whether it is regarded as a point on the first link or as a point on the second link. Therefore, the velocity of the point I_{bc} cannot be perpendicular to both lines I_{ab} I_{bc} and I_{ac} I_{bc} unless the point Ibc lies on the line joining the points I_{ab} and I_{ac}. Thus the three instantaneous centres (I_{ab}, I_{ac} and I_{bc}) must lie on the same straight line. The exact location of Ibc on line I_{ab} I_{ac} depends upon the directions and magnitudes of the angular velocities of B and C relative to A.

Acceleration Diagram for a Link

– Consider two points A and B on a rigid link as shown in Fig. (a). Let the point B moves with respect to A, with an angular velocity of **ω** rad/s and let **α** rad/s^2 be the angular acceleration of the link AB.

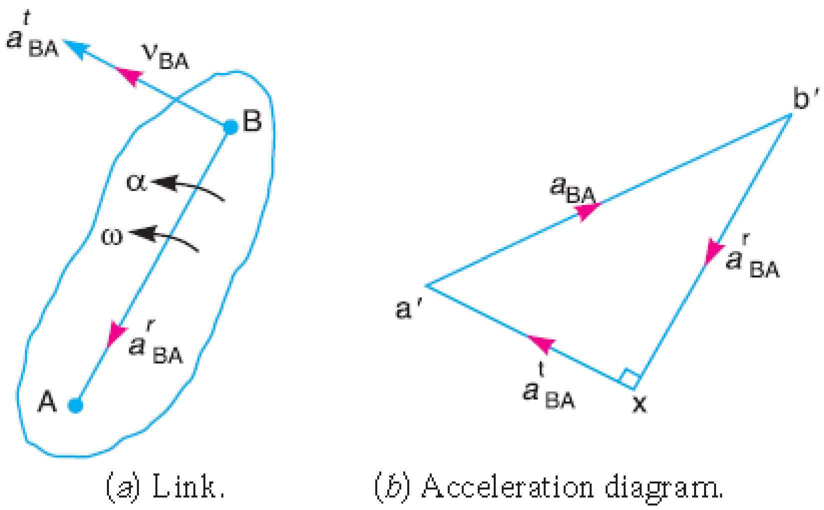


Fig. 3.16 Acceleration of a link

– We have already discussed that acceleration of a particle whose velocity changes both in magnitude and direction at any instant has the following two components.

1 The centripetal or radial component, which is perpendicular to the velocity of the particle at the given instant.

2 The tangential component, which is parallel to the velocity of the particle at the given instant.

- Thus for a link A B, the velocity of point B with respect to A (i.e. vBA) is perpendicular to the link A B as shown in Fig.(a). Since the point B moves with respect to A with an angular velocity of **ω** rad/s, therefore centripetal or radial component of the
  acceleration of B with respect to A

$$a^r_{BA} = \omega^2 \times Length\ of\ link\ AB = \omega^2 \times AB = {v^2_{BA}}/{AB}$$

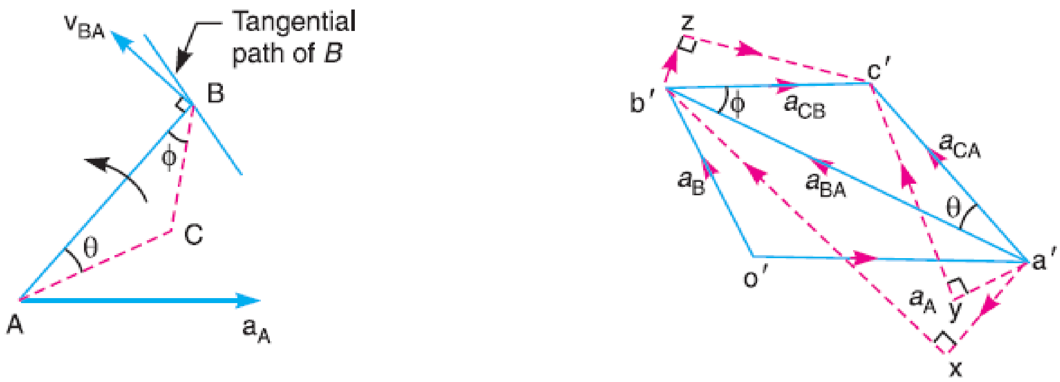
- This radial component of acceleration acts perpendicular to the velocity vBA, In other words, it acts parallel to the link AB.
  We know that tangential component of the acceleration of B with respect to A ,

$$a^t_{BA} = \alpha \times Length\ of\ link\ AB = \alpha \times AB$$

- This tangential component of acceleration acts parallel to the velocity vBA. In other words, it acts perpendicular to the link AB.
- In order to draw the acceleration diagram for a link A B, as shown in Fig. 3.16 (b), from any point b', draw vector b'x parallel to BA to represent the radial component of acceleration of B with respect to A.

Acceleration of a Point on a Link

- Consider two points A and B on the rigid link, as shown in Fig. 3.17 (a). Let the acceleration of the point A i.e. a_A is known in magnitude and direction and the direction of path of B is given. The acceleration of the point B is determined in magnitude and direction by drawing the acceleration diagram as discussed below.



(a) Points on a Link. (b) Acceleration diagram.

Fig. 3.17 acceleration of a point on a link

- From any point o', draw vector o'a' parallel to the direction of absolute acceleration at point A i.e. a_A , to some suitable scale, as shown in Fig. 3.17 (b).
- We know that the acceleration of B with respect to A i.e. a_{BA} has the following two components:

1 Radial component of the acceleration of B with respect to A i.e. a^r_{BA}
2 Tangential component of the acceleration B with respect to A i.e. a^t_{BA}

– Draw vector a''x parallel to the link AB such that,

$$vector\ a'x = a^r_{BA} = v^2_{BA}/AB$$

– From point x, draw vector xb'' perpendicular to AB or vector a''x and through o'' draw a line parallel to the path of B to represent the absolute acceleration of B i.e. a_B

– By joining the points a' and b' we may determine the total acceleration of B with respect to A i.e. a_{BA}. The vector a' b' is known as acceleration image of the link AB.

– For any other point C on the link, draw triangle a' b' c' similar to triangle ABC. Now vector b' c' represents the acceleration of C with respect to B i.e. a_{CB}, and vector a' c' represents the acceleration of C with respect to A i.e. a_{CA}. As discussed above, a_{CB} and a_{CA} will each have two components as follows :
   a. a_{CB} has two components; a^r_{CB} and a^t_{CB} as shown by triangle b''zc'' in fig.b
   b. a_{CA} has two components; a^r_{CA} and a^t_{CA} as shown by triangle a''yc''

– The angular acceleration of the link AB is obtained by dividing the tangential component of acceleration of B with respect to A to the length of the link.

$$\alpha_{AB} = a^t_{BA}/AB$$

Acceleration in Slider Crank Mechanism

– A slider crank mechanism is shown in Fig. 3.18 (a). Let the crank OB makes an angle θ with the inner dead centre (I.D.C) and rotates in a clockwise direction about the fixed point O with uniform angular velocity ω_{BO} rad/s

– Velocity of B with respect to O or velocity of B (because O is a fixed point),

$$v_{BO} = v_B = \omega_{BO} \times OB\ acting\ tangentially\ at\ B$$

– We know that centripetal or radial acceleration of B with respect to O or acceleration of B (Because O is a fixed point)

$$a^r_{BO} = a_B = \omega^2_{BO} \times OB = \frac{v^2_{BO}}{BO}$$

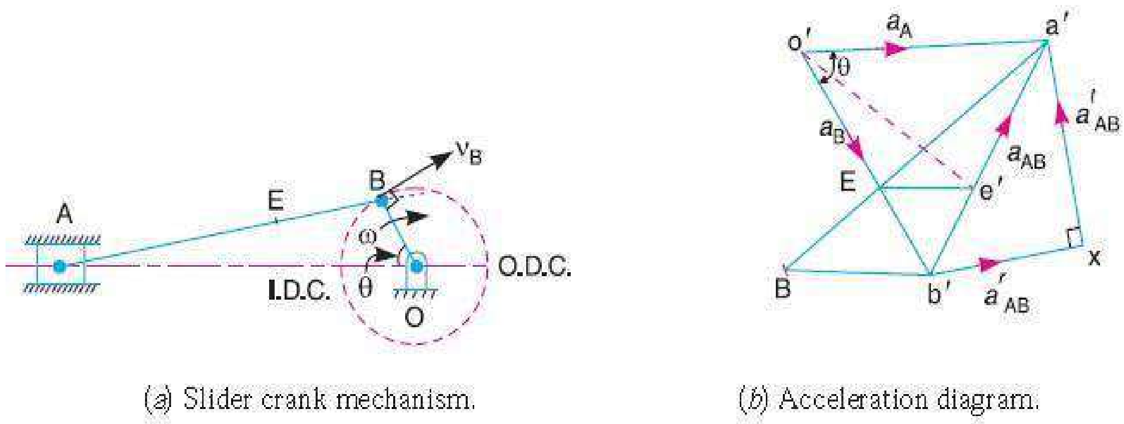


Fig. 3.18 acceleration in the slider crank mechanism

– The acceleration diagram, as shown in Fig. 3.18 (b), may now be drawn as discussed below:

1 Draw vector o' b' parallel to BO and set off equal in magnitude of a=a, to some BO suitable scale.

2 From point b', draw vector b'x parallel to BA. The vector b'x represents the radial component of the acceleration of A with respect to B whose magnitude is given by :

$$a^r_{AB} = v^2_{AB} / BA$$

3 From point x, draw vector xa" perpendicular to b"x. The vector xa" represents the tangential components of the acceleration of A with respect to B.

4 Since the point A reciprocates along AO, therefore the acceleration must be parallel to velocity. Therefore from o', draw o' a' parallel to A O, intersecting the vector xa' at a'.

5 The vector b' a', which is the sum of the vectors b' x and x a', represents the total acceleration of A with respect to B i.e. a_{AB}. The vector b'a' represents the acceleration of the connecting rod AB.

6 The acceleration of any other point on A B such as E may be obtained by dividing the vector b' a' at e' in the same ratio as E divides A B in Fig. 8.3 (a). In other words

$$a'e'/a'b' = AE/AB$$

7 The angular acceleration of the connecting rod A B may be obtained by dividing the tangential component of the acceleration of A with respect to B to the length of AB. In other words, angular acceleration of AB,

$$\alpha_{AB} = a^t_{AB} / AB$$

Examples Based on Acceleration

The crank of the slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine:

1. **Linear velocity and acceleration of the midpoint of the connecting rod, and**
2. **Angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position**

- **Given:**
- N_{BO} = 300 r.p.m. or $\omega_{BO} = 2\pi \times 300/60 = 31.42$ rad/s; OB = 150 mm = 0.15 m ; B A = 600 mm = 0.6 m
- We know that linear velocity of B with respect to O or velocity of B,

$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713\ m/s$$

- Draw vector ob perpendicular to BO, to some suitable scale, to represent the velocity of B with respect to O or simply velocity of B i.e. v_{BO} or vB, such that

vector ob = v_{BO} = v_B = 4.713 m/s

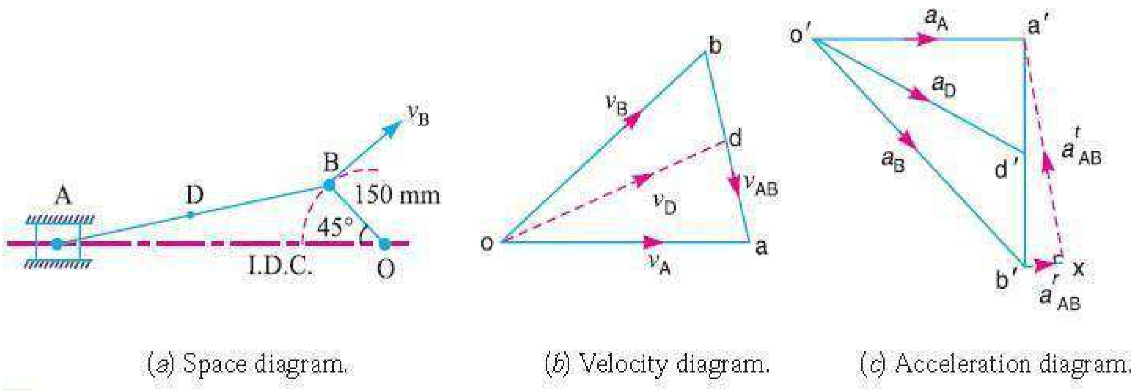


Fig. 3.19

- From point b, draw vector ba perpendicular to BA to represent the velocity of A with respect to B i.e. v_{AB} , and from point o draw vector oa parallel to the motion of A (which is along AO) to represent the velocity of A i.e. v_A. The vectors ba and oa intersect at a.
- By measurement we find the velocity A with respect to B,

$$v_{AB} = vector\ ba = 3.4\ m/s$$
$$v_A = \text{vector oa} = 4\ \text{m/s}$$

- In order to find the velocity of the midpoint D of the connecting rod A B, divide the vector ba at d in the same ratio as D divides A B, in the space diagram. In other words,

$$bd/ba = BD/BA$$

- By measurement, we find that

$$v_D = \text{vector od} = 4.1\ \text{m/s}$$

- We know that the radial component of the acceleration of B with respect to O or the acceleration of B,

$$a^r_{BO} = a_B = \frac{v^2_{BO}}{OB} = \frac{(4.713)^2}{0.15} = 148.1\ m/s^2$$

- And the radial component of the acceleration of A with respect to B,

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{(3.4)^2}{0.6} = 19.3\ m/s^2$$
$$vector\ o'b' = a^r_{BO} = a_B = 148.1\ m/s^2$$

- By measurement, we find that

$$a = vector\ o'd' = 117\ m/s^2$$

- We know that angular velocity of the connecting rod AB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67\ rad/s^2$$

- From the acceleration diagram, we find that

$$a^t_{AB} = 103\ \text{m/s}^2$$

- We know that angular acceleration of the connecting rod AB,

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67\ rad/s^2$$

An engine mechanism is shown in Fig. 3.20. The crank CB = 100 mm and the connecting rod BA = 300 mm with centre of gravity G, 100 mm from B. In the position shown, the crankshaft has a speed of 75 rad/s and an angular acceleration of 1200 rad/s^2. Find:

1. Velocity of G and angular velocity of AB, and

2. Acceleration of G and angular acceleration of AB.

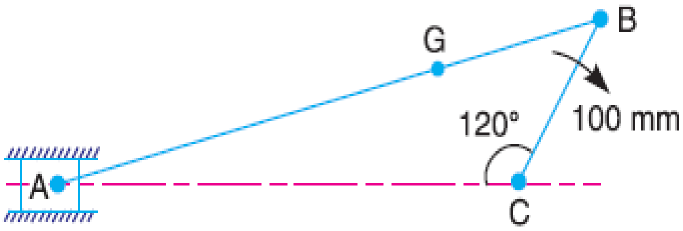


Fig. 3.20

– **Given :**

– ω_{BC} = 75 rad/s ; α_{BC} = 1200 rad/s^2, CB = 100 mm = 0.1 m; B A = 300 mm = 0.3 m

– We know that velocity of B with respect to C or velocity of B

$$v_{BC} = v_B = \omega_{BC} \times CB = 75 \times 0.1 = 7.5\ m/s$$

– Since the angular acceleration of the crankshaft, α_{BC} = 1200 rad/s^2, therefore tangential component of the acceleration of B with respect to C,

$$a^t_{BC} = \alpha_{BC} \times CB = 1200 \times 0.1 = 120\ m/s^2$$

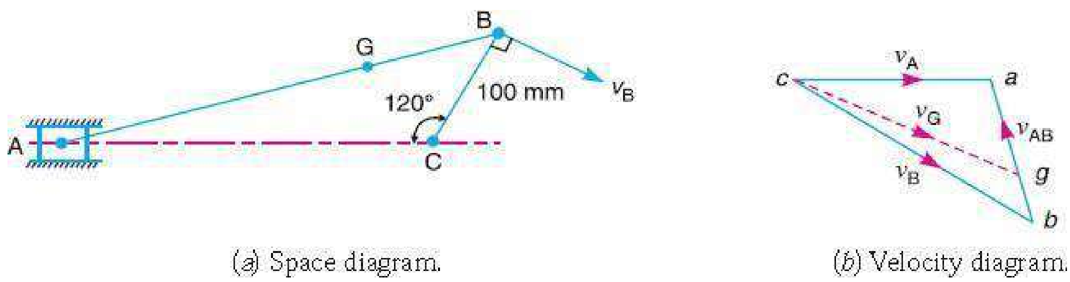
$$vector\ cb = v_{BC} = v_B = 7.5\ \frac{m}{s}$$

– By measurement, we find that velocity of G,

$$v_G = ector\ cg = 6.8\ m/s$$

– From velocity diagram, we find that the velocity of A with respect to B,

$$v_{AB} = vector\ ba = 4\ m/s$$

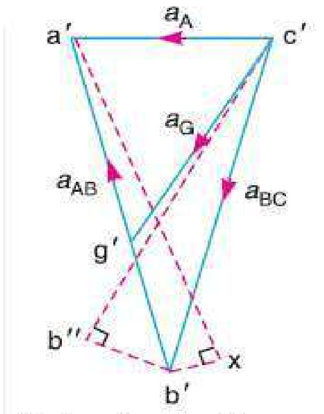


(a) Space diagram. (b) Velocity diagram.

Fig. 3.21

– We know that angular velocity of AB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{4}{0.3} = 13.3\ rad/s$$



(c) Acceleration diagram.

Fig. 3.22

- We know that radial component of the acceleration of B with respect to C

$$a^r_{BC} = \frac{v_{BC}^2}{CB} = \frac{(7.5)^2}{0.1} = 562.5\ m/s^2$$

- And radial component of the acceleration of A with respect to B,

$$a^r_{AB} = \frac{v_A^2}{CB} = \frac{(4)^2}{0.3} = 53.3\ m/s^2$$

$$vector\ c'b'' = a^r_{BC} = 562.5\ m/s^2$$

$$vector\ b''b' = a^t_{BC} = 120\ m/s^2$$

$$vector\ b'x = a^r_{AB} = 53.3\ m/s^2$$

- By measurement we find that acceleration of G,

$$a_G = vector\ xa' = 414\ m/s^2$$

- From acceleration diagram, we find that tangential component of the acceleration of A with respect to B,

$$a^t_{AB} = vector\ xa' = 546\ m/s^2$$

- Angular acceleration of AB

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{546}{0.3} = 1820\ rad/s^2$$

In the mechanism shown in Fig. 8.7, the slider C is moving to the right with a velocity of 1 m/s and an acceleration of 2.5 m/s2.The dimensions of various links are AB = 3 m inclined at 45° with the vertical and BC = 1.5 m inclined at 45° with the horizontal. Determine: 1. the magnitude of vertical and horizontal component of the acceleration of the point B, and 2. the angular acceleration of the links AB and BC.

- **Given:**
- v_C = 1 m/s ; a_C = 2.5 m/s2; AB = 3 m ; BC = 1.5 m
- Here,

$$vector\ d = v_{CD} = v_c = 1 m/s$$

- By measurement, we find that velocity of B with respect to A

$$v_{BA} = vector\ ab = 0.72\ m/s$$

- Velocity of B with respect to C

$$v_{BC} = vector\ cb = 0.72\ m/s$$

– We know that radial component of acceleration of B with respect to C,

$$a^r_{BC} = \frac{v_{BC}^2}{CB} = \frac{(0.72)^2}{1.5} = 0.346\ m/s^2$$

– And radial component of acceleration of B with respect to A,

$$a^r_{BA} = \frac{v_{BA}^2}{AB} = \frac{(0.72)^2}{3} = 0.173\ m/s^2$$

$$vectro\ d'c' = a_{cd} = a_c = 2.5\ m/s^2$$

$$vecto\ 'x = a^r_{BC} = 0.346\ \frac{m}{s^2}$$

$$vecto\ 'y = a^r_{BA} = 0.173\ m/s^2$$

– By measurement,

$$vector\ b'b'' = 1.13\ m/s^2$$

– By measurement from acceleration diagram, we find that tangential component of acceleration of the point B with respect to A

$$a^t_{BA} = ector\ yb' = 1.41\ m/s^2$$

– And tangential component of acceleration of the point B with respect to C,

$$a^t_{BC} = vector\ xb' = 1.94\ m/s^2$$

– we know that angular velocity of AB,

$$\alpha_{AB} = \frac{v^t_{BA}}{AB} = 0.47\ rad/s^2$$

– And aglular acceleration ofBC,

$$\alpha_{BC} = \frac{a^t_{BC}}{CB} = \frac{1.94}{1.5}\ rad/s^2$$

LECTURE 1

MOTION OF LINK IN MACHINE

LIVE TO LEARN & LEARN TO SHARE

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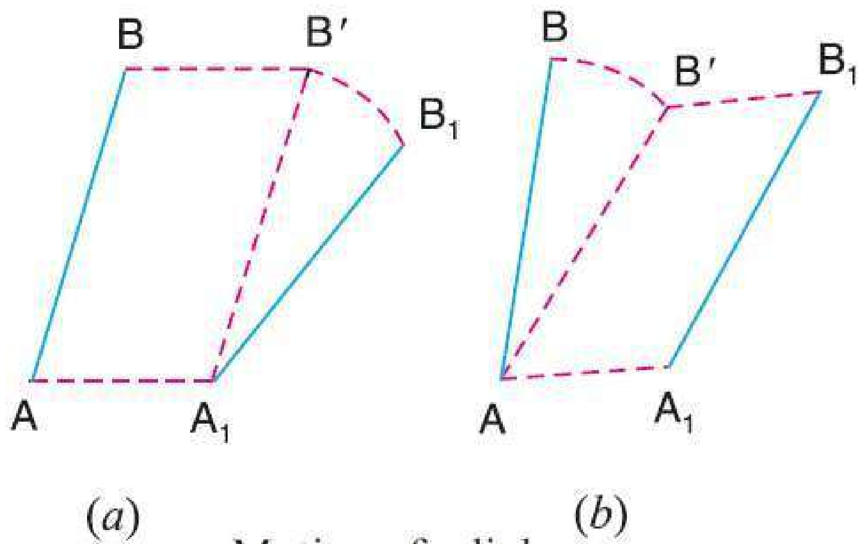
DEPARTMENT OF MECHANICAL ENGINEERING

INTRODUCTION



Motion of a link.

Source : R. S. Khurmi

Motion of link *AB to A*1*B*1 is an example of combined motion of rotation and translation, it being immaterial whether the motion of rotation takes first, or the motion of translation.

METHODS FOR DETERMINING THE VELOCITY OF A POINT ON A LINK

1. Relative velocity method

   Can be used in any configuration

2. Instantaneous centre method

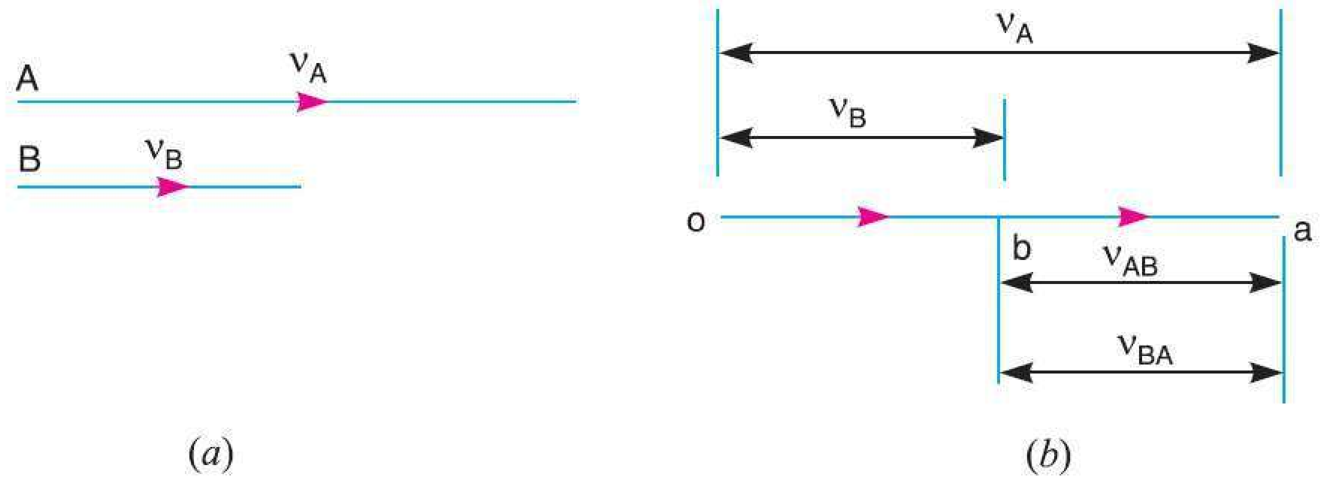
   convenient and easy to apply in simple mechanisms

RELATIVE VELOCITY METHOD

From Fig., the relative velocity of A with respect to B (*i.e.* v_{AB}) may be written in the vector form as follows :

$$\overline{ba} = \overline{oa} - \overline{ob}$$

Source : R. S. Khurmi



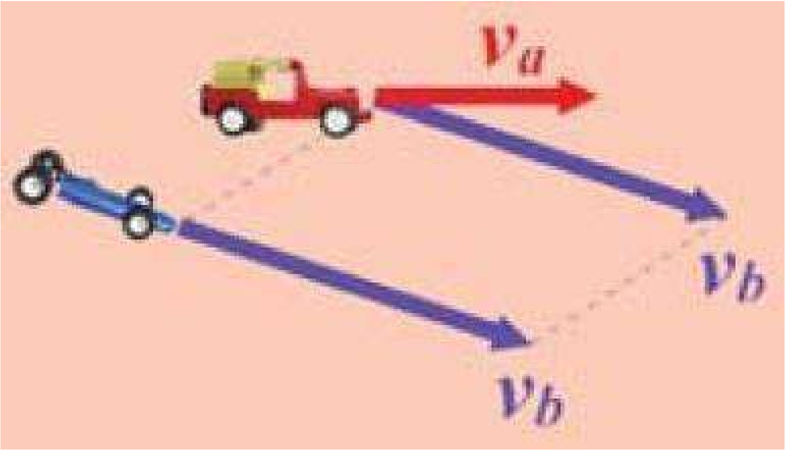
Relative velocity of two bodies moving along parallel lines.

Similarly, the relative velocity of B with respect to A ,

v_{BA} = Vector difference of v_B and $v_A = \overline{v_B} - \overline{v_A}$

$$\overline{ab} = \overline{ob} - \overline{oa}$$

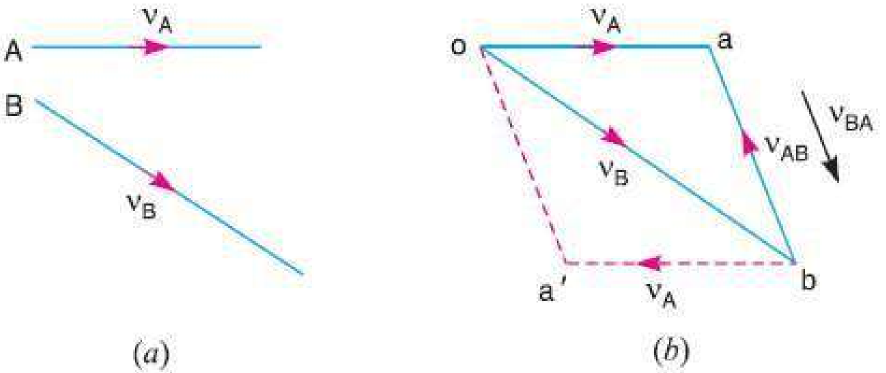
RELATIVE VELOCITY



v_{AB} = Vector difference of v_A and $v_B = \overline{v_A} - \overline{v_B}$

$\overline{ba} = \overline{oa} - \overline{ob}$

Source : R. S. Khurmi



Relative velocity of two bodies moving along inclined lines.

Similarly, the relative velocity of *B* with respect to *A*,

v_{BA} = Vector difference of v_B and $v_A = \overline{v_B} - \overline{v_A}$

$\overline{ab} = \overline{ob} - \overline{oa}$

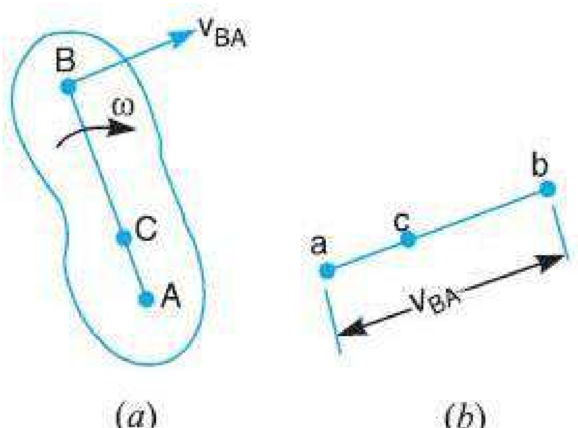
From above, we conclude that the relative velocity of point *A* with respect to *B* (v_{AB}) and the relative velocity of point *B* with respect *A* (v_{BA}) are equal in magnitude but opposite in direction, *i.e.*

$$v_{AB} = -v_{BA} \quad \text{or} \quad \overline{ba} = -\overline{ab}$$

Note: It may be noted that to find v_{AB}, start from point *b* towards *a* and for v_{BA}, start from point *a* towards *b*.

MOTION OF A LINK

Source : R. S. Khurmi



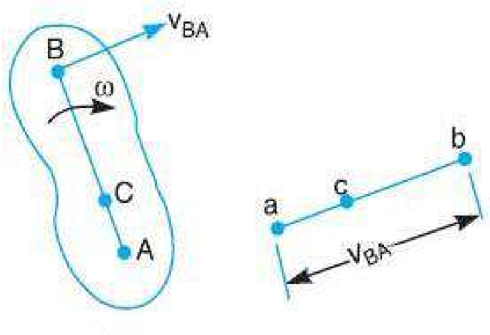
Motion of a Link.

- Let one of the extremities (B) of the link move relative to A, in a clockwise direction.
- No relative motion between A and B, along the line AB
- relative motion of B with respect to A must be perpendicular to AB.

Hence velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.

MOTION OF A LINK

Source : R. S. Khurmi



Motion of a Link.

Let ω = Angular velocity of the link AB about A.

We know that the velocity of the point B with respect to A,

$$v_{BA} = \overline{ab} = \omega . AB \qquad ...(i)$$

Similarly, the velocity of any point C on AB with respect to A,

$$v_{CA} = \overline{ac} = \omega . AC \qquad ...(ii)$$

From equations (i) and (ii),

$$\frac{v_{CA}}{v_{BA}} = \frac{\overline{ac}}{\overline{ab}} = \frac{\omega . AC}{\omega . AB} = \frac{AC}{AB} \qquad ...(iii)$$

Thus, we see from equation (iii), that the point c on the vector ab divides it in the same ratio as C divides the link AB.

LECTURE 2

VELOCITY AND ACCELERATION DIAGRAMS

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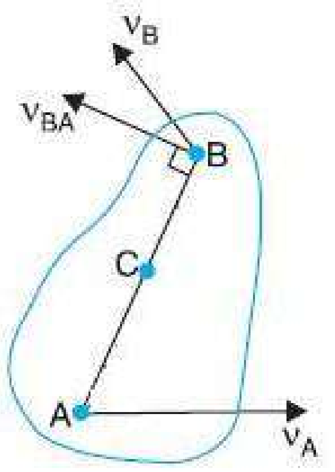
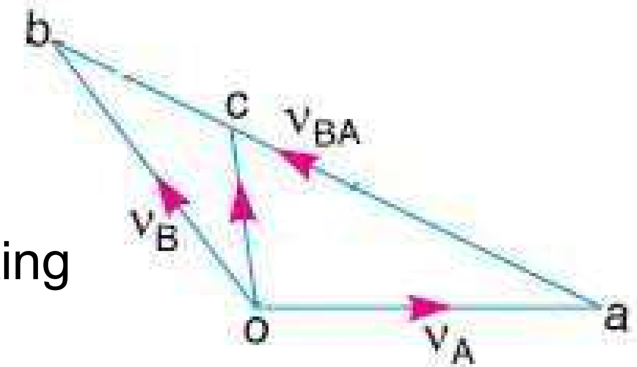
VELOCITY OF A POINT ON A LINK BY RELATIVE VELOCITY METHOD



Motion of points on a link.

Source : R. S. Khurmi

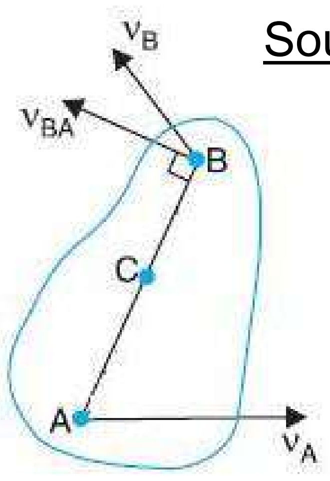
- V_A is known in magnitude and direction
- absolute velocity of the point *B i.e. V_B* is known in direction only
- V_B be determined by drawing the velocity diagram



Velocity diagram.

- With suitable scale, Draw oa = V_A
- Through a, draw a line perpendicular to AB
- Through o, draw a line parallel to V_B intersecting the line of V_{BA} at b
- Measure ob, which gives the required velocity of point B (V_B), to the scale
- ab = velocity of the link AB

VELOCITY OF A POINT ON A LINK BY RELATIVE VELOCITY METHOD



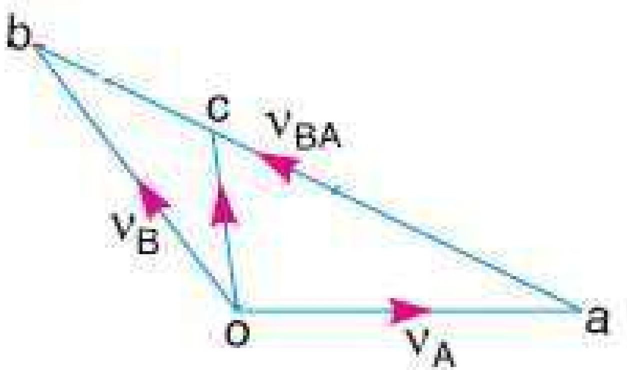
Motion of points on a link.

Source : R. S. Khurmi

- *How to find V_C ?*

Fix 'c' on the velocity diagram, using

$$\frac{ac}{ab} = \frac{AC}{AB}$$



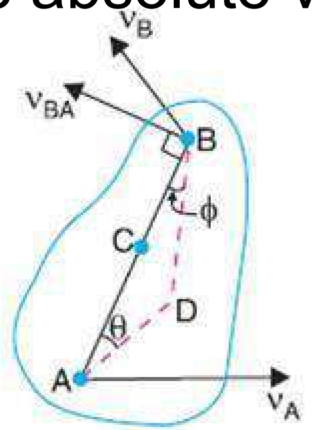
Velocity diagram.

- oc = V_C = Absolute velocity of C
- the vector ac represents the velocity of C with respect to A i.e. V_{CA}.

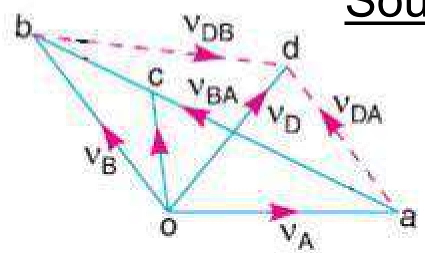
VELOCITY OF A POINT ON A LINK BY RELATIVE VELOCITY METHOD

How to find the absolute velocity of any other point D outside AB?

Source : R. S. Khurmi



(*a*) Motion of points on a link.



(*b*) Velocity diagram.

Construct triangle ABD in the space diagram

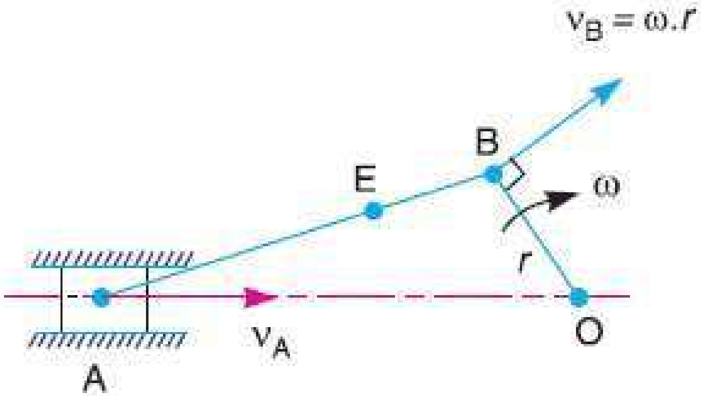
Completing the velocity triangle abd:

- Draw VDA perpendicular to AD;
- Draw VDB perpendicular to BD, intersection is ‘d’.
- od = absolute velocity of D.

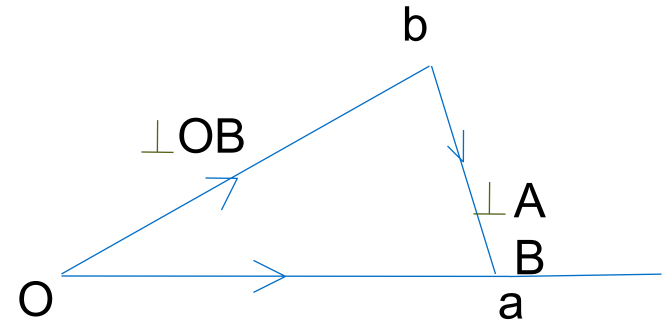
The angular velocity of the link AB $= \omega_{AB} = \dfrac{v_{BA}}{AB} = \dfrac{ab}{AB}$

VELOCITIES IN SLIDER CRANK MECHANISM

Source : R. S. Khurmi

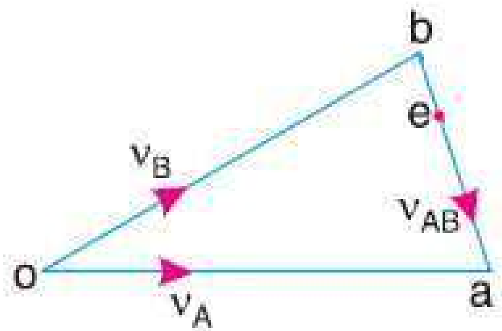


Slider crank mechanism.

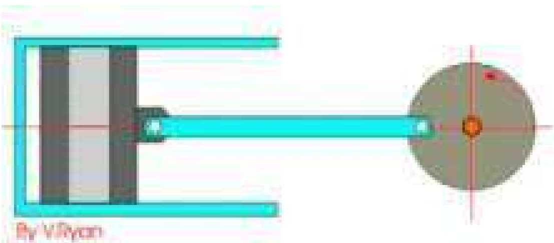


Fix 'e', based on the ratio

$$be/ba = BE/BA$$



Velocity diagram.



V_E = length 'oe' = absolute vel. Of E

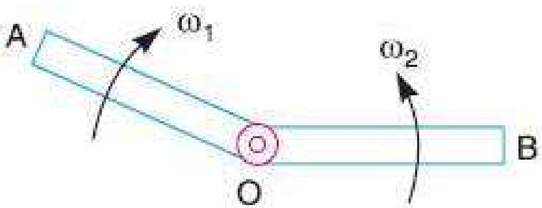
The angular velocity of the connecting rod AB (ω_{AB}) may be determined as follows:

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB} \qquad \text{(Anticlockwise about A)}$$

RUBBING VELOCITY AT A PIN JOINT

The rubbing velocity is defined as the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of the pin.

Source : R. S. Khurmi



Links connected by pin joints.

Let ω_1 = Angular velocity of the link OA or the angular velocity of the point A with respect to O.

ω_2 = Angular velocity of the link OB or the angular velocity of the point B with respect to O, and

r = Radius of the pin.

According to the definition,

Rubbing velocity at the pin joint O

$= (\omega_1 - \omega_2)\, r$, if the links move in the same direction

$= (\omega_1 + \omega_2)\, r$, if the links move in the opposite direction

LECTURE 3

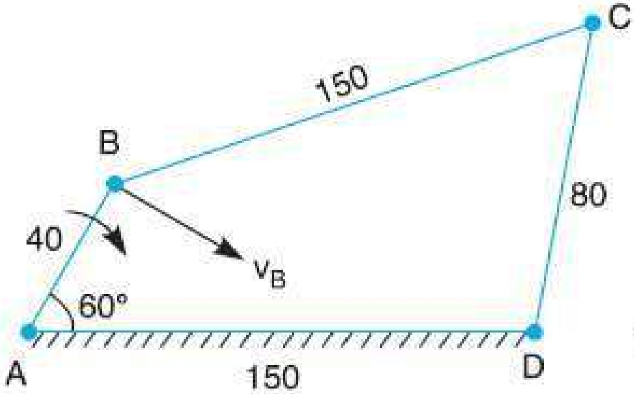
GRAPHICAL METHOD

DEPARTMENT OF MECHANICAL ENGINEERING

NUMERICAL EXAMPLE-1

In a four bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40 mm long and rotates at 120 r.p.m. clockwise, while the link CD = 80 mm oscillates about D. BC and AD are of equal length. Find the angular velocity of link CD when angle BAD = 60°.

Step-1 : Draw Space diagram with suitable scale



Space diagram (All dimensions in mm).

Step-2 : Identify Given data & convert it into SI units

$$N_{BA} = 120 \text{ r.p.m. or } \omega_{BA} = 2\pi \times 120/60 = 12.568 \text{ rad/s}$$

AB = 0.04 m ; BC = 0.15 m; CD = 0.08 m; AD = 0.15 m

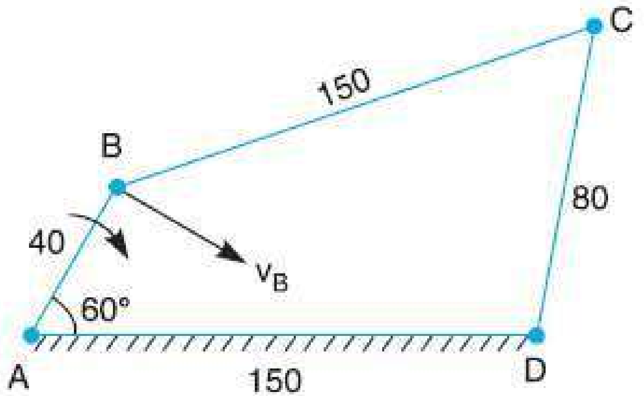
Step-3 : Calculate V_B

$$v_{BA} = v_B = \omega_{BA} \times AB = 12.568 \times 0.04 = 0.503 \text{ m/s}$$

Source : R. S. Khurmi

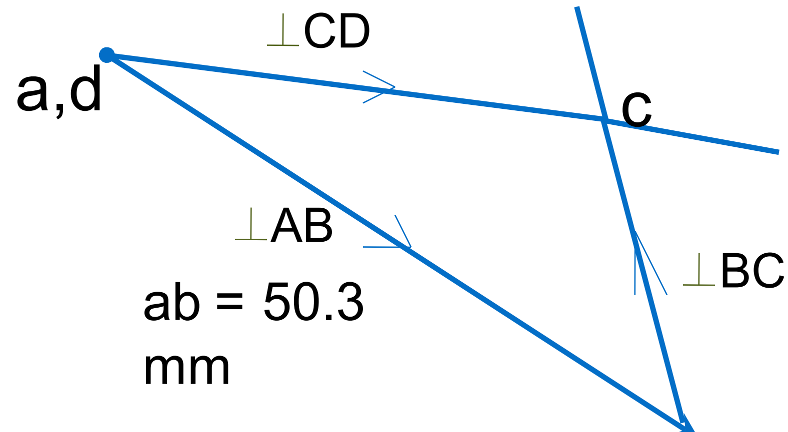
NUMERICAL EXAMPLE-1

Scale 1:100; i.e. V_B = 0.503 m/s = 50.3 mm



Space diagram (All dimensions in mm).

Source : R. S. Khurmi



Question: Find the angular velocity of link CD

V_{CD} = cd = 38.5 mm (by measurement) = 0.385 m/s , CD = 0.08 m

∴ Angular velocity of link *CD*,

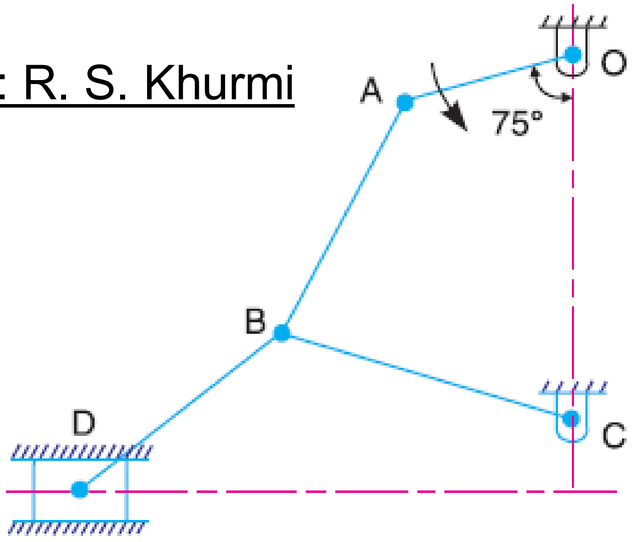
$$\omega_{CD} = \frac{v_{CD}}{CD} = \frac{0.385}{0.08} = 4.8 \text{ rad/s (clockwise about } D\text{)} \textbf{ Ans.}$$

NUMERICAL EXAMPLE -2

In the given Fig., the angular velocity of the crank OA is 600 r.p.m. Determine the linear velocity of the slider D and the angular velocity of the link BD, when the crank is inclined at an angle of 75° to the vertical. The dimensions of various links are : OA = 28 mm ; AB = 44 mm ; BC 49 mm ; and BD = 46 mm. The centre distance between the centres of rotation O and C is 65 mm. The path of travel of the slider is 11 mm below the fixed point C. The slider m path and OC is vertical.

Source : R. S. Khurmi

Find: V_D, ω_{BD}



Solution. Given: $N_{AO} = 600$ r.p.m. or

$\omega_{AO} = 2\pi \times 600/60 = 62.84$ rad/s

Since $OA = 28$ mm $= 0.028$ m, therefore velocity of A with respect to O or velocity of A (because O is a fixed point),

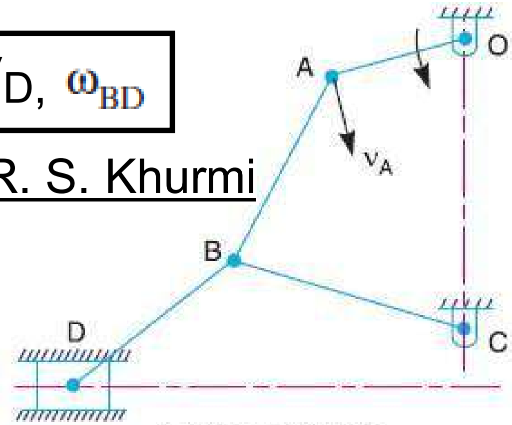
$$v_{AO} = v_A = \omega_{AO} \times OA = 62.84 \times 0.028 = 1.76 \text{ m/s}$$

... (Perpendicular to OA)

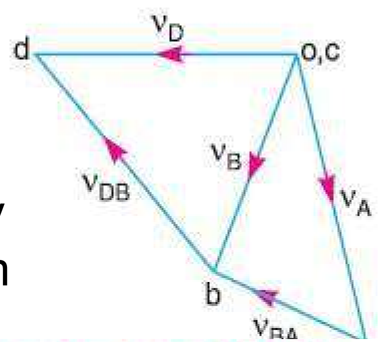
NUMERICAL EXAMPLE -2

Find: V_D, ω_{BD}

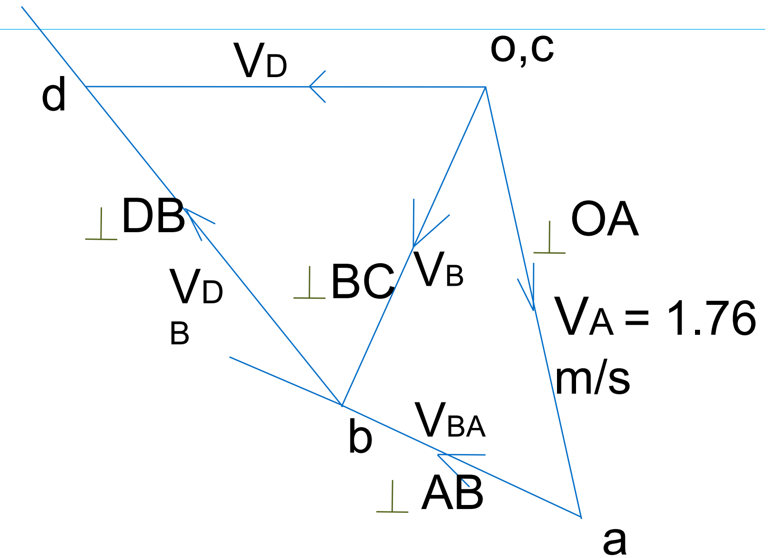
Source : R. S. Khurmi



(*a*) Space diagram.



Velocity diagram



Velocity diagram

By measurement, cd = od = V_D = 1.6 m/s

Angular velocity of the link BD

By measurement from velocity diagram, we find that velocity of *D* with respect to *B*,

$$v_{DB} = \text{vector } bd = 1.7 \text{ m/s}$$

Since the length of link *BD* = 46 mm = 0.046 m, therefore angular velocity of the link *BD*,

$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{1.7}{0.046} = 36.96 \text{ rad/s (Clockwise about } B\text{)} \quad \textbf{Ans.}$$

NUMERICAL EXAMPLE -3

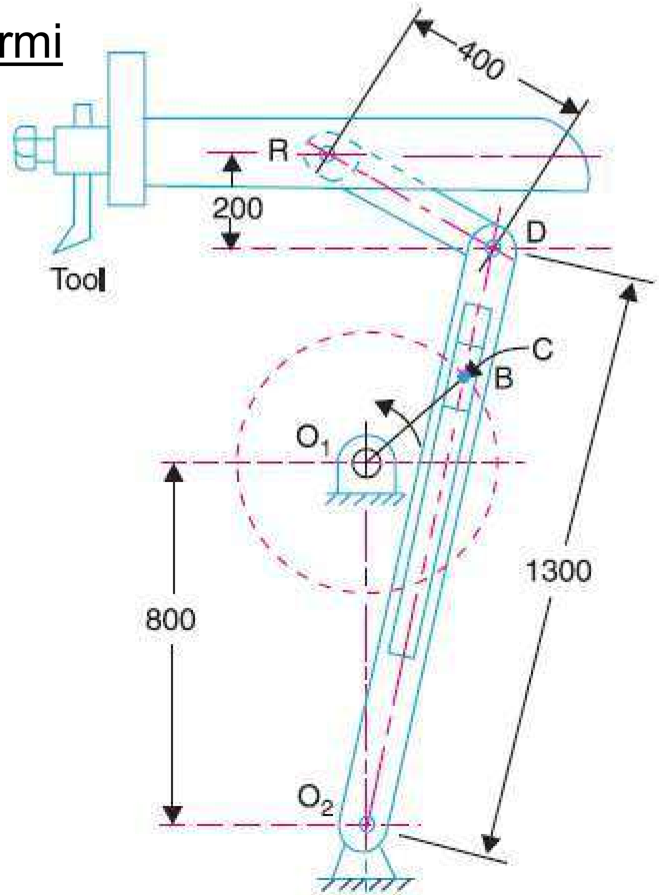
Source : R. S. Khurmi

A quick return mechanism of the crank and slotted lever type shaping machine is shown in the Fig. The dimensions of the various links are as follows :

O_1O_2 = 800 mm ; O_1B = 300 mm ; O_2D = 1300 mm ; DR = 400 mm.

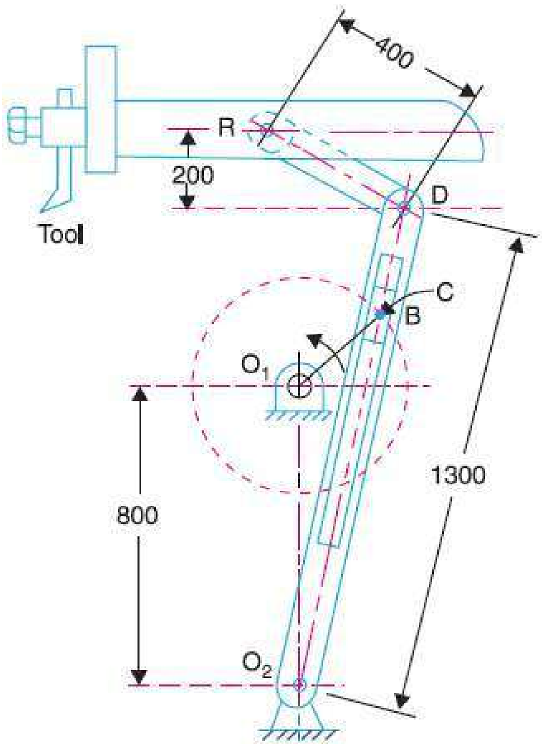
The crank O_1B makes an angle of 45° with the vertical and rotates at 40 r.p.m. in the counter clockwise direction.

Find : 1. velocity of the ram R, or the velocity of the cutting tool, and 2. angular velocity of link O_2D.

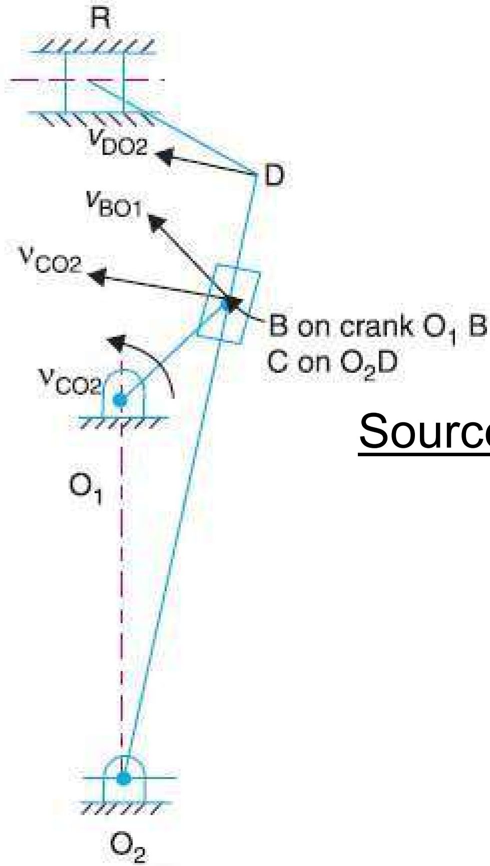


All dimensions in mm.

NUMERICAL EXAMPLE -3



All dimensions in mm.



Space diagram

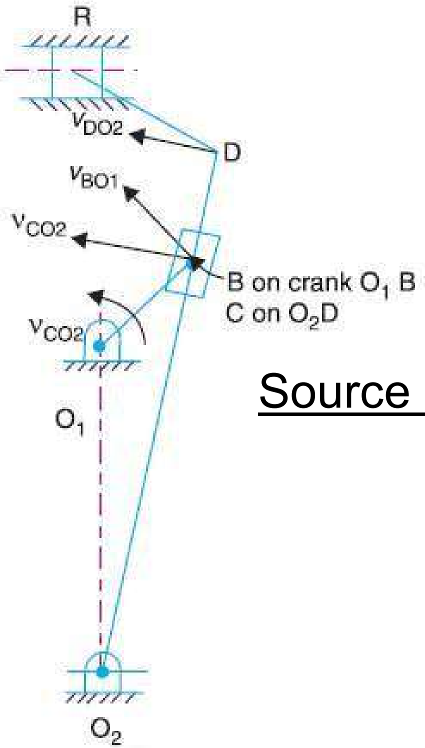
Source : R. S. Khurmi

Solution. Given: $N_{BO1} = 40$ r.p.m. or $\omega_{BO1} = 2\pi \times 40/60 = 4.2$ rad/s

$$v_{BO1} = v_B = \omega_{BO1} \times O_1B = 4.2 \times 0.3 = 1.26 \text{ m/s}$$

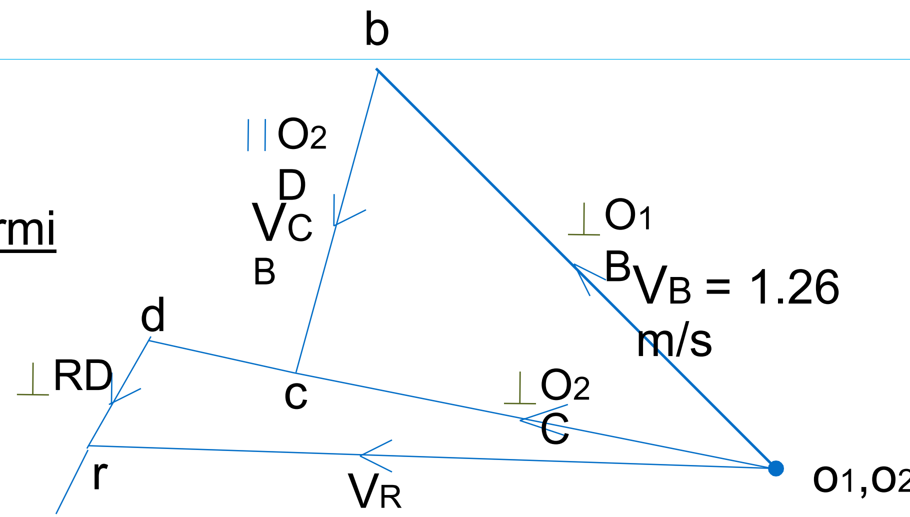
. . . (Perpendicular to O_1B)

AL EXAMPLE -3



Space diagram

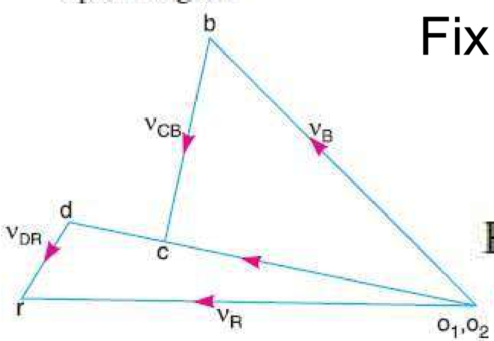
Source : R. S. Khurmi



Draw bc parallel to O_2D, to intersect at 'c'

Fix 'd' using the ratio $cd / o_2d = CD/O_2D$ ⟹ $\frac{cd}{cd + co_2} = \frac{CD}{O_2D}$

Find V_R & ω_{DO_2}



Velocity diagram.

By measurement, velocity of the ram R, v_R = vector o_1r = 1.44 m/s **Ans.**

Angular velocity of link O_2D

By measurement, $v_{DO2} = v_D$ = vector o_2d = 1.32 m/s

$$\omega_{DO_2} = \frac{v_{DO2}}{O_2D} = \frac{1.32}{1.3} = 1.015 \text{ rad/s (Anticlockwise about } O_2)$$ **Ans.**

TUTORIAL PROBLEM

Fig. 7.22 shows the structure of Whitworth quick return mechanism used in reciprocating machine tools. The various dimensions of the tool are as follows :

OQ = 100 mm ; OP = 200 mm, RQ = 150 mm and RS = 500 mm.

The crank OP makes an angle of 60° with the vertical. Determine the velocity of the slider S (cutting tool) when the crank rotates at 120 r.p.m. clockwise.

Find also the angular velocity of the link RS and the velocity of the sliding block T on the slotted lever QT.

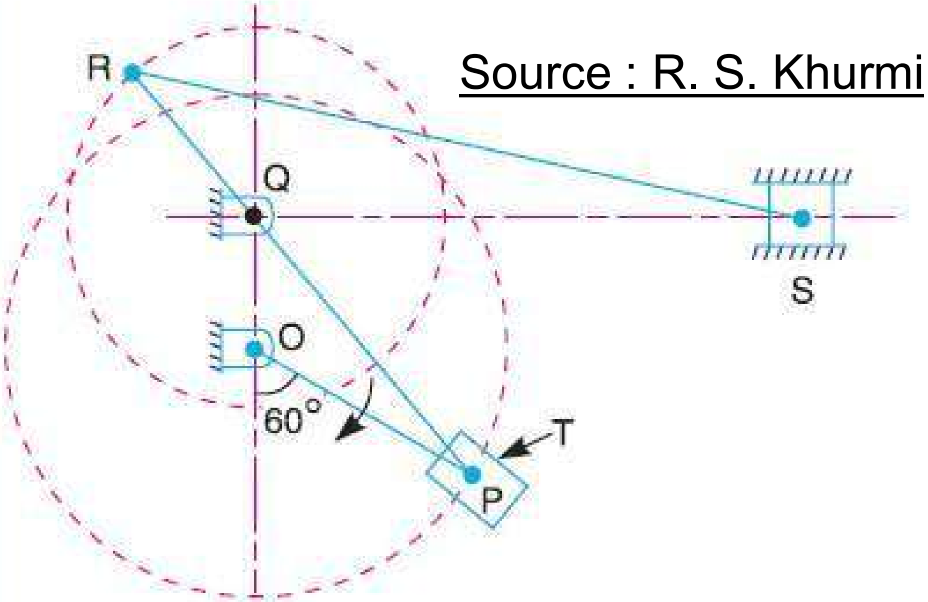
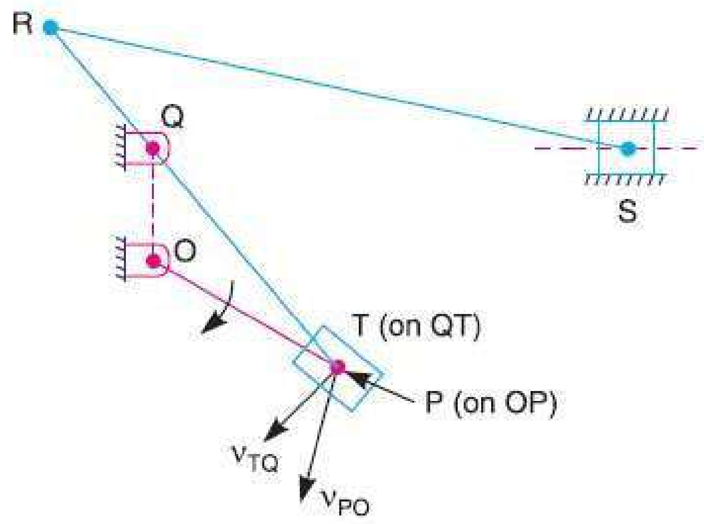


Fig. 7.22

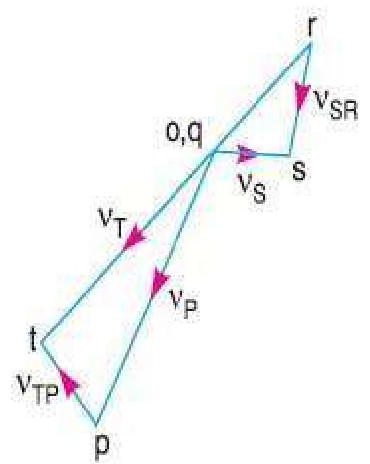
Source : R. S. Khurmi

TUTORIAL PROBLEM (SOLUTION)

Source : R. S. Khurmi



(*a*) Space diagram.



(*b*) Velocity diagram.

v_S = vector os = 0.8 m/s **Ans.**

Angular velocity of link RS

$$\omega_{RS} = \frac{v_{SR}}{RS} = \frac{0.96}{0.5} = 1.92 \text{ rad/s}$$ **Ans.**

Velocity of the sliding block T on the slotted lever QT

v_{TP} = vector pt = 0.85 m/s **Ans.**

LECTURE 4

INSTANTANEOUS CENTRE OF ROTATION

LIVE TO LEARN & LEARN TO SHARE

MRCET CAMPUS

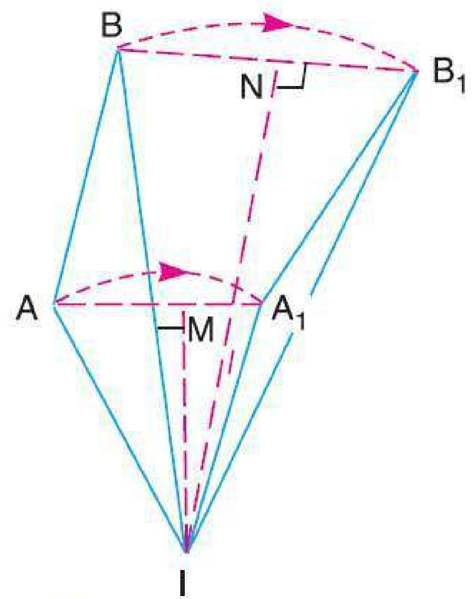
UGC Autonomous

DEPARTMENT OF MECHANICAL ENGINEERING

INSTANTANEOUS CENTRE METHOD

Translation of the link AB may be assumed to be a motion of pure rotation about some centre I, known as the instantaneous centre of rotation (also called centro or virtual centre).

The position of the centre of rotation must lie on the intersection of the right bisectors of chords *AA*1 and *BB*1. these bisectors intersect at *I as shown in Fig., which is the instantaneous centre of rotation or* virtual centre of the link *AB.*
(also called centro or virtual centre).



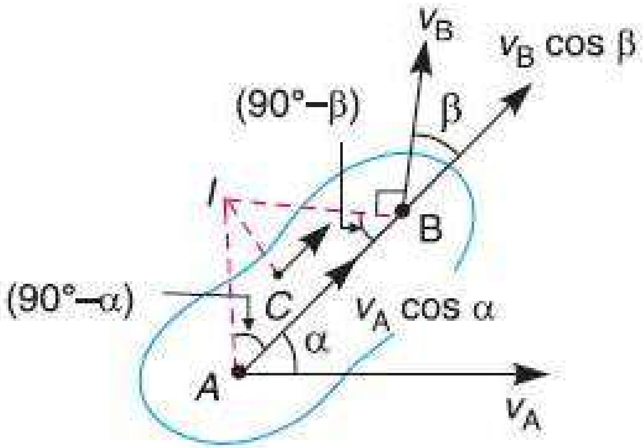
Instantaneous centre of rotation.

Source : R. S. Khurmi

VELOCITY OF A POINT ON A LINK BY INSTANTANEOUS CENTRE METHOD

V_A is known in Magnitude and direction V_B direction alone known How to calculate Magnitude of V_B using instantaneous centre method ?

Source : R. S. Khurmi



Velocity of a point on a link.

Draw AI and BI perpendiculars to the directions V_A and V_B respectively to intersect at I, which is known as instantaneous centre of the link.

VELOCITY OF A POINT ON A LINK BY INSTANTANEOUS CENTRE METHOD

Since A and B are the points on a rigid link, there cannot be any relative motion between them along the line AB.

Source : R. S. Khurmi

Now resolving the velocities along AB,

$$v_A \cos \alpha = v_B \cos \beta$$

or

$$\frac{v_A}{v_B} = \frac{\cos \beta}{\cos \alpha} = \frac{\sin (90° - \beta)}{\sin (90° - \alpha)} \quad \ldots(i)$$

Applying Lami's theorem to triangle ABI,

$$\frac{AI}{\sin (90° - \beta)} = \frac{BI}{\sin (90° - \alpha)}$$

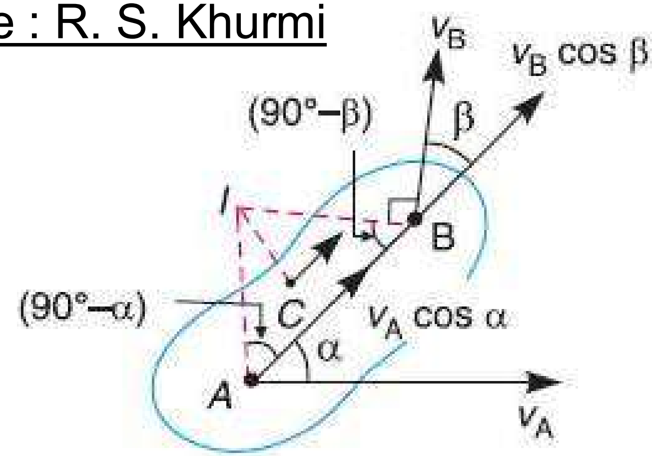
or

$$\frac{AI}{BI} = \frac{\sin (90° - \beta)}{\sin (90° - \alpha)} \quad \ldots(ii)$$

From equation (i) and (ii),

$$\frac{v_A}{v_B} = \frac{AI}{BI} \quad \text{or} \quad \frac{v_A}{AI} = \frac{v_B}{BI} = \omega \quad \ldots(iii)$$

where ω = Angular velocity of the rigid link.



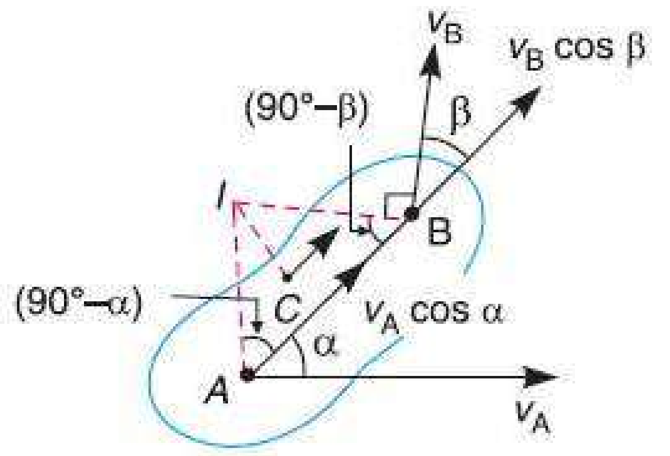
Velocity of a point on a link.

VELOCITY OF A POINT ON A LINK BY INSTANTANEOUS CENTRE METHOD

Source : R. S. Khurmi

If C is any other point on the link, then

$$\frac{v_A}{AI} = \frac{v_B}{BI} = \frac{v_C}{CI} \qquad \ldots(iv)$$

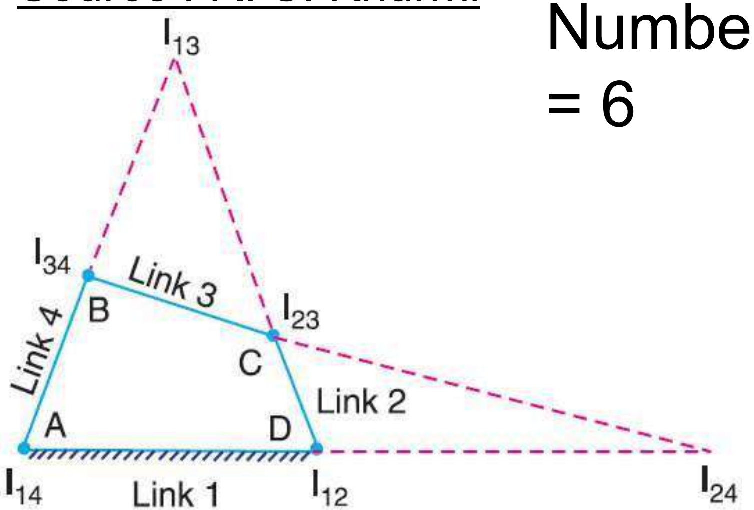


Velocity of a point on a link.

If V_A is known in magnitude and direction and V_B in direction only, then velocity of point B or any other point C lying on the same link may be determined (Using *iv*) in magnitude and direction.

TYPES OF INSTANTANEOUS CENTRES

Source : R. S. Khurmi



Types of instantaneous centres.

Number of Instantaneous Centres = N = 6

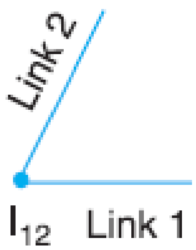
The instantaneous centres I_{12} and I_{14} ***fixed instantaneous centres***

The instantaneous centres I_{23} and I_{34} ***permanent instantaneous centres*** as they move when the mechanism moves, but the joints are of permanent nature.

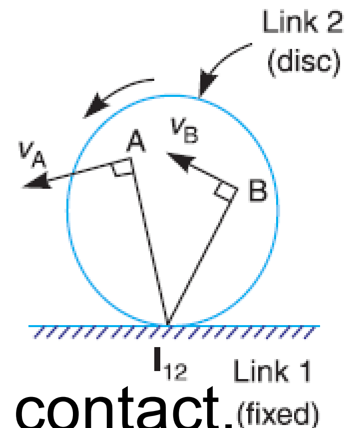
I_{13} and I_{24} are ***neither fixed nor permanent instantaneous centres*** as they vary with the configuration of the mechanism.

LOCATION OF INSTANTANEOUS CENTRES

<u>Source : R. S. Khurmi</u>

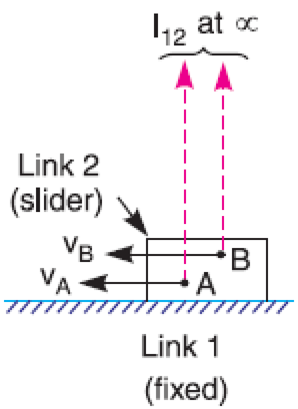


When the two links are connected by a pin joint (or pivot joint), the instantaneous centre lies on the centre of the pin



Pure rolling contact (i.e. link 2 rolls without slipping), the instantaneous centre lies on their point of contact.

$$\frac{v_A}{v_B} = \frac{I_{12}A}{I_{12}B}$$

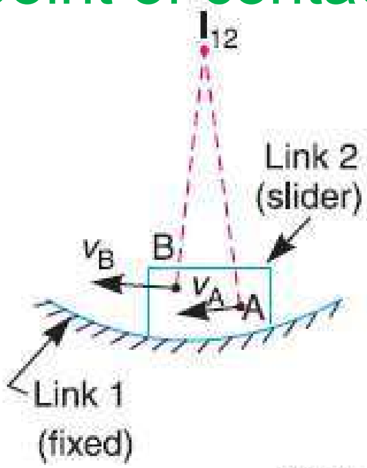


When the two links have a sliding contact, the instantaneous centre lies on the common normal at the point of contact.

The instantaneous centre lies at infinity and <u>each point on the slider have the same velocity</u>.

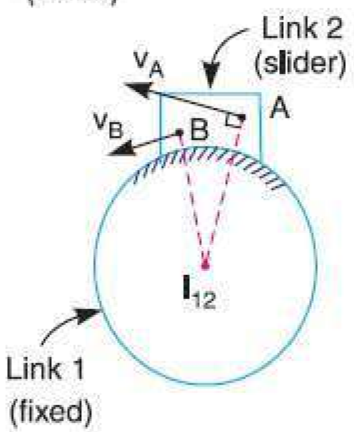
LOCATION OF INSTANTANEOUS CENTRES

When the two links have a sliding contact, the instantaneous centre lies on the common normal at the point of contact.



The instantaneous centre lies on the centre of curvature of the curvilinear path in the configuration at that instant.

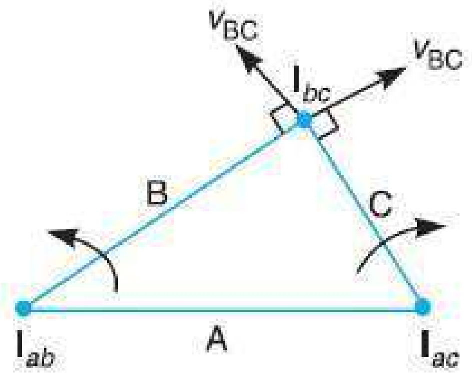
Source : R. S. Khurmi



When the link 2 (slider) moves on fixed link 1 having constant radius of curvature, the instantaneous centre lies at the centre of curvature i.e. the centre of the circle, for all configuration of the links.

ARONHOLD KENNEDY (OR THREE CENTRES IN LINE) THEOREM

It states that if three bodies move relatively to each other, they have three instantaneous centres and lie on a straight line.



Aronhold Kennedy's theorem.

Source : R. S. Khurmi

the velocity of the point I_{bc} cannot be perpendicular to both lines $I_{ab}\,I_{bc}$ and $I_{ac}\,I_{bc}$ unless the point I_{bc} lies on the line joining the points I_{ab} and I_{ac}.

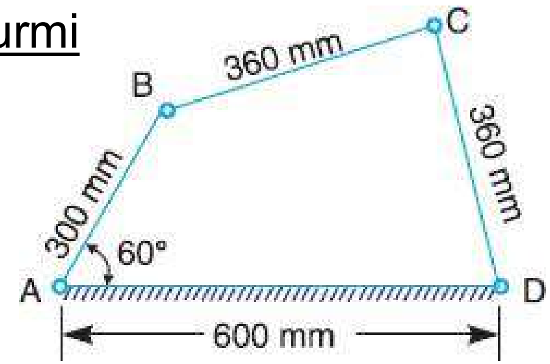
Thus the three instantaneous centres (I_{ab}, I_{ac} and I_{bc}) must lie on the same straight line.

The exact location of I_{bc} on line $I_{ab}\,I_{ac}$ depends upon the directions and magnitudes of the angular velocities of B and C relative to A.

NUMERICAL EXAMPLE-1

In a pin jointed four bar mechanism, as shown in Fig. AB = 300 mm, BC = CD = 360 mm, and AD = 600 mm. The angle BAD = 60°. The crank AB rotates uniformly at 100 r.p.m. Locate all the instantaneous centres and find the angular velocity of the link BC

<u>Source : R. S. Khurmi</u>



Solution. Given : $N_{AB} = 100$ r.p.m or

$$\omega_{AB} = 2\pi \times 100/60 = 10.47 \text{ rad/s}$$

Since the length of crank $AB = 300$ mm $= 0.3$ m,

therefore velocity of point B on link AB,

$$v_B = \omega_{AB} \times AB = 10.47 \times 0.3 = 3.141 \text{ m/s}$$

Location of instantaneous centres:

1. Find number of Instantaneous centres

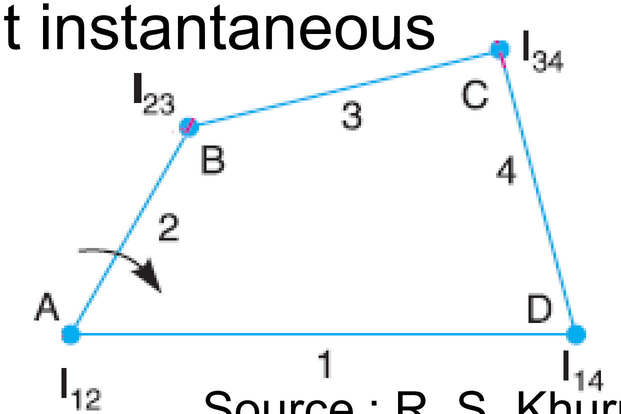
$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

NUMERICAL EXAMPLE-1

2. List the Ins. centres

Links	1	2	3	4
Ins. Centres	12	13	14	
		23	24	
			34	

3. Draw <u>configuration (space) diagram with suitable scale</u>. And, Locate the fixed and permanent instantaneous centres by inspection



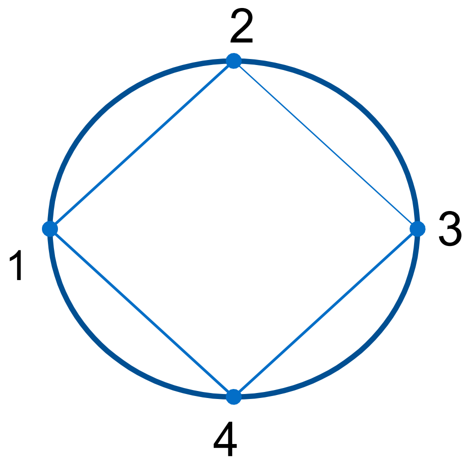
<u>Source : R. S. Khurmi</u>

I_{12}, I_{14} – Fixed centres;
I_{23}, I_{34} – Permanent centres

How to locate I_{13}, I_{24} – Neither fixed nor Permanent centres

NUMERICAL EXAMPLE-1

4. Locate the neither fixed nor permanent instantaneous centres by Aronhold Kennedy's theorem.



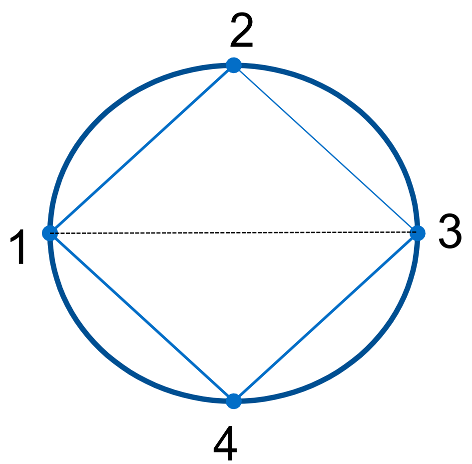
Source : R. S. Khurmi

Draw a circle with any arbitrary radius

At equal distance locate Links 1, 2, 3 & 4 as points on the circle.

NUMERICAL EXAMPLE-1

Locating I_{13}

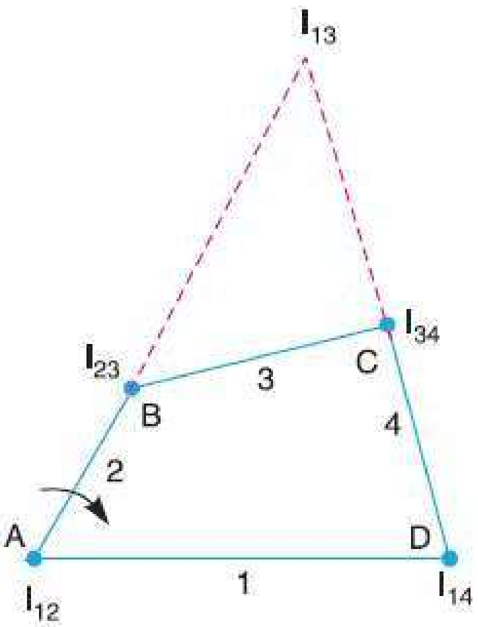


Source : R. S. Khurmi

13 is common side to Triangle 134 & 123

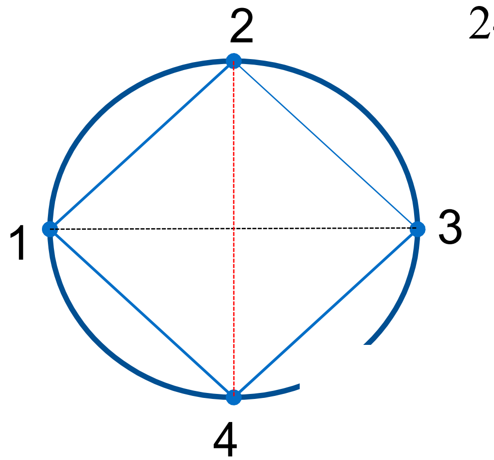
I_{13} → I_{14} I_{34}
I_{13} → I_{12} I_{23}

Therefore, I_{13} lies on the intersection of the lines joining the points $I_{14}I_{34}$ & I_{12} I_{23}



NUMERICAL EXAMPLE-1

Locating I_{24}

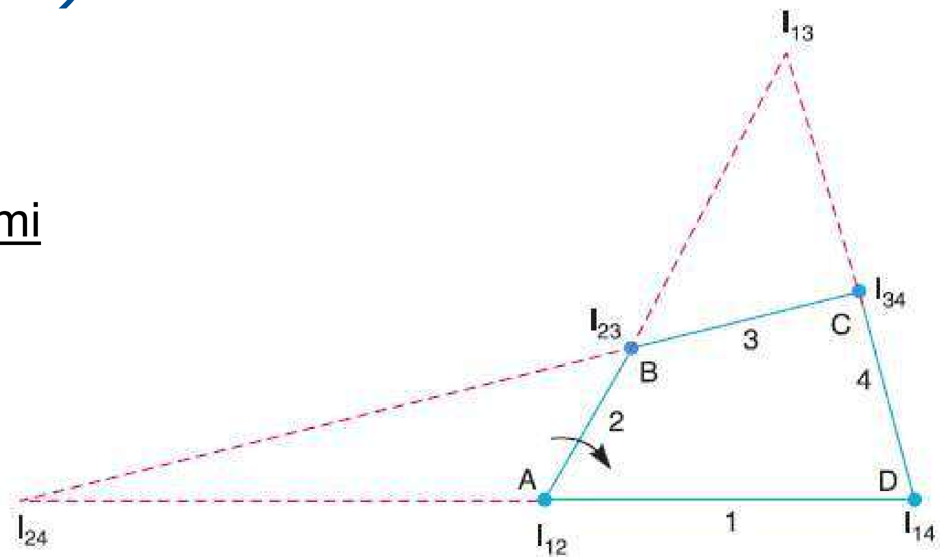


24 is common side to Triangle 124 & 234

I_{24} → I_{12} I_{14} ; I_{34} I_{23}

Therefore, I_{24} lies on the intersection of the lines joining the points $I_{12}I_{14}$ & I_{34} I_{23}

Source : R. S. Khurmi



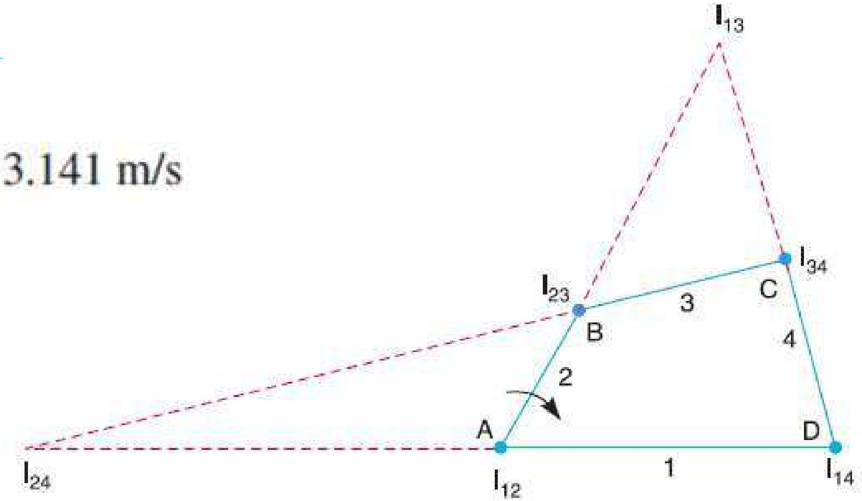
Thus all the six instantaneous centres are located.

NUMERICAL EXAMPLE-1

Find Angular velocity of the link BC

$$v_B = \omega_{AB} \times AB = 10.47 \times 0.3 = 3.141 \text{ m/s}$$

We know that:



Source : R. S. Khurmi

Let ω_{BC} = Angular velocity of the link *BC*.

Since *B* is also a point on link *BC*, therefore velocity of point *B* on link *BC*,

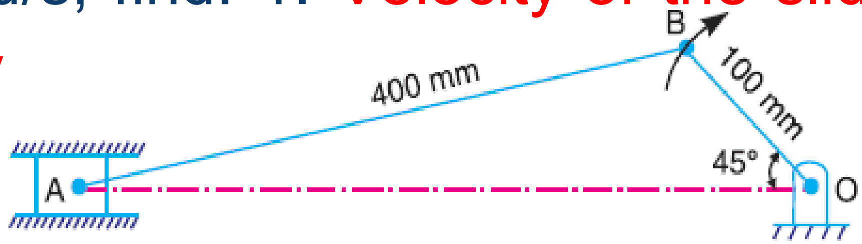
$$v_B = \omega_{BC} \times I_{13}B$$

By measurement, we find that $I_{13}B = 500 \text{ mm} = 0.5 \text{ m}$

$$\therefore \quad \omega_{BC} = \frac{v_B}{I_{13}B} = \frac{3.141}{0.5} = 6.282 \text{ rad/s}$$ **Ans.**

NUMERICAL EXAMPLE-2

Locate all the instantaneous centres of the slider crank mechanism as shown in the Fig. The lengths of crank OB and connecting rod AB are 100 mm and 400 mm respectively. If the crank rotates clockwise with an angular velocity of 10 rad/s, find: 1. Velocity of the slider A, and 2. Angular velocity



Source : R. S. Khurmi

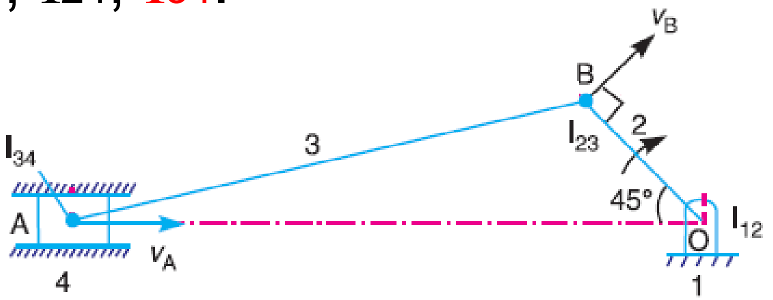
Solution. Given : $\omega_{OB} = 10$ rad/ s; $OB = 100$ mm $= 0.1$ m

We know that linear velocity of the crank OB,

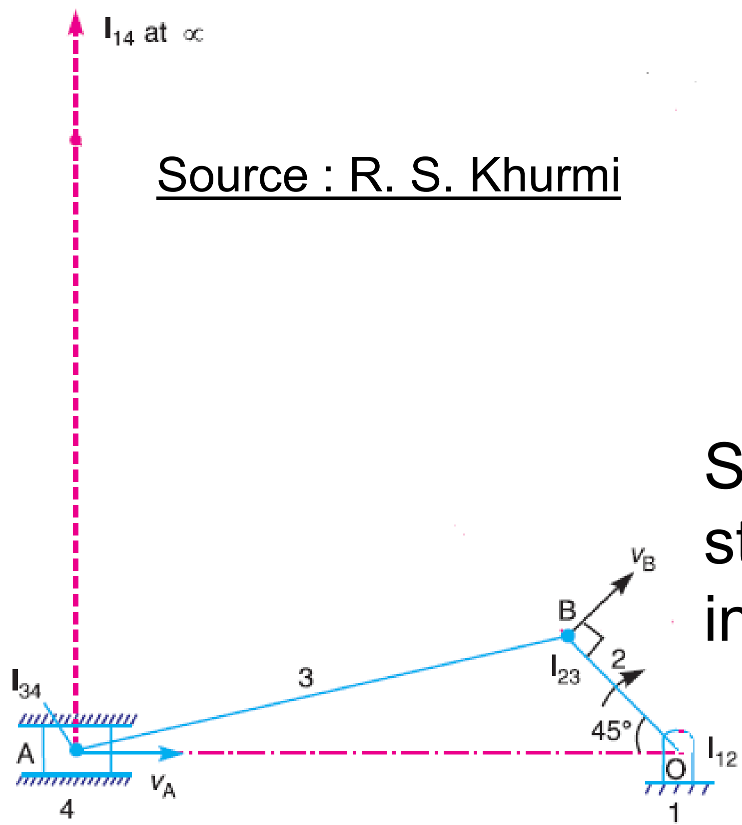
$$v_{OB} = v_B = \omega_{OB} \times OB = 10 \times 0.1 = 1 \text{ m/s}$$

NUMERICAL EXAMPLE-2

- Draw configuration diagram with suitable scale.
- Locate Ins. Centres (Here, n = 4; No. of Ins. Centres N = 6)
- Ins. Centers are I_{12}, I_{13}, I_{14}, I_{23}, I_{24}, I_{34}.



By inspection Locate I_{12}, I_{23} & I_{34}.

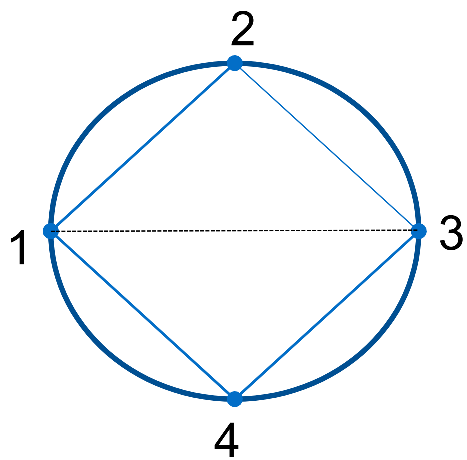


Source : R. S. Khurmi

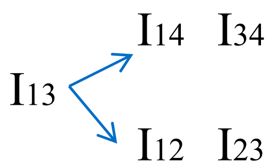
Since the slider (link 4) moves on a straight surface (link 1), I_{14}, will be at infinity.

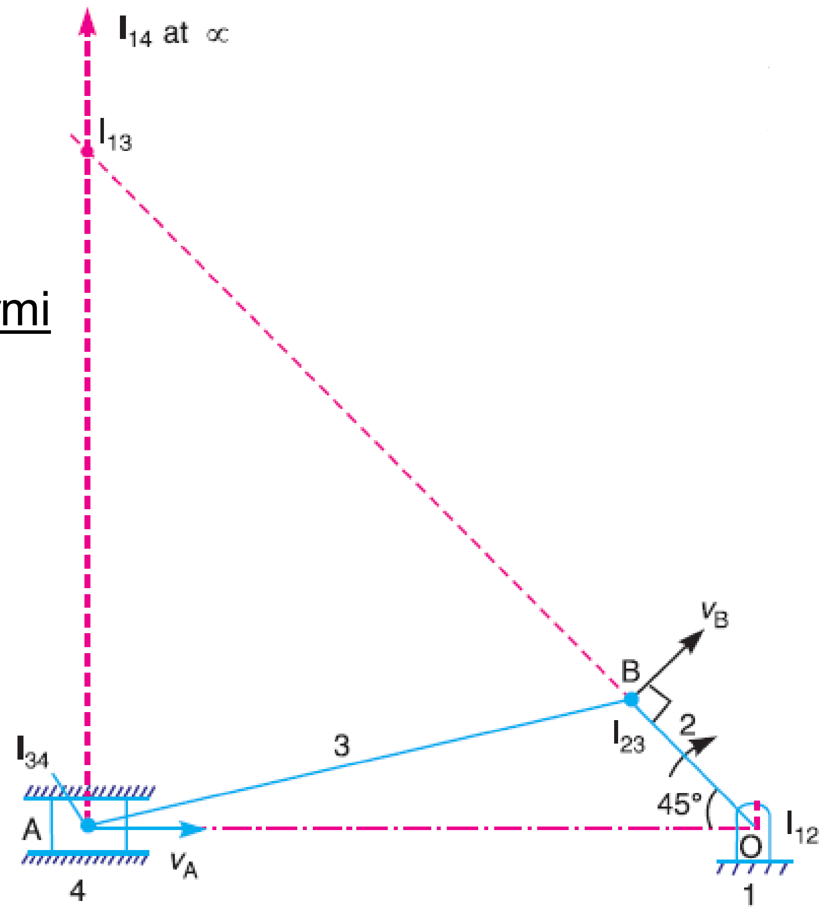
NUMERICAL EXAMPLE-2

- Ins. Centers are I_{12}, I_{13}, I_{14}, I_{23}, I_{24}, I_{34}.
- Fixing I_{13}…… ?



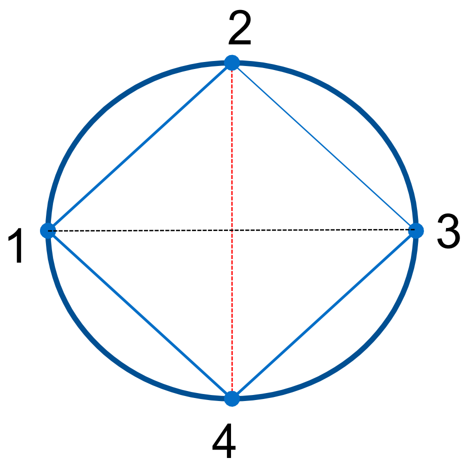
Source : R. S. Khurmi





NUMERICAL EXAMPLE-2

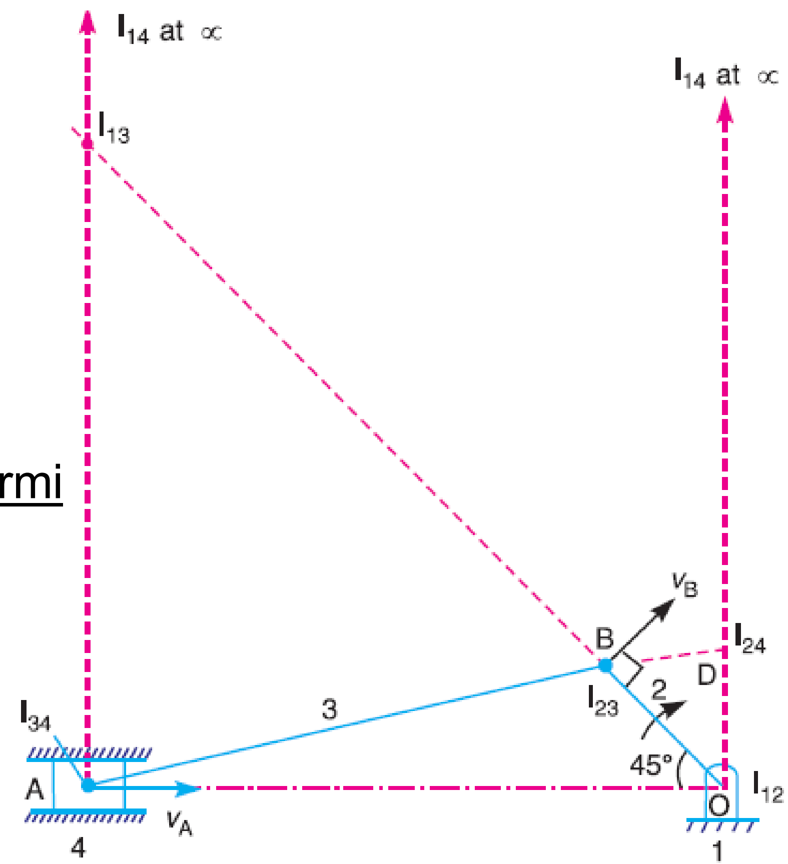
- Ins. Centers are I_{12}, I_{13}, I_{14}, I_{23}, I_{24}, I_{34}.
- Fixing I_{24}…… ?



I_{24} → I_{12} I_{14}

I_{24} → I_{34} I_{23}

Source : R. S. Khurmi

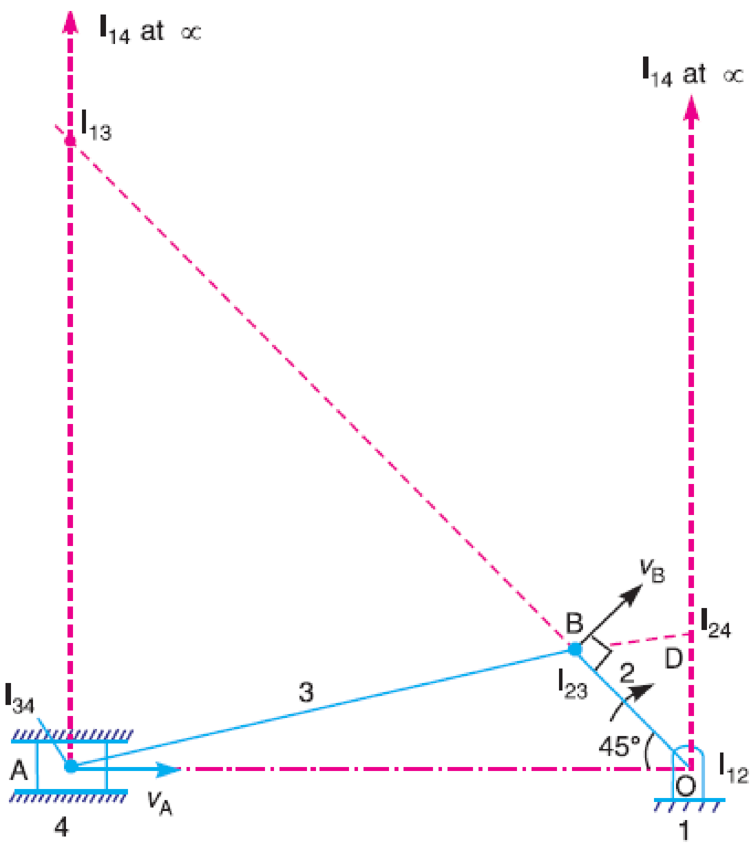


I_{14} can be moved to any convenient joint

NUMERICAL EXAMPLE-2

Solution:

Source : R. S. Khurmi



By measurement, we find that

$I_{13}A = 460 \text{ mm} = 0.46 \text{ m}$; and $I_{13}B = 560 \text{ mm} = 0.56 \text{ m}$

1. *Velocity of the slider A*

Let v_A = Velocity of the slider A.

We know that $\frac{v_A}{I_{13}A} = \frac{v_B}{I_{13}B}$

or $v_A = v_B \times \frac{I_{13}A}{I_{13}B} = 1 \times \frac{0.46}{0.56} = 0.82 \text{ m/s}$ **Ans.**

2. *Angular velocity of the connecting rod AB*

Let ω_{AB} = Angular velocity of the connecting rod AB.

We know that $\frac{v_A}{I_{13}A} = \frac{v_B}{I_{13}B} = \omega_{AB}$

$\therefore \omega_{AB} = \frac{v_B}{I_{13}B} = \frac{1}{0.56} = 1.78 \text{ rad/s}$ **Ans.**

VISIT THE FOLLOWING VIDEOS IN YOUTUBE

- https://www.youtube.com/watch?v=-tgruur8O0Q
- https://www.youtube.com/watch?v=WNh5Hp0lgms
- https://www.youtube.com/watch?v=ha2PzDt5SbE

EXERCISE-1

The mechanism of a wrapping machine, as shown in Fig. 6.18, has the following dimensions :

O_1A = 100 mm; AC = 700 mm; BC = 200 mm; O_3C = 200 mm; O_2E = 400 mm; O_2D = 200 mm and BD = 150 mm.

The crank O_1A rotates at a uniform speed of 100 rad/s. Find the velocity of the point E of the bell crank lever by instantaneous centre method.

Source : R. S. Khurmi

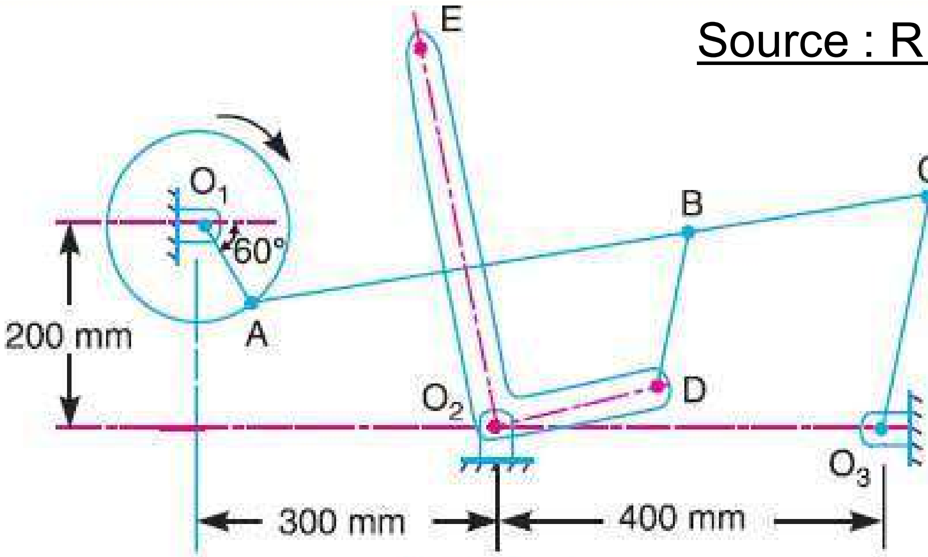
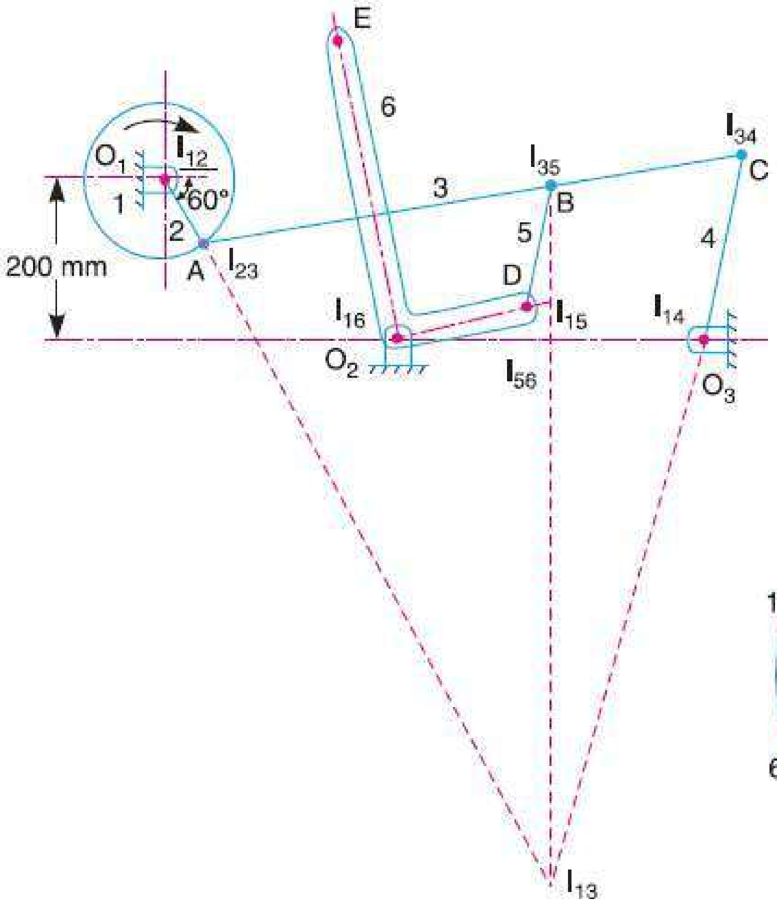


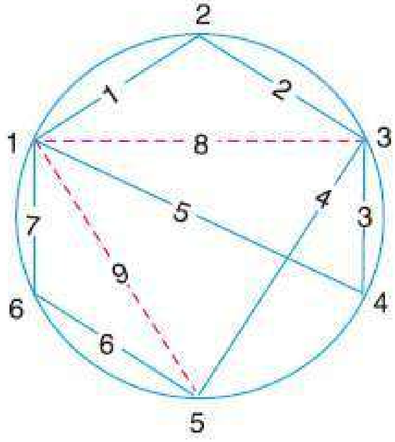
Fig. 6.18

EXERCISE-1: SOLUTION

$$N = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = 15$$



Source : R. S. Khurmi



EXERCISE-1: ANSWER

Velocity of point E on the bell crank lever

Let v_E = Velocity of point E on the bell crank lever,

v_B = Velocity of point B, and

v_D = Velocity of point D.

$$v_B = \frac{v_A}{I_{13} A} \times I_{13} B = \frac{10}{0.91} \times 0.82 = 9.01 \text{ m/s} \quad \textbf{Ans.}$$

$$v_D = \frac{v_B}{I_{15} B} \times I_{15} D = \frac{9.01}{0.13} \times 0.05 = 3.46 \text{ m/s} \quad \textbf{Ans.}$$

$$v_E = \frac{v_D}{I_{16} D} \times I_{16} E = \frac{3.46}{0.2} \times 0.4 = 6.92 \text{ m/s} \quad \textbf{Ans.}$$

LECTURE 5

ACCELERATION IN MECHANISMS

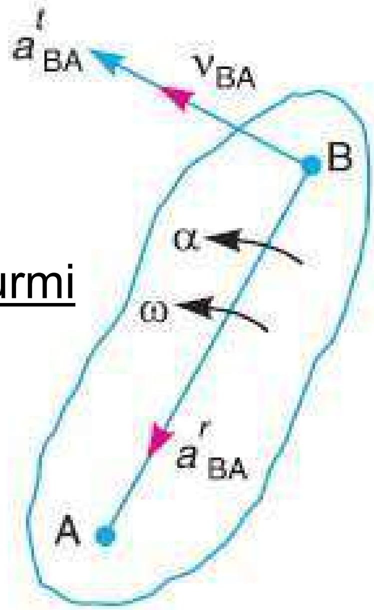
MRCET CAMPUS
UGC Autonomous

DEPARTMENT OF MECHANICAL ENGINEERING

ACCELERATION IN MECHANISMS

Acceleration analysis plays a very important role in the development of machines and mechanisms

Let the point B moves with respect to A, with an angular velocity of ω rad/s and let α rad/s² be the angular acceleration of the link AB.



Source : R. S. Khurmi

Acceleration for a link.

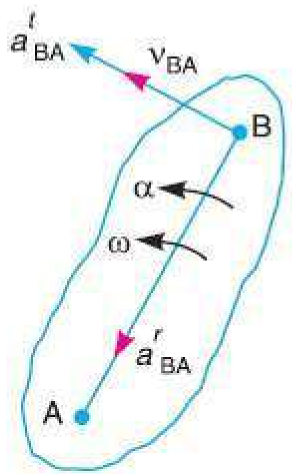
1. The centripetal or radial component of acceleration, which is perpendicular to the velocity (i.e. parallel to link AB) of the particle at the given instant.

$$a^r_{BA} = \omega^2 \times \text{Length of link } AB = \omega^2 \times AB = v^2_{BA} / AB \qquad \ldots \left(\because \omega = \frac{v_{BA}}{AB} \right)$$

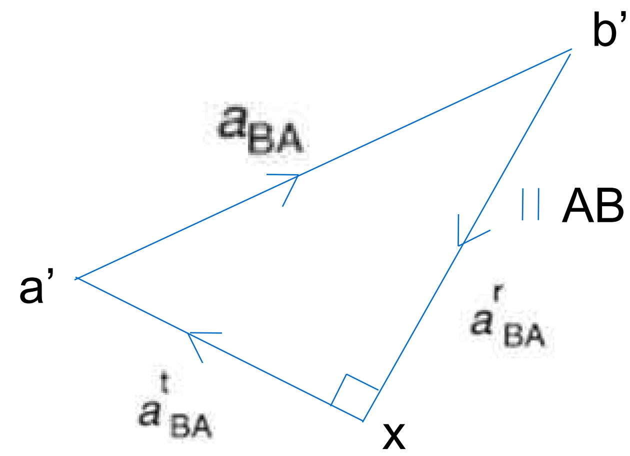
2. The tangential component, which is parallel to the velocity (i.e. Perpendicular to Link AB) of the particle at the given instant.

$$a^t_{BA} = \alpha \times \text{Length of the link } AB = \alpha \times AB$$

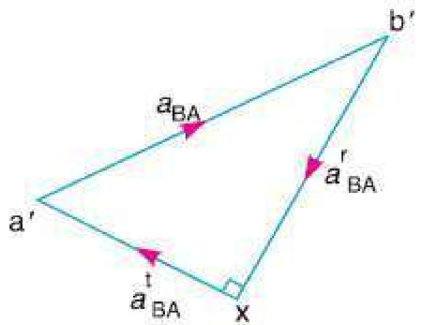
ACCELERATION DIAGRAM FOR A LINK



Acceleration for a link.



Source : R. S. Khurmi



Acceleration diagram.

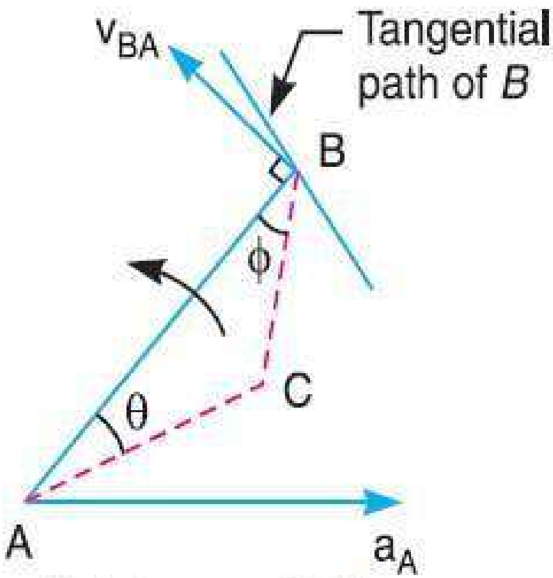
Total acceleration of B with respect to A is the vector sum of radial component and tangential component of acceleration

$$\vec{a}_{BA} = \vec{a}^{\,r}_{BA} + \vec{a}^{\,t}_{BA}$$

ACCELERATION OF A POINT ON A LINK

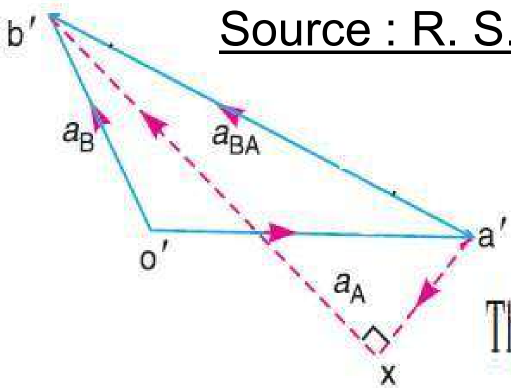
Let the acceleration of the point A i.e. a_A <u>is known in magnitude and direction</u> and the direction of path of B is given.

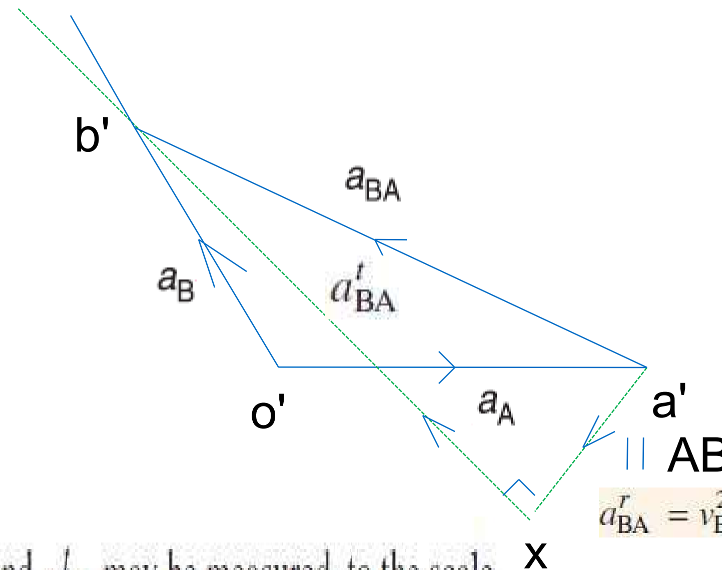
How to determine a_B ?
Draw acceleration diagram.



Points on a Link.

Source : R. S. Khurmi





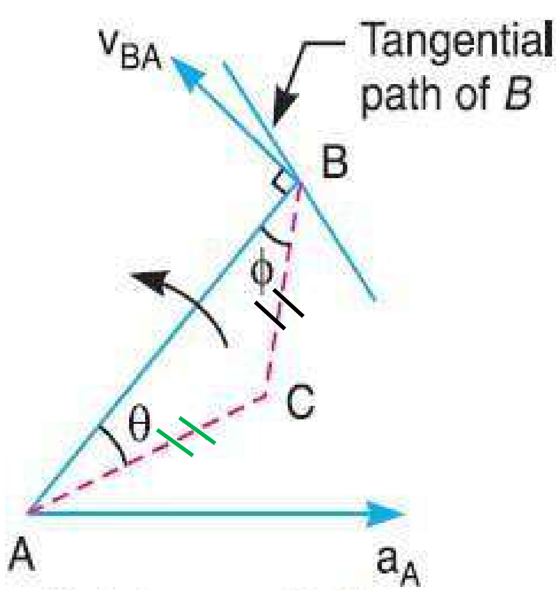
$a^r_{BA} = v^2_{BA} / AB$

The values of a_B, a_{BA} and a^t_{BA} may be measured, to the scale.

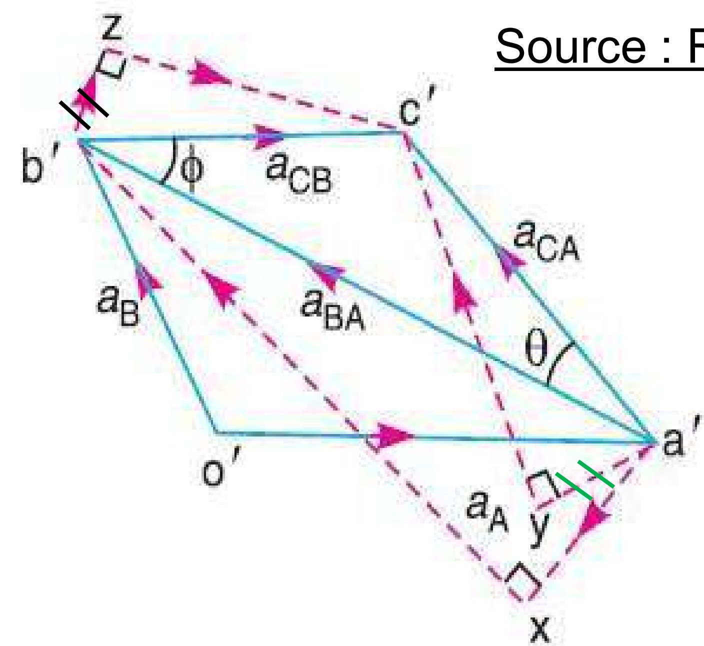
ACCELERATION OF A POINT ON A LINK

For any other point C on the link, draw triangle a'b'c' similar to triangle ABC.

Source : R. S. Khurmi



Points on a Link.

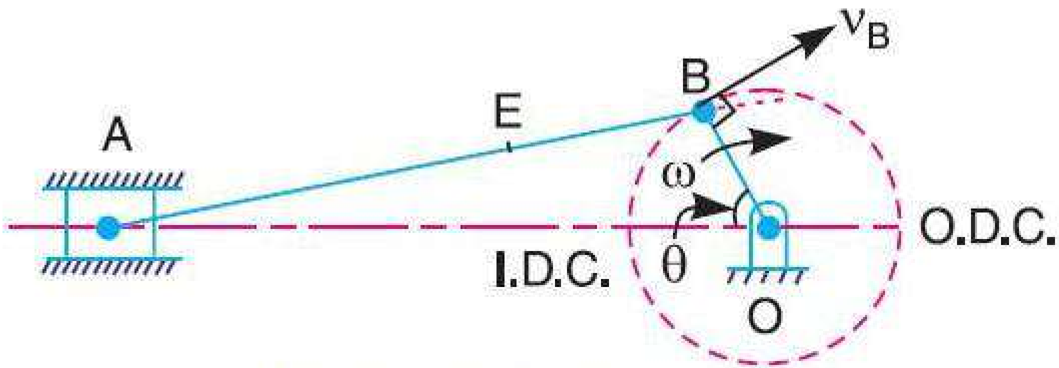


Acceleration diagram.

Mathematically, angular acceleration of the link AB,

$$\alpha_{AB} = a^t_{BA} / AB$$

ACCELERATION IN SLIDER CRANK MECHANISM



Slider crank mechanism.

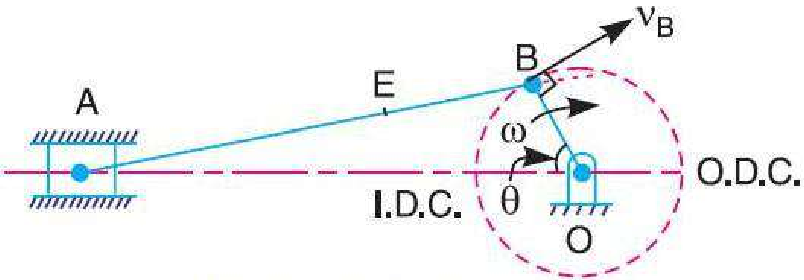
Source : R. S. Khurmi

$$v_{BO} = v_B = \omega_{BO} \times OB, \text{ acting tangentially at } B.$$

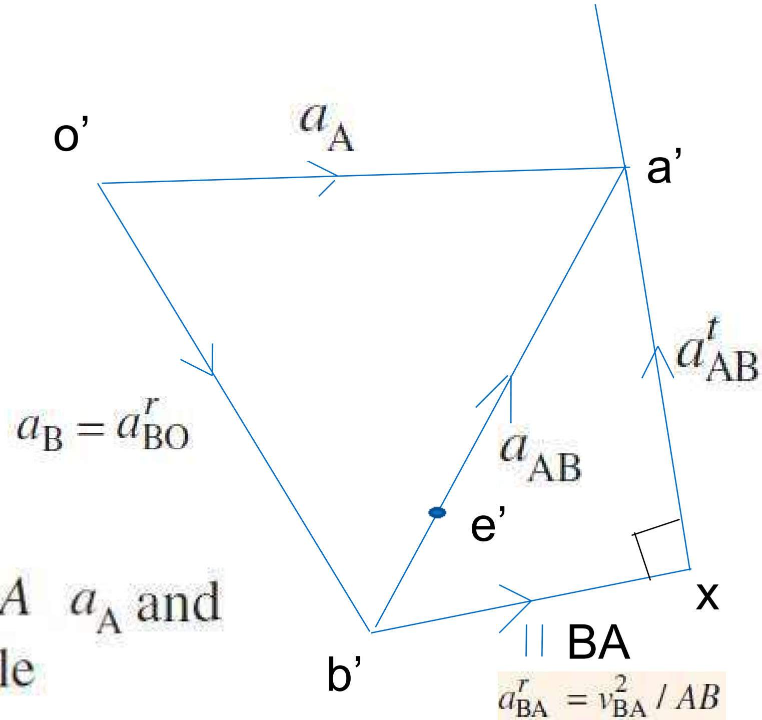
$$a^r_{BO} = a_B = \omega^2_{BO} \times OB = \frac{v^2_{BO}}{OB}$$

A point at the end of a link which moves with constant angular velocity has no tangential component of acceleration.

ACCELERATION IN SLIDER CRANK MECHANISM



Slider crank mechanism.



Source : R. S. Khurmi

acceleration of the piston or the slider A a_A and a^t_{AB} may be measured to the scale

Point e’ can be fixed using $a'e'/a'b' = AE/AB$

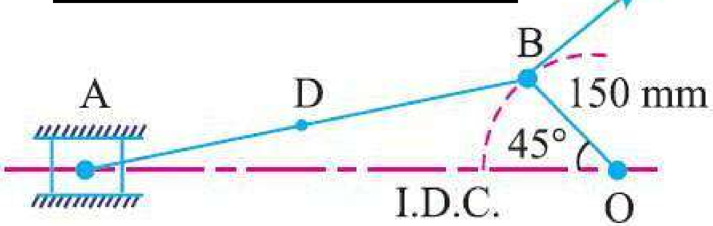
angular acceleration of AB, $\alpha_{AB} = a^t_{AB}/AB$

NUMERICAL EXAMPLE -1

The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine : 1. linear velocity and acceleration of the midpoint of the connecting rod, and 2. angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.

NUMERICAL EXAMPLE -1

Source : R. S. Khurmi



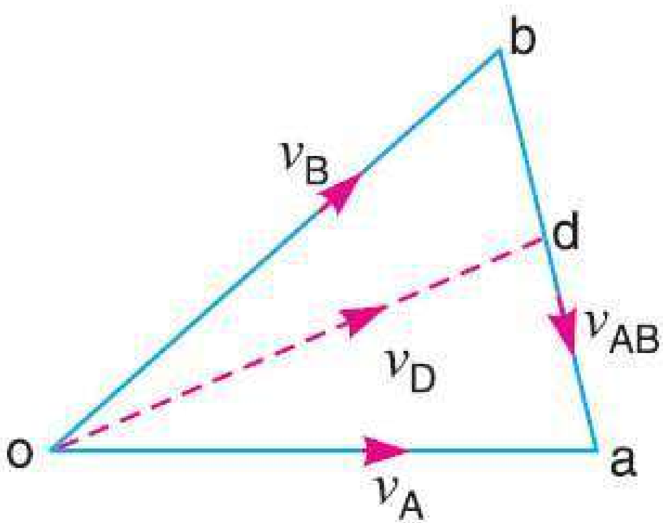
Space diagram.

Solution.

Given : $N_{BO} = 300$ r.p.m. or $\omega_{BO} = 2\pi \times 300/60 = 31.42$ rad/s;

$OB = 150$ mm $= 0.15$ m ; $BA = 600$ mm $= 0.6$ m

$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713$ m/s



Velocity diagram.

By measurement, v_{AB} = vector $ba = 3.4$ m/s

v_A = vector $oa = 4$ m/s

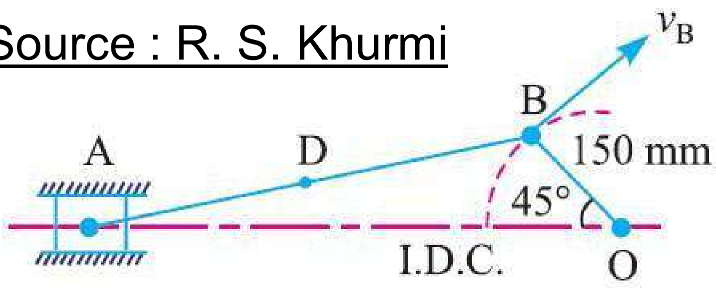
Since D is the midpoint of AB, d is also midpoint of vector ba.

velocity of the midpoint D

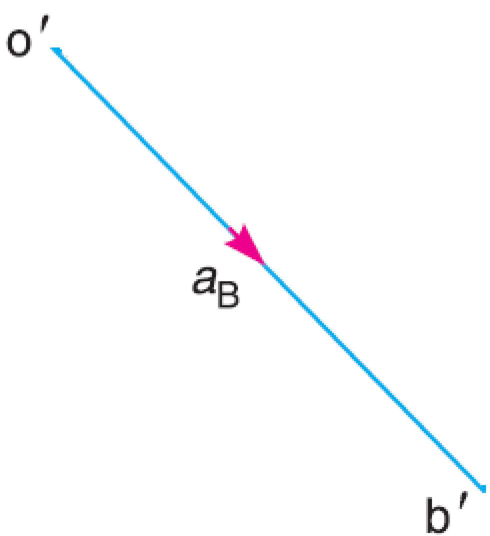
v_D = vector $od = 4.1$ m/s **Ans.**

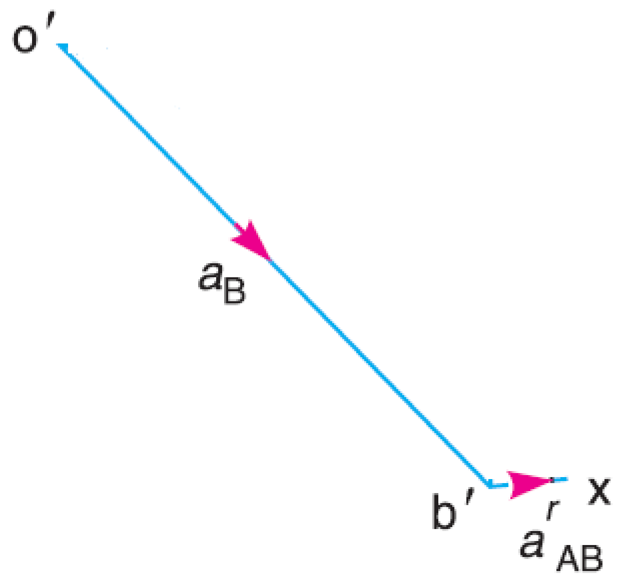
NUMERICAL EXAMPLE -1

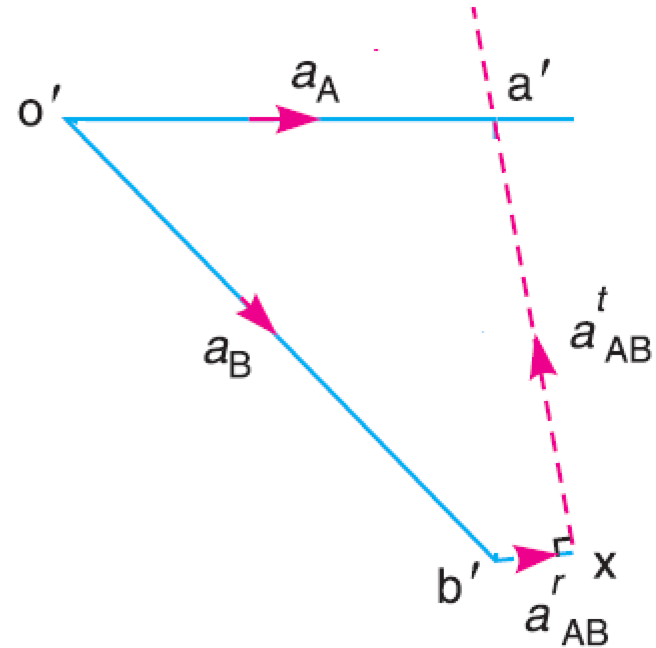
Source : R. S. Khurmi



Space diagram.



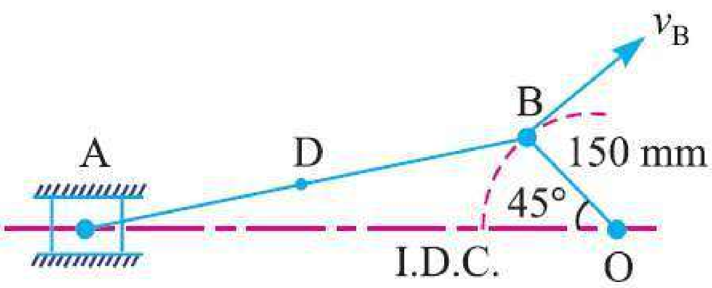




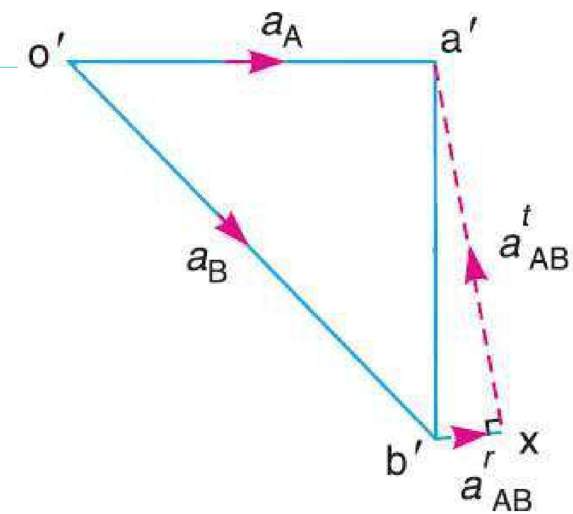
$$a^r_{BO} = a_B = \frac{v^2_{BO}}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{(3.4)^2}{0.6} = 19.3 \text{ m/s}^2$$

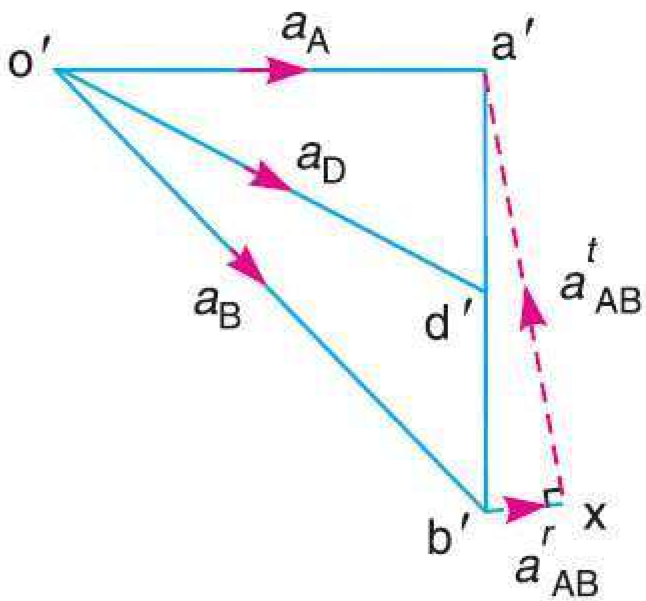
NUMERICAL EXAMPLE -1



Space diagram.



Source : R. S. Khurmi



By measurement, a_D = vector $o'\,d'$ = 117 m/s² **Ans.**

Angular velocity of the connecting rod

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 \text{ rad/s}^2 \quad \textbf{Ans.}$$

Angular acceleration of the connecting rod

From the acceleration diagram, $a^t_{AB} = 103 \text{ m/s}^2$

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67 \text{ rad/s}^2 \quad \textbf{Ans.}$$

TUTORIAL PROBLEM-1

The dimensions and configuration of the four bar mechanism, shown in Fig. 8.10, are as follows :

P_1A = 300 mm; P_2B = 360 mm; AB = 360 mm, and P_1P_2 = 600 mm.

The angle $AP_1P_2 = 60°$. The crank P_1A has an angular velocity of 10 rad/s and an angular acceleration of 30 rad/s², both clockwise. Determine the angular velocities and angular accelerations of P_2B, and AB and the velocity and acceleration of the joint B.

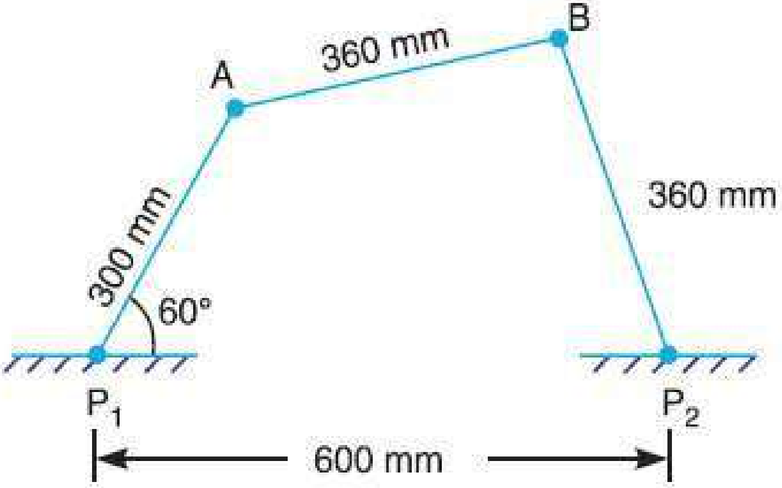


Fig. 8.10

Source : R. S. Khurmi

$$v_{BP2} = v_B = 2.2 \text{ m/s} \quad \textbf{Ans.}$$

$$\omega_{P2B} = \frac{v_{BP_2}}{P_2B} = \frac{2.2}{0.36} = 6.1 \text{ rad/s} \quad \textbf{Ans.}$$

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{2.05}{0.36} = 5.7 \text{ rad/s} \quad \textbf{Ans.}$$

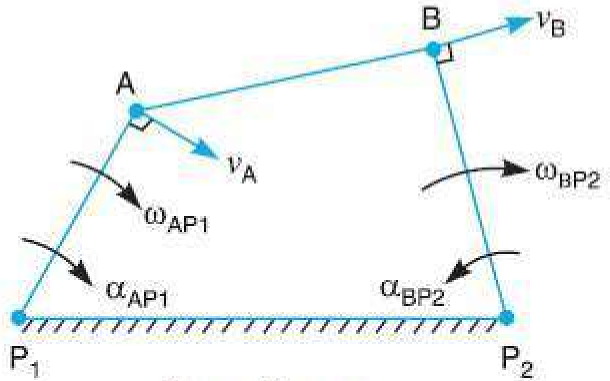
$$a_B = 29.6 \text{ m/s}^2 \quad \textbf{Ans.}$$

$$\alpha_{P2B} = \frac{a^t_{BP_2}}{P_2B} = \frac{26.6}{0.36} = 73.8 \text{ rad/s}^2 \quad \textbf{Ans.}$$

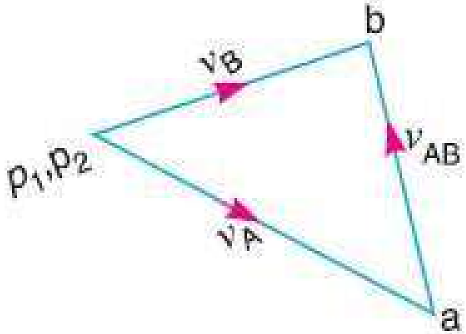
$$\alpha_{AB} = \frac{a^t_{BA}}{AB} = \frac{13.6}{0.36} = 37.8 \text{ rad/s}^2 \quad \textbf{Ans.}$$

TUTORIAL PROBLEM-1

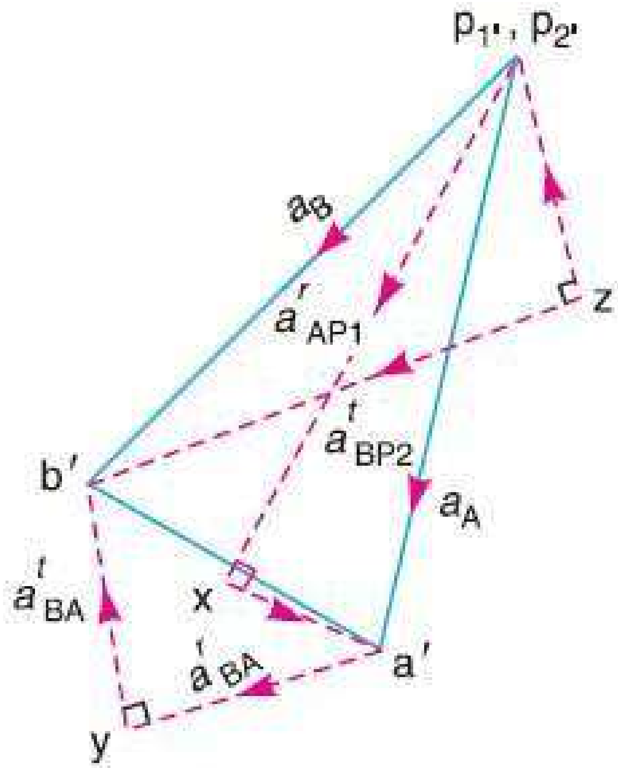
Source : R. S. Khurmi



Space diagram.

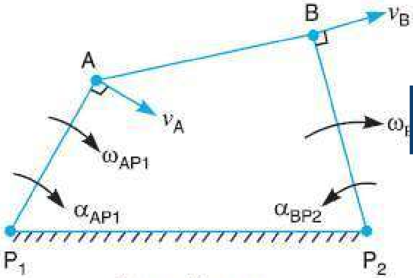


Velocity diagram.



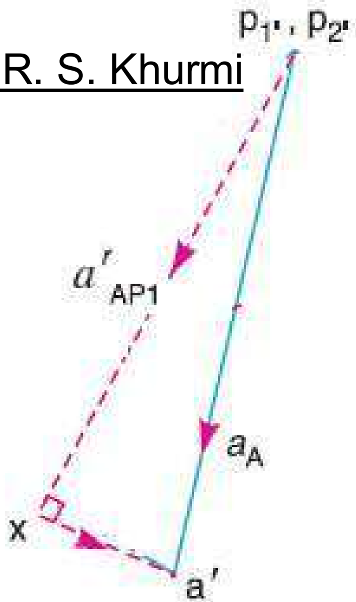
Acceleration diagram

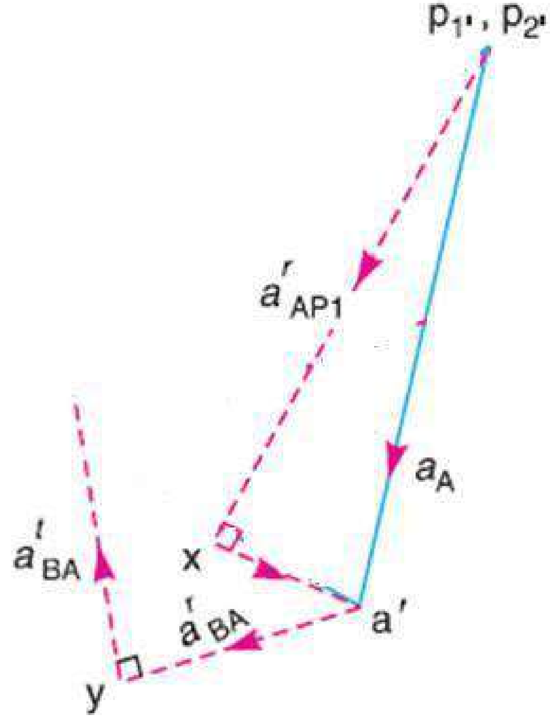
RIAL PROBLEM-1

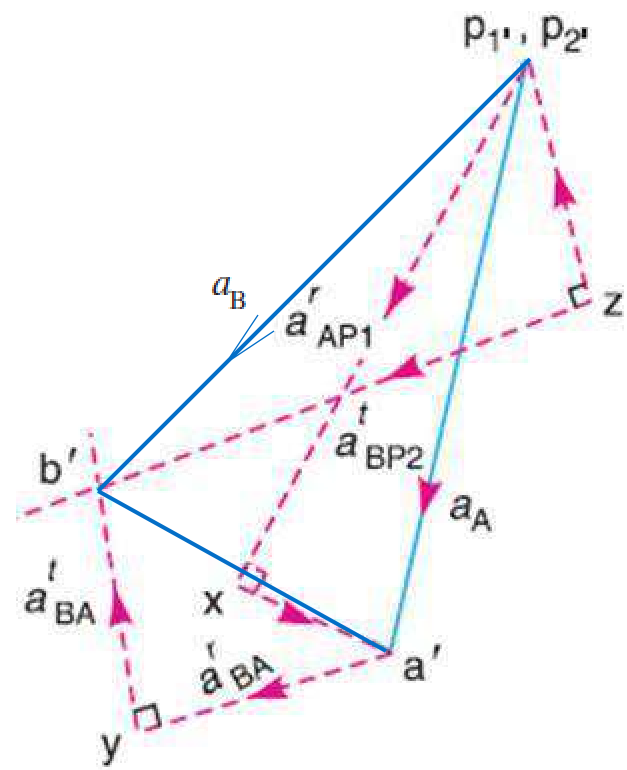


Space diagram.

Source : R. S. Khurmi







vector $p_1' x = a^r_{AP_1} = 30 \text{ m/s}^2$

vector $xa' = a^t_{AP_1} = 9 \text{ m/s}^2$

By measurement,

$a_A = a_{AP1} = 31.6 \text{ m/s}^2$

vector $a'y = a^r_{BA} = 11.67 \text{ m/s}^2$

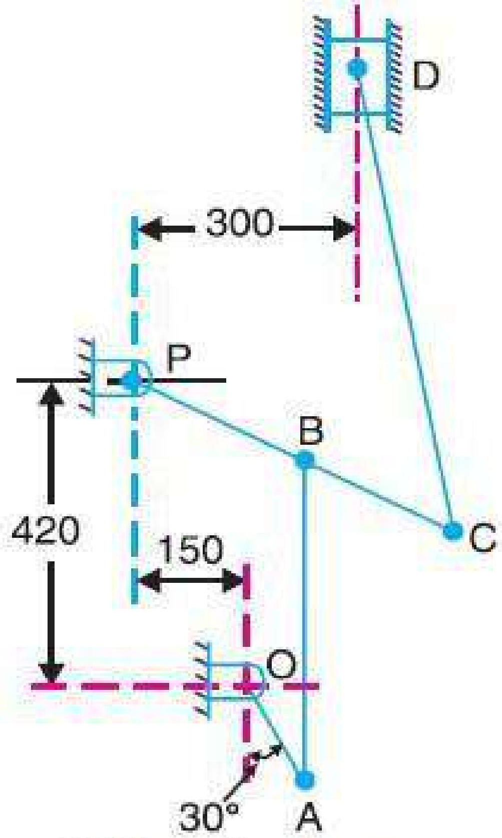
a^t_{BA} magnitude is yet unknown

$p_2' z = a^r_{BP_2} = 13.44 \text{ m/s}^2$

EXERCISE-1

Find out the acceleration of the slider D and the angular acceleration of link CD for the engine mechanism shown in Fig. 8.14.

The crank OA rotates uniformly at 180 r.p.m. in clockwise direction. The various lengths are: OA = 150 mm ; AB = 450 mm; PB = 240 mm ; BC = 210 mm ; CD = 660 mm.



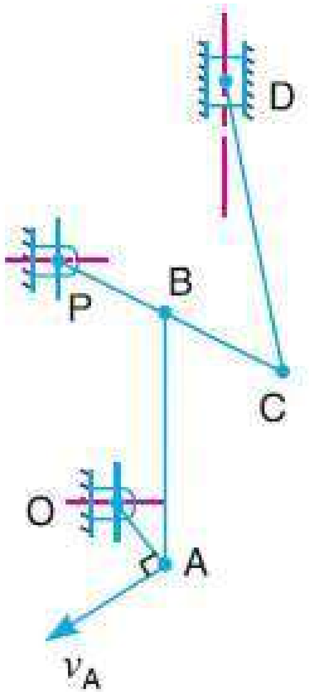
All dimensions in mm.

Fig. 8.14

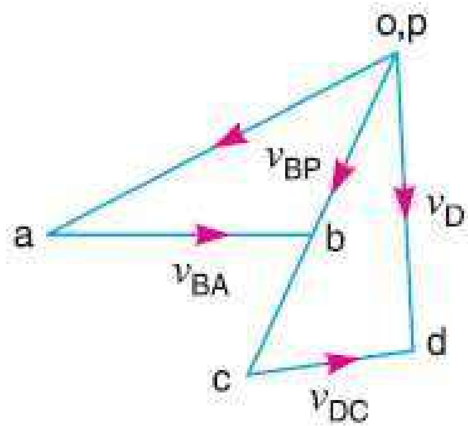
Source : R. S. Khurmi

ANSWER

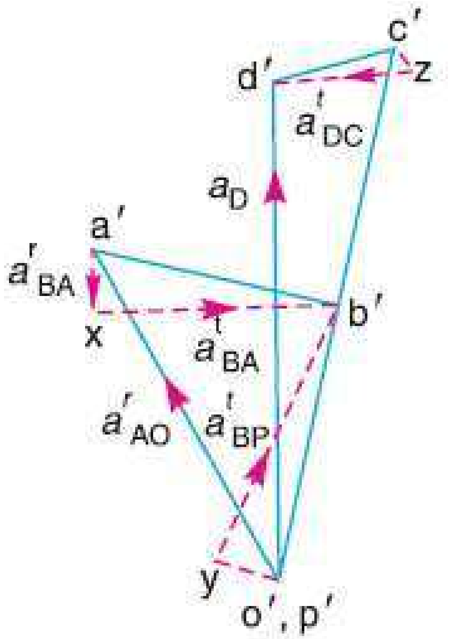
Source : R. S. Khurmi



(*a*) Space diagram.



(*b*) Velocity diagram.



(*c*) Acceleration diagram.

a_D = vector $o'd'$ = 69.6 m/s² **Ans.**

$$\alpha_{CD} = \frac{a^t_{DC}}{CD} = \frac{17.4}{0.66} = 26.3 \text{ rad/s}^2$$ **Ans.**

LECTURE 6

CORIOLIS COMPONENT OF ACCELERATION

MRCET CAMPUS

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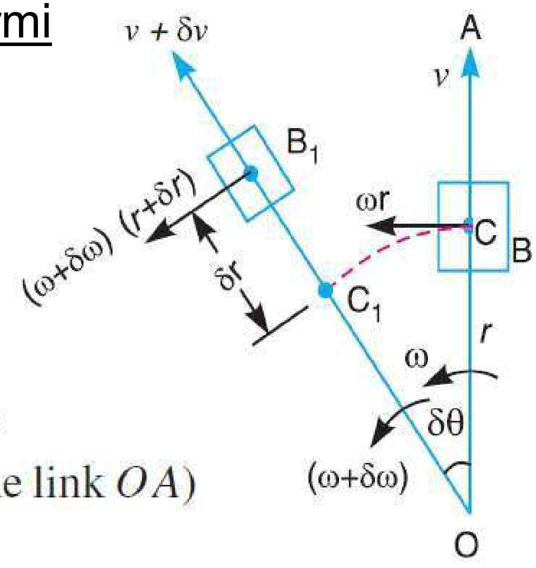
DEPARTMENT OF MECHANICAL ENGINEERING

Where?

When a point on one link is sliding along another rotating link, such as in quick return motion mechanism

Source : R. S. Khurmi



Let ω = Angular velocity of the link OA at time t seconds.

v = Velocity of the slider B along the link OA at time t seconds.

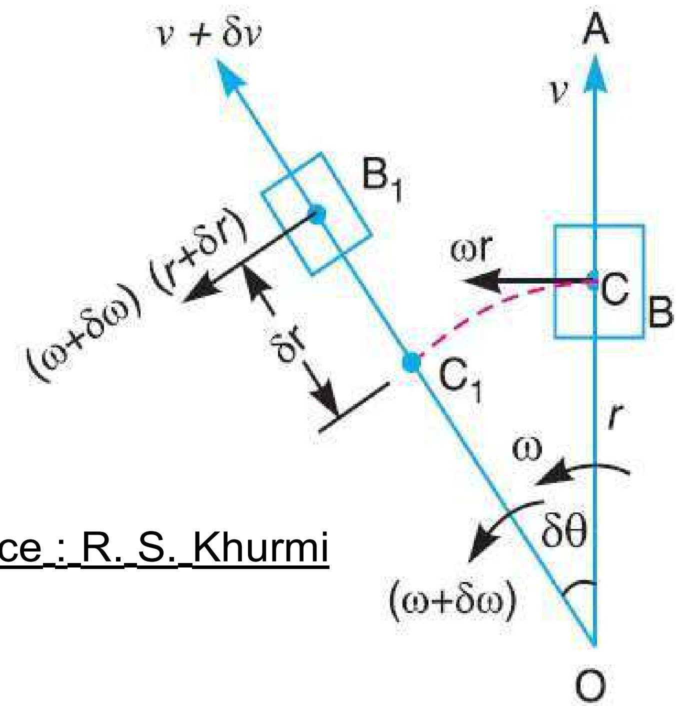
$\omega . r$ = Velocity of the slider B with respect to O (perpendicular to the link OA) at time t seconds, and

$(\omega + \delta\omega)$, $(v + \delta v)$ and $(\omega + \delta\omega)(r + \delta r)$

= Corresponding values at time $(t + \delta t)$ seconds.

CORIOLIS COMPONENT OF ACCELERATION

The tangential component of acceleration of the slider B with respect to the coincident point C on the link is known as coriolis component of acceleration and is always perpendicular to the link.



Source : R. S. Khurmi

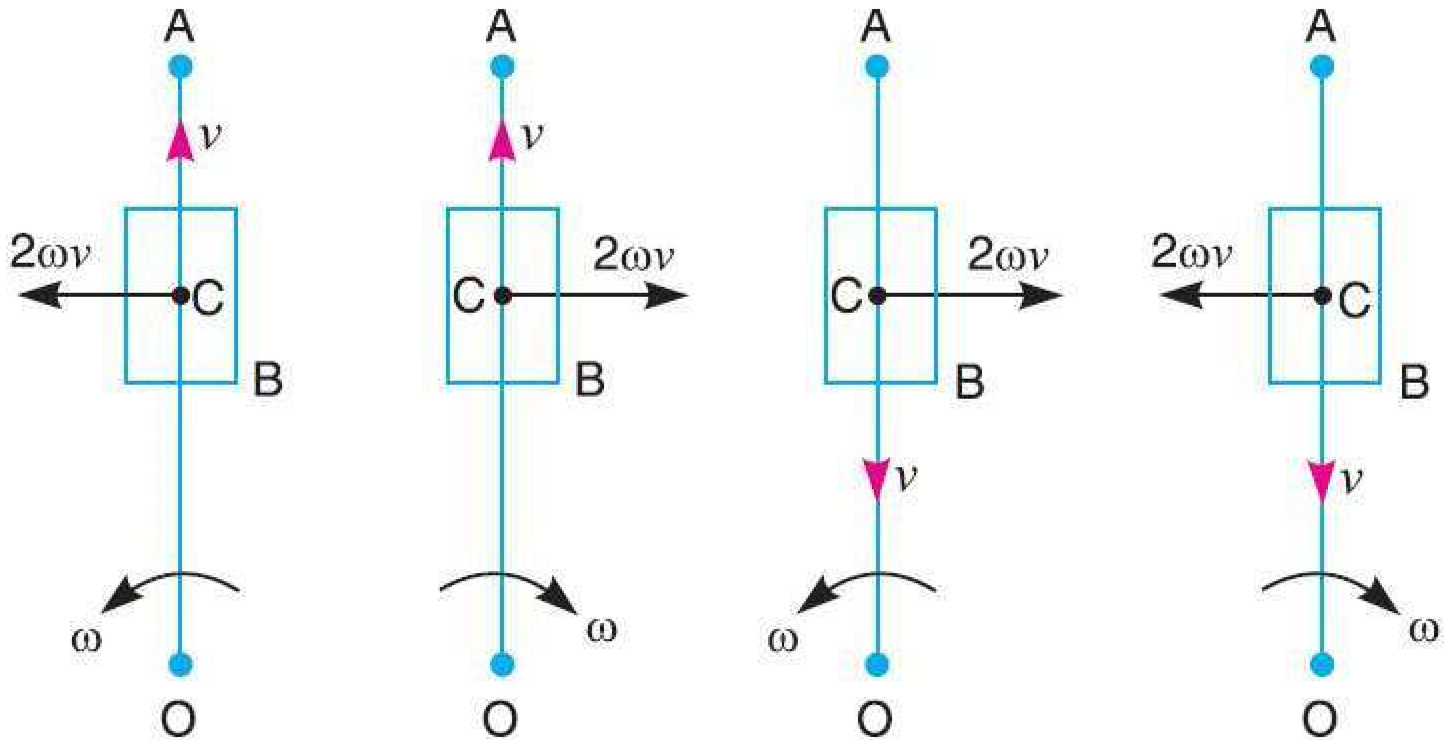
∴ Coriolis component of the acceleration of B with respect of C,

$$a_{BC}^{c} = a_{BC}^{t} = 2\,\omega.v$$

where ω = Angular velocity of the link OA, and

v = Velocity of slider B with respect to coincident point C.

CORIOLIS COMPONENT OF ACCELERATION

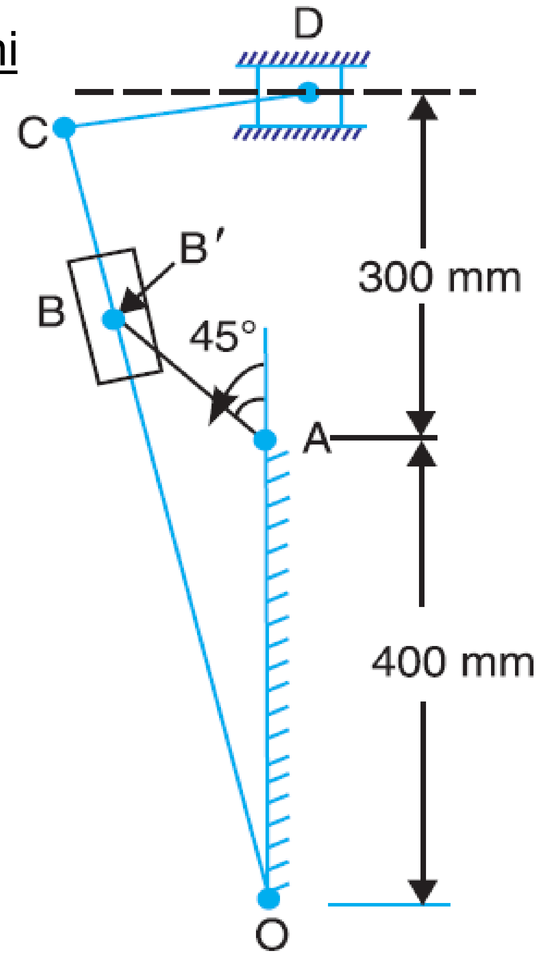


Direction of coriolis component of acceleration.

<u>Source : R. S. Khurmi</u>

NUMERICAL EXAMPLE -1

Source : R. S. Khurmi



A mechanism of a crank and slotted lever quick return motion is shown in the Fig. If the crank rotates counter clockwise at 120 r.p.m., determine for the configuration shown, the velocity and acceleration of the ram D. Also determine the angular acceleration of the slotted lever.

Crank, AB = 150 mm ; Slotted arm, OC = 700 mm and link CD = 200 mm.

NUMERICAL EXAMPLE -1 (CONSTRUCTION OF VELOCITY DIAGRAM)

Solution. Given : $N_{BA} = 120$ r.p.m or $\omega_{BA} = 2\pi \times 120/60 = 12.57$ rad/s ; $AB = 150$ mm $= 0.15$ m; $OC = 700$ mm $= 0.7$ m; $CD = 200$ mm $= 0.2$ m

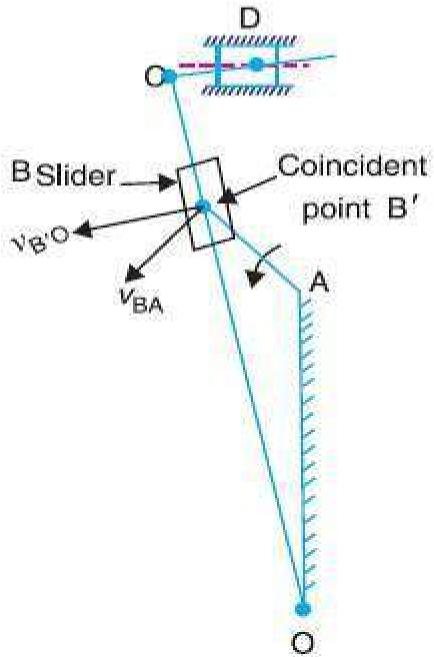
We know that velocity of B with respect to A,

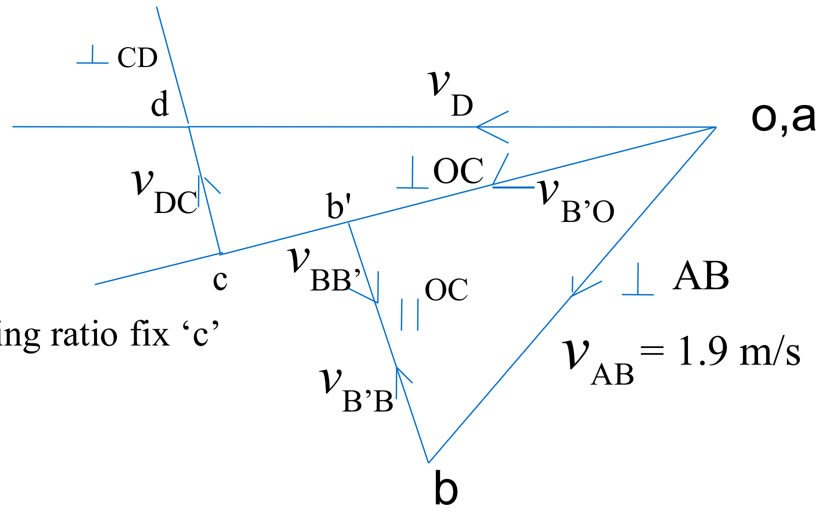
$$v_{BA} = \omega_{BA} \times AB$$

$$= 12.57 \times 0.15 = 1.9 \text{ m/s}$$

...(Perpendicular to AB)

Source : R. S. Khurmi





Using ratio fix ‘c’

NUMERICAL EXAMPLE -1

v_D = vector od = 2.15 m/s **Ans.**

From velocity diagram by measurement :

From velocity diagram, we also find that

Velocity of B with respect to B',

$v_{BB'}$ = vector $b'b$ = 1.05 m/s

Velocity of D with respect to C,

v_{DC} = vector cd = 0.45 m/s

Velocity of B' with respect to O

$v_{B'O}$ = vector ob' = 1.55 m/s

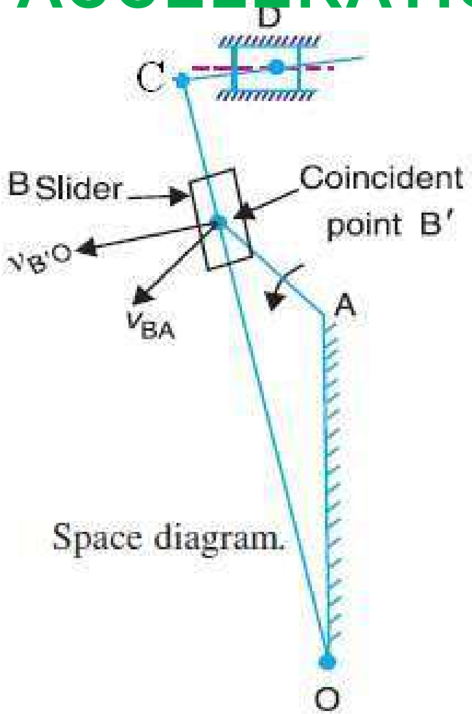
Velocity of C with respect to O,

v_{CO} = vector oc = 2.15 m/s

∴ Angular velocity of the link OC or OB',

$$\omega_{CO} = \omega_{B'O} = \frac{v_{CO}}{OC} = \frac{2.15}{0.7} = 3.07 \text{ rad/s}$$

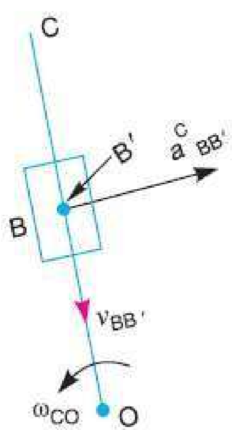
NUMERICAL EXAMPLE -1 (CONSTRUCTION OF ACCELERATION DIAGRAM)



Space diagram.

Link	Radial accel.	Tangen. accel.	Coriolis Accel.
AB	$a^r_{BA} = \omega^2_{BA} \times AB$ $= (12.57)^2 \times 0.15$ $= 23.7\ m/s^2$	Zero	Nil
BB'	Direction $a^r_{BB'}$ wn.	-	$a^c_{BB'} = 2\omega.v$ $= 2\omega_{CO}.v_{BB'}$ $= 2 \times 3.07 \times 1.05 = 6.45$ m/
DC	$a^r_{DC} = \frac{v^2_{DC}}{CD}$ $= \frac{(0.45)^2}{0.2} = 1.01\ m/s^2$	Direction known a^t_{DC}	Nil
B'O	$a^r_{B'O} = \frac{v^2_{B'O}}{B'O}$ $= \frac{(1.55)^2}{0.52} = 4.62\ m/s^2$	Direction known.	Nil

$a^t_{B'O}$



Direction of coriolis component.

Source : R. S. Khurmi