K. D.K. College of Engineering, Nagpur

Department of Mechanical Engineering

Subject: - HEAT TRANSFER

Unit No-5

K. D. K. COLLEGE OF ENGINEERING, Nagpur.

DEPARTMENT OF MECHANICAL ENGINEERING

RADIATION HT NOTES

UNIT V :

Radiation, Nature Of Thermal Radiation, Black Body Radiation, Radiation Intensity, Law Of Radiation – Kirchoffs, Planks, Weins Displacement, Stefan Boltzmann And Lamberts Co-Sine Law. Emissivity, Absorbtivity, Transmissivity, Reflectivity, Radiosity, Emissive Power, Irradiation. Radiation Network, Radiation Exchange Between Surfaces, Idea Of Shape Factor And Reciprocity Theorem, Radiation Between Parallel Plates, Cylinder And Spheres. Radiation Shields, Effect Of Radiation On Temp Measurement.

UNIT– V

RADIATION HEAT TRANSFER

Thermal radiation is the process by which the surface of an object radiates its thermal energy as a result of a temperature difference, **ΔT, by the mechanism of photon emission or electromagnetic wave emission.**

Because the mechanism of transmission is photon emission, unlike conduction and convection, there need be no intermediate matter to enable transmission.

The significance of this is that radiation will be the only mechanism for heat transfer whenever a vacuum is present.

An example of thermal radiation is the infrared radiation emitted by a common household radiator or electric heater.

An example a person near a raging bonfire will feel the radiated heat of the fire, even if the surrounding air is very cold. Thermal radiation is generated when heat from the movement of charged particles within atoms is converted to electromagnetic radiation.

Solar radiation heats the earth during the day, while at night the earth re-radiates some heat back into space.

Thermodynamic considerations show that an ideal thermal radiator, or blackbody, will emit energy at a rate proportional to the fourth power of the absolute temperature of the body and directly proportional to its surface area. Thus

$$q_{\text{emitted}} = \sigma A T^4 \quad \text{…………… Eq}^n\ 1$$

where σ is the proportionality constant and is called the Stefan-Boltzmann constant with the value of $\sigma = 5.669 \times 10^{-8}$ **W/m² K⁴**. Eqⁿ 1 is called the Stefan-Boltzmann law of thermal radiation, and it applies and governs only radiation emitted by a blackbody.

The net radiant exchange between two surfaces will be proportional to the difference in absolute temperatures to the fourth power; i.e.,

$$\frac{q_{\text{net exchange}}}{A} \propto \sigma(T_1^4 - T_2^4)$$

We call such a body black because black surfaces, such as a piece of metal covered with carbon black.

Solar radiation:

A body that emits some radiation in the visible range is called a light source. The sun is obviously our primary light source. The electromagnetic radiation emitted by the sun is known as **solar radiation,** and nearly all of it falls into the wavelength band 0.3–3 μm. Almost *half* of solar radiation is light (i.e., it falls into the visible range), with the remaining being ultraviolet and infrared.

The radiation emitted by bodies at room temperature falls into the **infrared** region of the spectrum, which extends from 0.76 to 100 μm. Bodies start emitting noticeable visible radiation at temperatures above 800 K.

The **ultraviolet** radiation includes the low-wavelength end of the thermal radiation spectrum and lies between the wavelengths 0.01 and 0.40 μm. Ultraviolet rays are to be avoided since they can kill microorganisms and cause serious damage to humans and other living organisms. About 12 percent of solar radiation is in the ultraviolet range.

Microwave ovens utilize electromagnetic radiation in the microwave region of the spectrum generated by microwave tubes called magnetrons. Microwaves in the range of 102–105 μm are very suitable for use in cooking since they are reflected by metals, transmitted by glass and plastics, and absorbed by food (especially water) molecules. Thus, the electric energy converted to radiation in a microwave oven eventually becomes part of the internal energy of the food.

The wavelength of the electromagnetic waves used in radio and TV broadcasting usually ranges between 1 and 1000 m in the radio wave region of the spectrum.

RADIATION PROPERTIES

When radiant energy strikes a material surface, part of the radiation is reflected, part is absorbed, and part is transmitted, as shown in Figure 1. We define the **reflectivity** ρ as the fraction reflected, the **absorptivity** α as the fraction absorbed, and the **transmissivity** τ as the fraction transmitted. Thus,

$$\rho + \alpha + \tau = 1$$

Absorptivity: The rate at which radiant energy is absorbed by a surface is indicated by the absorptivity of the surface, α, with values, $0 \leq \alpha \leq 1$. Opaque surfaces can absorb or reflect incident radiation. Generally, dull, rough surfaces absorb more radiant energy than bright, polished surfaces, and bright surfaces reflect more than dull surfaces.

Transmissivity: The rate at which radiant energy is transmitted by a surface is indicated by the transmissivity of the material, τ, with values, $0 \leq \tau \leq 1$. Some substances, such as gases and liquids, glass, etc., are capable of transmitting radiation. Most gases have high transmissivity, i.e. $\tau \approx 1$ and $\rho = \alpha = 0$ (like air at atmospheric pressure). However, some other gases (water vapour, CO_2 etc.) may be highly absorptive to thermal radiation, at least at certain wavelength.

Reflectivity: And finally, the rate at which radiant energy is reflected by a surface is indicated by the reflectivity of the material, ρ, with values, $0 \leq \rho \leq 1$. Bright surfaces reflect incident radiation more than dull surfaces.

Emissivity (ϵ) is the ratio of a surface's ability to emit radiant energy compared with the ability of a perfect black body of the same area at the same temperature. The ratio $\mathbf{E/E_b}$ is known as emissivity (ϵ) of a surface. Emissivity is a dimensionless constant having values between 0 and 1 i.e. $0 \leq \epsilon \leq 1$. Perfect reflectors would have $\epsilon = \mathbf{0}$. Perfect absorbers would have $\epsilon = \mathbf{1}$. Where, E_b is called the blackbody emissive power and the emissive power of a body E is defined as the energy emitted by the body per unit area and per unit time..

A material with high emissivity is efficient in both absorbing radiation energy as well as emitting it. Therefore a good absorber is also a good emitter

Two types of reflection phenomena may be observed when radiation strikes a surface. If the angle of incidence is equal to the angle of reflection, the reflection is called *specular*. On the other hand, when an incident beam is distributed uniformly in all directions after reflection, the reflection is called *diffuse*.

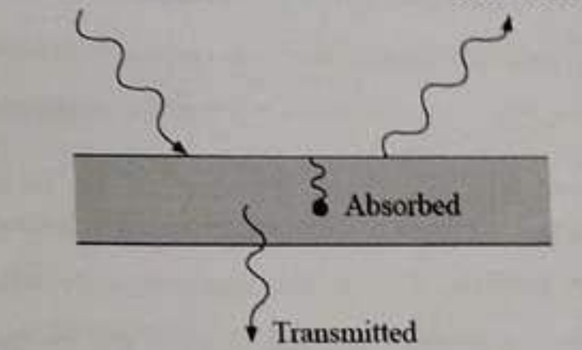
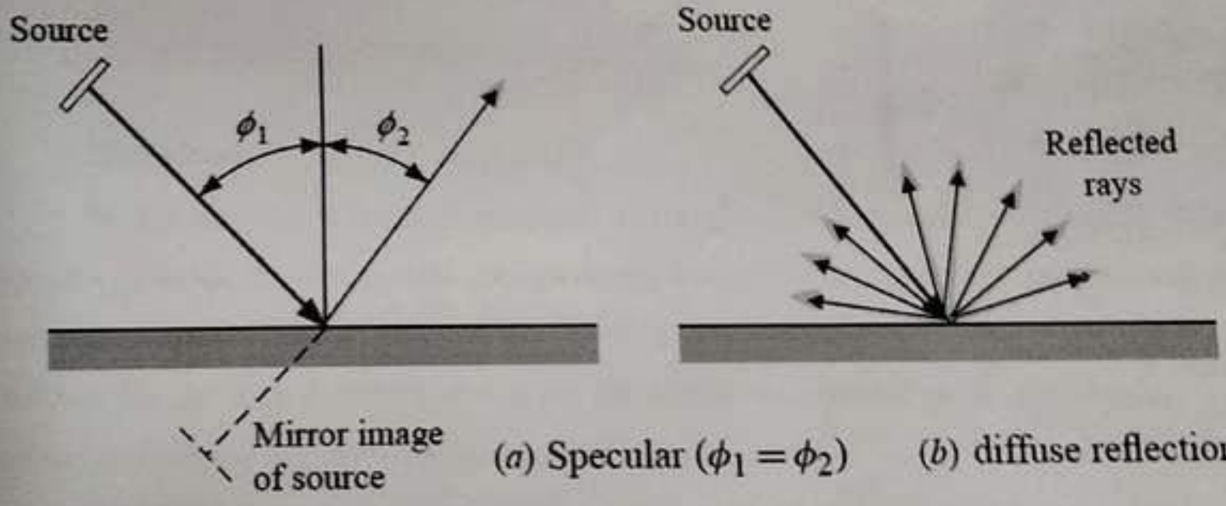


Figure 2 | (*a*) Specular ($\phi_1 = \phi_2$) and (*b*) diffuse reflection.



(*a*) Specular ($\phi_1 = \phi_2$) (*b*) diffuse reflection.

Opaque Body:

A body is considered **transparent** if it can transmit some of the radiation waves falling on its surface. Opaque surfaces can absorb or reflect incident radiation. If electromagnetic waves are not transmitted through the substance it is therefore called opaque. When radiation waves hit the surface of an opaque body, some of the waves are reflected back while the other waves are absorbed by a thin layer of the material close to the surface. For engineering purposes all materials are thick enough that they can be considered for opaque surfaces, $\tau = 0$ and equation $\rho + \alpha + \tau = 1$ reducing to: $\alpha + \rho = 1$.

The Gray Body:

A *gray body* is defined such that the monochromatic emissivity ϵ_λ of the body is independent of wavelength. The *monochromatic emissivity* is defined as the ratio of the monochromatic emissive power of the body to the monochromatic emissive power of a blackbody at the same wavelength and temperature. Thus

$$\epsilon_\lambda = \frac{E_\lambda}{E_{b\lambda}}$$

Blackbody

Within the visual band of radiation, any material, which absorbs all visible light, appears as black. Extending this concept to the much broader thermal band, we speak of surfaces with $\alpha = 1$ (therefore $\alpha = 0$ **and** $\rho = 0$) as also being "black" or "thermally black". It follows that for such a surface, $\varepsilon = 1$ and the surface will behave as an ideal emitter. The terms ideal surface and black surface are used interchangeably.

A **blackbody** is defined as ***a perfect emitter and absorber of radiation***. At a specified temperature and wavelength, no surface can emit more energy than a blackbody. A blackbody absorbs ***all*** incident radiation, regardless of wavelength and direction. Also, a blackbody emits radiation energy uniformly in all directions per unit area normal to direction of emission. That is, a blackbody is a ***diffuse*** emitter. The term *diffuse* means "independent of direction."

The radiation energy emitted by a blackbody per unit time and per unit surface area was expressed as $E_b(T) = \sigma T^4$. $E_b \rightarrow W/m^2$

where σ = 5.67 x 10^{-8} W/m^2 · K^4 is the **Stefan–Boltzmann constant** and T is the absolute temperature of the surface in K and E_b is called the **blackbody emissive power.**

We call such a body black because black surfaces, such as a piece of metal covered with carbon black.

Emissive power

It is the emitted thermal radiation leaving a system per unit time, per unit area of surface. The total emissive power of a surface is all the emitted energy, summed over all the direction and all wavelengths, and is usually denoted as E. The total emissive power is found to be dependent upon the temperature of the emitting surface, the subsystem which this system is composed, and the nature of the surface structure or texture. The monochromatic emissive power Eλ, is defined as the rate, per unit area, at which the surface emits thermal radiation at a particular wavelength λ. Thus the total and monochromatic hemispherical emissive power are related by

$$E = \int_0^\infty E_\lambda d\lambda \qquad E \rightarrow W/m^2$$

and the functional dependency of E_λ on λ must be known to evaluate E.

Laws of Radiation

1) Stefan–Boltzmann law:

Thermodynamic considerations show that an ideal thermal radiator, or blackbody, will emit energy at a rate proportional to the fourth power of the absolute temperature of the body and directly proportional to its surface area. Thus

$Q_{emitted} \rightarrow watt$

$E_b \rightarrow W/m^2$

$$Q_{emitted} = \sigma A T^4 \text{ or } E_b = \sigma T^4 \text{............... Eq}^n\ 1$$

where σ is the proportionality constant and is called the Stefan-Boltzmann constant with the value of σ = **5.669×10^{-8} W/m^2 K^4**. E_b is the energy radiated per unit time and per unit area by the ideal radiator, where $E_b = \frac{Q_{emitted}}{A}$ is in watts per square meter and T is in degrees

Quantum Theory : Individual unit of energy of that wavelength of light is called or treated as Quanta.

plank found that radiation from glowing body changes in color from Red to orange & finally to blue as its temperature rises. He assumed that each light ray has different wavelength at different temp. This is same in monochromatic light. That theory named as Quantum theory.

Prob: Determine the maximum ~~emissive~~ monochromatic emissive power at temp 1400 K. Also find the value of monochromatic emissive power at 4 μm at same temp.

Sol:
$$E_{b\lambda} = \frac{C_1 \lambda_{max}^{-5}}{[\exp(C_2/\lambda_{max} T) - 1]}$$

from wein's law $\lambda_{max} T = 2.898 \times 10^{-3}$ mk.

$\lambda_{max} = 2.898 \times 10^{-3}/1400 = 2.07 \times 10^{-6}$ m

$\therefore E_{b\lambda max} = 6.922 \times 10^{10}$ w/m^3

$E_{b\lambda} =$ at $\lambda = 4$ μm & $T = 1400$ k,

$\therefore E_{b\lambda} = 3.031 \times 10^{10}$ w/m^3

Kelvin. The subscript b in Eq^n 1 denotes that this is the radiation from a blackbody. We call this blackbody radiation because materials that obey this law appear black to the eye; they appear black because they do not reflect any radiation. Thus a blackbody is also considered as one that absorbs all radiation incident upon it. E_b is called the emissive power of a blackbody.

Eq^n 1 is called the Stefan-Boltzmann law of thermal radiation which states that heat radiated is proportional to the fourth power of the absolute temperature of the surface, and it applies and governs only radiation emitted by a blackbody.

2) Planck's law:

The *monochromatic emissivity* is defined as the ratio of the monochromatic emissive power of the body to the monochromatic emissive power ($E_{b\lambda}$) of a blackbody at the same wavelength λ and temperature T. Thus

$$\epsilon_\lambda = \frac{E_\lambda}{E_{b\lambda}} = \frac{\text{Monochromatic emissive power of any body}}{\text{Monochromatic emissive power of black body}}$$

The variation of monochromatic emissive power with wave length is given by Planck's law derived in 1900 using quantum theory. The monochromatic emissive power at any given wavelength and temperature is given by

$$E_{b\lambda}(\lambda, T) = \frac{C_1}{\lambda^5\left[exp\left(\frac{C_2}{\lambda T}\right)-1\right]} \quad W/m^2/m \quad \text{.............. Eq}^n\ 2$$

$C_1 = 3.742 \times 10^{-16}$ *W.m2*

$C_2 = 1.4388 \times 10^{-2}$ *mK*

Also, T is the absolute temperature of the surface, λ is the wavelength of the radiation emitted,

The variation of the spectral blackbody emissive power with wavelength is plotted in Figure for selected temperatures. Several observations can be made from this figure:

1. The emitted radiation is a continuous function of *wavelength.* At any specified temperature, it increases with wavelength, reaches a peak, and then decreases with increasing wavelength.

Prob ① Calculate the radiant flux density from black body at 400°C. If the emitted radiant energy is to be doubled, to what temp. surface of the black body need to be raised?

Soln: from stefan boltzmann Law,

$E_b = \sigma_b T^4$; $\sigma_b = 5.67 \times 10^{-8}\ W/m^2K^4$

$T = 400 + 273 = 673\ K$

$E_b = 11631.7\ W/m^2$

For doubled radiant energy, Abs. temp will be,

$2 \times 11631.7 = \sigma_b \times T^4$ $\sigma_b = 5.67 \times 10^{-8}\ W/m^2K^4$

$T = 800.3\ K = 527.3°C$

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prob on weins Law

A furnace emits radiation at 2000K. Treating it as a black body radiation, calculate the,

① monochromatic radiant flux density at 1 µm wavelength

② wavelength at which emission is maximum ~~corresponding radiant flux density~~.

③ Total emissive power

④ wavelength λ such that emission ~~and the corresponding radiant flux density~~ from 0 to λ equal to the emission from λ to ∞.

Soln: from plank's Law

① $$E_{b\lambda} = \frac{C_1 \lambda^{-5}}{\exp\left(C_2/\lambda T\right) - 1} = 2.81 \times 10^{7}\ W/m^2$$ per m wavelength

② from wein's displacement Law

$$\lambda_{max} T = 2.898 \times 10^{-3}\ mK.$$

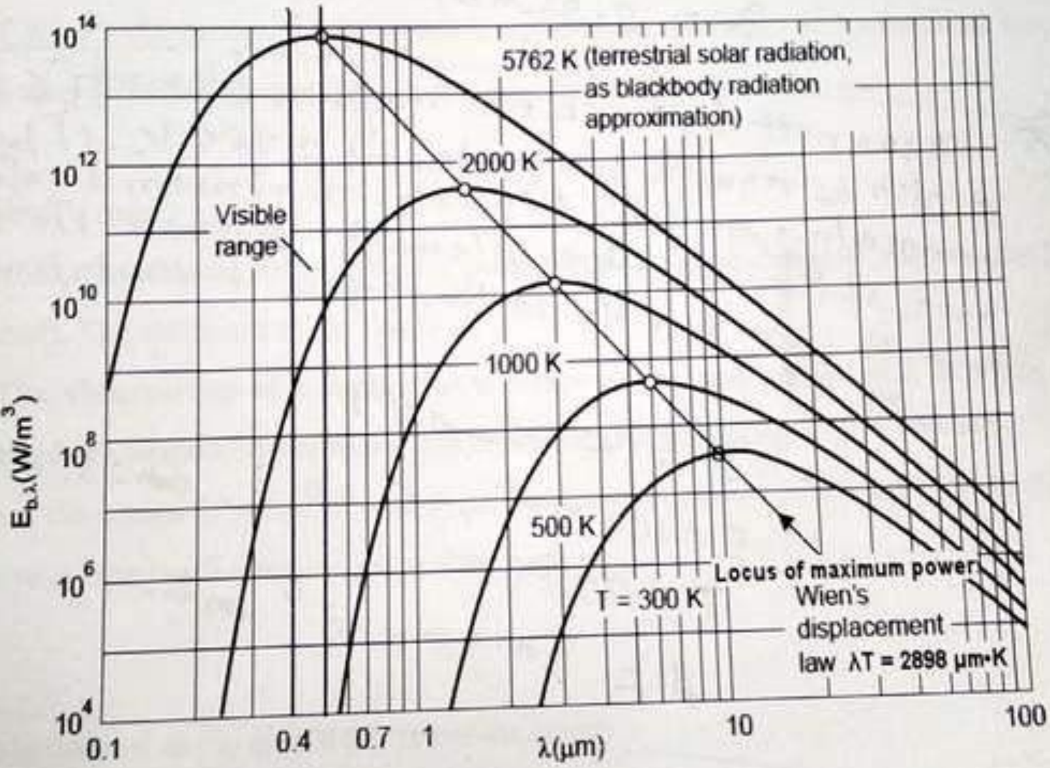
$$\lambda_{max} = \frac{2.898 \times 10^{-3}}{2000} = 1.449 \times 10^{-6}\ m$$

$$\lambda_{max} = 1.449\ \mu m$$

2. At any wavelength, the amount of emitted radiation *increases* with increasing temperature.

3. As the temperature increases, the maximum value is found to occur at lower wavelengths. Consequently, a larger fraction of the radiation is emitted at *shorter wavelengths* at higher temperatures.

4. The radiation emitted by the *sun,* which is considered to be a blackbody at 5780 K (or roughly at 5800 K), reaches its peak in the visible region of the spectrum. Therefore, the sun is in tune with our eyes. On the other hand, surfaces at $T < 800$ K emit almost entirely in the infrared region and thus are not visible to the eye unless they reflect light coming from other sources.



(3) Total Emissive power

$$E = \sigma T^4 \quad \sigma = 5.67 \times 10^{-8} \, W/m^2 K^4$$

$$E = 907200 \, W/m^2$$

(4) The emission in the band width 0 to λ is half of the total emission from 0 to ∞

Therefore, fractional emissive power is 0.5

$\therefore f_{0-\lambda} = 0.5$ at this fraction

$\lambda T = 4100 \quad \& \; T = 2000K.$

$\therefore \lambda = 2.05 \, \mu m$

prob (3) A 100 watt light bulb has tungsten filament ($\epsilon = 0.3$) which is required to operate at 2780 K. If bulb is completely evacuated. calculate minimum surface area of the tungsten filament. Assume no radiant energy strikes the bulb surface & heat steady state condns.

$$E = \epsilon \sigma_b A T^4$$

$E = 100$ watt, $\epsilon = 0.3$, $\sigma_b = 5.67 \times 10^{-8}$

$T = 2780$

$A = 0.98 \times 10^{-4} \, m^2$

3) Wien's displacement law:

The wavelength at which the maximum value of monochromatic emissive power ($E_{b\lambda}$) occurs at any given temperature can be determined by differentiating the Eq^n 2 with respect to λ while holding T constant and equating it to zero. As the temperature increases, the peak of the curve in Figure shifts toward shorter wavelengths. The wavelength at which the peak occurs for a specified temperature is given by Wien's displacement law as

$$\lambda_{max} T = 2.898 \times 10^{-3} \text{ mK}$$

Wien's Displacement law states that the wavelength λmax corresponding to the maximum energy is inversely proportional to the absolute temperature T of the hot body.
A plot of Wien's displacement law, which is the locus of the peaks of the radiation emission curves, is also given in Figure. It is seen that λ_{max} decreases as T increases.

4) Kirchhoff's law:

An important relationship between emissivity (ϵ) and absorbtivity (α) was postulated by Kirchhoff. The statement of the law is

"The absorptivity of a surface at a temperature and wavelength is equal to the emissivity of the surface at the same temperature and wavelength".
the ratio of the emissive power of a body to the emissive power of a blackbody at the same temperature is equal to the absorptivity of the body.

$$\frac{E}{E_b} = \alpha$$

This ratio is defined as the emissivity (ε) of the body,

$$\frac{E}{E_b} = \epsilon$$

so that $\epsilon = \alpha$ is called Kirchhoff's identity.

Equation $\epsilon = \alpha$ is the **Kirchhoff's law, which states that the emissivity of a body which is in thermal equilibrium with its surrounding is equal to its absorptivity of the body.** It should be noted that the source temperature is equal to the temperature of the irradiated surface. However, in practical purposes it is assumed that emissivity and absorptivity of a system are equal even if it is not in thermal equilibrium with the surrounding. The reason being the absorptivity of most real surfaces is relatively insensitive to temperature and wavelength. This particular assumption leads to the concept of grey body. The emissivity is considered to be independent of the wavelength of radiation for grey body.

5) Lambert's Cosine Law:

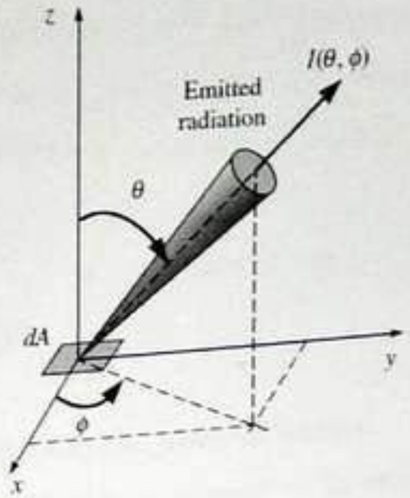
The law states that the total emissive power E_θ of a radiating plane surface in any direction is directly proportional to the cosine of the angle of emission. The angle of emission θ is the angle subtended by the normal to the radiating surface and the direction vector of emission of the receiving surface. If E_n be the total emissive power of radiating surface in the direction of its normal, then $\boldsymbol{E_\theta = E_n \cos \theta}$

The above equation is true only for diffused radiation surface. The radiation emanating from a point on a surface is termed diffused in the intensity, *I*, is constant. The law is also known Lambert's Cosine Law on diffuse radiation.

Irradiation:

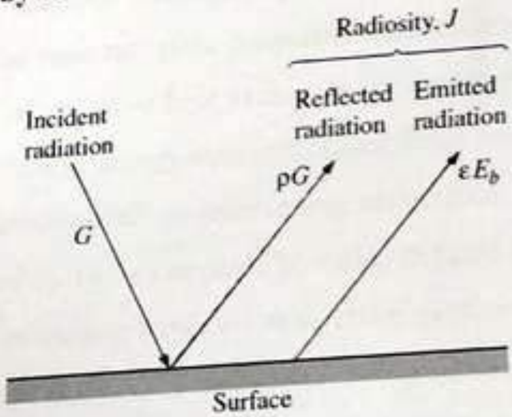
All surfaces emit radiation, but they also receive radiation emitted or reflected by other surfaces. The intensity of incident radiation Ii (θ, φ) is defined as *the rate at which radiation energy dG is incident from the (θ, φ) direction per unit area of the receiving surface normal to this direction and per unit solid angle about this direction* (Fig.). Here **θ** is the zenith angle between the direction of incident radiation and the normal of the surface and φ is the azimuth angle.

The radiation flux incident on a surface from *all directions* is called **irradiation** *G*.



Radiosity

Surfaces emit radiation as well as reflect it, and thus the radiation leaving a surface consists of emitted and reflected parts. The calculation of radiation heat transfer between surfaces involves the total radiation energy streaming away from a surface, with no regard for its origin. **The total radiation energy leaving a surface per unit time and per unit area is the radiosity and is denoted by J.**



As shown in Figure, the radiosity is the sum of the energy emitted and the energy reflected when no energy is transmitted, or

For a surface that is *gray* and *opaque*, the radiosity (J) can be expressed as and where ϵ is the emissivity and E_b is the blackbody emissive power and G is the irradiation i.e. total radiation incident upon a surface per unit time and per unit area. Since the transmissivity is assumed to be zero, the reflectivity (ρ) may be expressed as

$$\rho = 1 - \alpha = 1 - \epsilon$$

So that $J = \epsilon E_b + (1 - \epsilon)G$ W/m^2

Where $E_b = \sigma T^4$ is the blackbody emissive power of surface.

That is, the radiosity of a blackbody is equal to its emissive power. This is expected, since a blackbody does not reflect any radiation, and thus radiation coming from a blackbody is due to emission only.

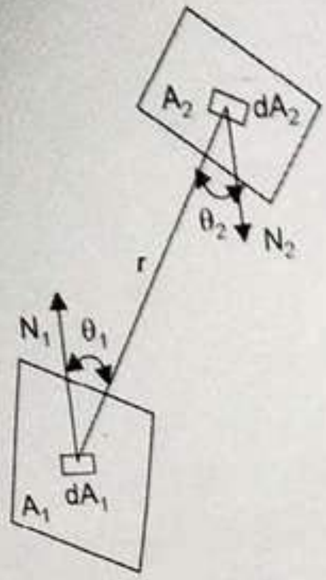
Shape Factors, Angle factor, view factor, configuration factor:

Radiation heat transfer between surfaces depends on the orientation of the surfaces relative to each other as well as their radiation properties and temperatures. Radiation shape factor is useful in the analysis of radiative heat exchange between two surfaces. Shape factor is defined as the fraction of radiative energy that is diffused from one surface of the element and strikes the other surface directly with no intervening reflections. The view factor from a surface *i* to a surface *j* is denoted by $F_{i \rightarrow j}$ or just F_{ij}, and is defined as

F_{ij} _ the fraction of the radiation leaving surface i that strikes surface j directly.

the view factor F_{12} represents the fraction of radiation leaving surface 1 that strikes surface 2 directly, and F_{21} represents the fraction of the radiation leaving surface 2 that strikes surface 1 directly.

Reciprocity Theorm:



Consider two black surfaces A_1 and A_2, as shown in Figure. To obtain a general expression for the energy exchange between these surfaces when they are maintained at different temperatures. The problem becomes essentially one of determining the amount of energy that leaves one surface and reaches the other. To solve this problem the radiation shape factors are defined as fraction of radiant energy emission form surface 1 reaching the surface 2.

The energy leaving surface 1 and arriving at surface 2 is

$$Q_{1-2} = E_{b1}A_1F_{1-2}$$

And rate of total energy radiated by A_1 surface is $Q_1 = A_1\sigma T_1^4 = E_{b1}A_1$

where, Emissive Power $E_b = \sigma T^4$
Q = Radiation H.T. rate

and the energy leaving surface 2 and arriving at surface 1 is

$$Q_{2-1} = E_{b2}A_2F_{2-1}$$

And rate of total energy radiated by A_2 surface is $Q_2 = A_2\sigma T_2^4 = E_{b2}A_2$

Hence the fraction of the rate of energy leaving area A_1 and impinging on area A_2 is given by

$$\frac{Q_{1-2}}{Q_1} = \frac{E_{b1}A_1F_{1-2}}{E_{b1}A_1} = F_{1-2}$$

And the fraction of the rate of energy leaving area A_2 and impinging on area A_1 is given by

$$\frac{Q_{2-1}}{Q_2} = \frac{E_{b2}A_2F_{2-1}}{E_{b2}A_2} = F_{2-1}$$

F_{1-2} or F_{2-1} is known as **Shape Factors, Angle factor, view factor, configuration factor.**

Since the surfaces are black, all the incident radiation will be absorbed, and the net energy exchange is

$$E_{b1}A_1F_{1-2} - E_{b2}A_2F_{2-1} = Q_{1-2}$$

If both surfaces are at the same temperature, there can be no heat exchange, that is, $Q_{1-2} = 0$.

Also, for $T_1 = T_2$

$$E_{b1} = E_{b2}$$

so that

$$A_1F_{12} = A_2F_{21}$$

Equation is known as a **reciprocity relation**, and it applies in a general way for any two surfaces i and j: $A_iF_{ij} = A_jF_{ji}$

Radiation shield:

The radiative heat transfer from one surface to another with any interfering surface (an interfering shield) in between, which is termed as radiation shield. A radiation shield is a barrier wall of low emissivity placed between two surfaces which reduce the radiation between the bodies. In fact, the radiation shield will put additional resistance to the radiative heat transfer between the surfaces. Radiation shields are used to reduce the rate of radiation heat transfer to or from an object that must be thermally isolated. The lower the emissivity of the shield, the higher the resistance. Radiation shields are commonly used on spacecraft, where radiation is the sole mechanism for heat transfer. Radiation shields are also used in temperature measurements of fluids to reduce the error caused by the radiation effect when the temperature sensor is exposed to surfaces that are much hotter or colder than the fluid itself.

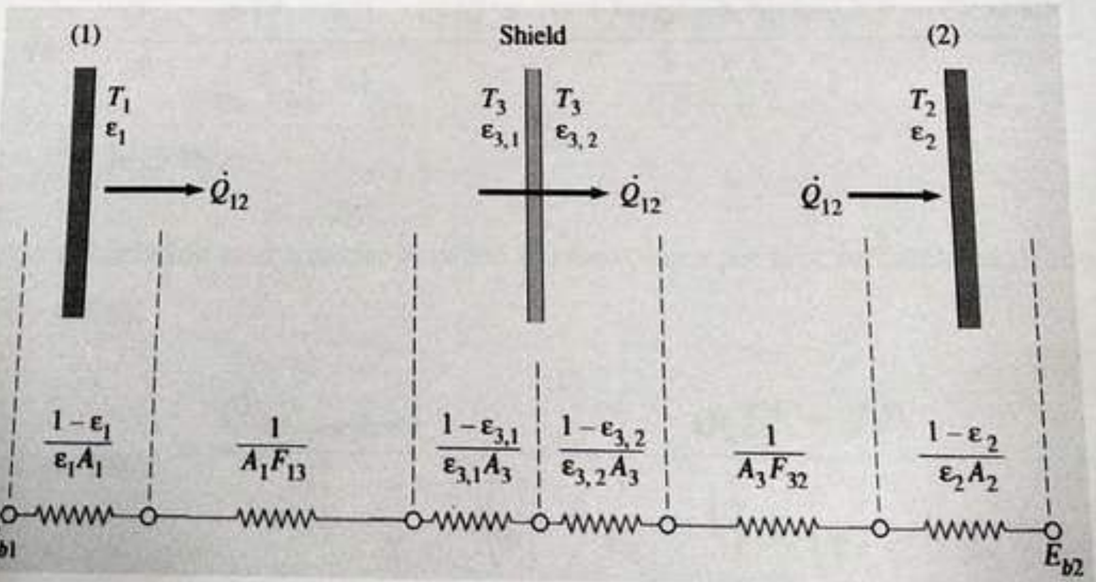


FIGURE: The radiation shield placed between two parallel plates and the radiation network associated with it.

Ques. 1

A thin aluminum sheet with an emissivity of 0.1 on both sides is placed between two very large parallel plates that are maintained at uniform temperatures $T_1 = 800$ K and $T_2 = 500$ K and have emissivities $\varepsilon_1 = 0.2$ and $\varepsilon_2 = 0.7$, respectively. Determine the net rate of radiation heat transfer between the two plates per unit surface area of the plates and compare the result to that without the shield.

Solution:

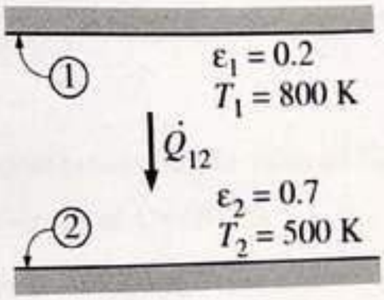


Fig.: The two parallel plates without shield

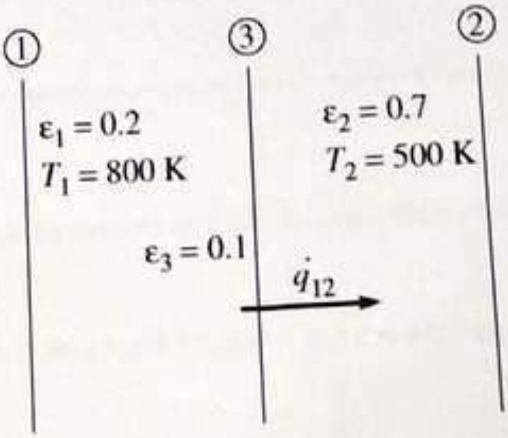


Fig.: The two parallel plates with shield

$$\dot{q}_{12} = \frac{\dot{Q}_{12}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)[(800\ \text{K})^4 - (500\ \text{K})^4]}{\frac{1}{0.2} + \frac{1}{0.7} - 1}$$

$$= \mathbf{3625\ W/m^2}$$

the net rate of radiation heat transfer between the two plates per unit surface area of the plates without the shield.

$$\dot{q}_{12,\text{ one shield}} = \frac{\dot{Q}_{12,\text{ one shield}}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right)}$$

$$= \frac{(5.67 \times 10^{-8}\ W/m^2 \cdot K^4)[(800\ K)^4 - (500\ K)^4]}{\left(\frac{1}{0.2} + \frac{1}{0.7} - 1\right) + \left(\frac{1}{0.1} + \frac{1}{0.1} - 1\right)}$$

$$= 806\ \mathbf{W/m^2}$$

Note that the rate of radiation heat transfer reduces to about onefourth of what it was as a result of placing a radiation shield between the two parallel plates.

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Ques:

Determine the maximum value of $E_{b\lambda}$ at a temperature of 1400 K. Also determine the value at a wavelength of 4×10^{-6} m.

Solution: Using Wien's law $\lambda_{max} = 2.898 \times 10^{-3}/1400 = \mathbf{2.07 \times 10^{-6}\ m}$
substituting in equation

$$E_{b\lambda}(\lambda, T) = \frac{C_1}{\lambda^5\left[exp\left(\frac{C_2}{\lambda T}\right) - 1\right]} \qquad \mathbf{W/m^2/\ m}$$

$$C_1 = 3.742 \times 10^{-16} \quad W.m2/$$

$$C_2 = 1.4388 \times 10^{-2} \quad mK$$

$$E_{b\lambda} = \frac{3.7415 \times 10^{-16}}{(2.07 \times 10^{-6})^5\ [e^{(1.4388 \times 10^{-2}/2.898 \times 10^{-3})} - 1]} = 6.919 \times 10^{10}\ W/m^2/m$$

Check: Using eqn. (13.2 (*b*))

$$E_{b\lambda\, max} = 12.87 \times 10^{-6}\ (1400)^5 = \mathbf{6.922 \times 10^{10}\ W/m^2/m}$$

At 4 micrometer

$$E_{b\lambda} = \frac{3.7415 \times 10^{-16}}{(4 \times 10^{-6})^5[e^{(1.4388 \times 10^{-2}/1400 \times 4 \times 10^{-6})} - 1]} = \mathbf{3.031 \times 10^{10}\ W/m^2/m.}$$

Ques: Consider a 20 cm diameter spherical ball at 800 K suspended in air. Assuming the ball closely approximates a blackbody; determine (a) the total blackbody emissive power, (b) the total amount of radiation emitted by the ball in 5 min, and (c) the spectral blackbody emissive power at a wavelength of 3 μm.

Analysis (*a*) The total blackbody emissive power is determined from the Stefan–Boltzmann law to be

$$E_b = \sigma T^4 = (5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(800\ \text{K})^4 = 23.2 \times 10^3\ \text{W/m}^2 = 23.2\ \text{kW/m}^2$$

That is, the ball emits 23.2 kJ of energy in the form of electromagnetic radiation per second per m^2 of the surface area of the ball.

(*b*) The total amount of radiation energy emitted from the entire ball in 5 min is determined by multiplying the blackbody emissive power obtained above by the total surface area of the ball and the given time interval:

$$A_s = \pi D^2 = \pi(0.2\ \text{m})^2 = 0.1257\ \text{m}^2$$

$$\Delta t = (5\ \text{min})\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = 300\ \text{s}$$

$$Q_{\text{rad}} = E_b A_s\, \Delta t = (23.2\ \text{kW/m}^2)(0.1257\ \text{m}^2)(300\ \text{s})\left(\frac{1\ \text{kJ}}{1000\ \text{W} \cdot \text{s}}\right)$$

$$= 876\ \text{kJ}$$

That is, the ball loses 876 kJ of its internal energy in the form of electromagnetic waves to the surroundings in 5 min, which is enough energy to raise the temperature of 1 kg of water by 50°C. Note that the surface temperature of the ball cannot remain constant at 800 K unless there is an equal amount of energy flow to the surface from the surroundings or from the interior regions of the ball through some mechanisms such as chemical or nuclear reactions.

(*c*) The spectral blackbody emissive power at a wavelength of 3 μm is determined from Planck's distribution law to be

$$E_{b\lambda} = \frac{C_1}{\lambda^5\left[\exp\left(\dfrac{C_2}{\lambda T}\right) - 1\right]} = \frac{3.743 \times 10^8\ \text{W} \cdot \mu\text{m}^4/\text{m}^2}{(3\ \mu\text{m})^5\left[\exp\left(\dfrac{1.4387 \times 10^4\ \mu\text{m} \cdot \text{K}}{(3\ \mu\text{m})(800\ \text{K})}\right) - 1\right]}$$

$$= 3848\ \text{W/m}^2 \cdot \mu\text{m}$$

Unit No-6

Unit no: 6

HEAT EXCHANGERS

In many industrial applications heat has to be transferred from one flowing fluid to another through a solid barrier separating these fluids. The equipments used for this purpose are called **Heat Exchangers**.

Heat exchangers are devices that facilitate the exchange of heat i.e. **thermal energy (enthalpy)** between two fluids that are at different temperatures while keeping them from mixing with each other.

The application of convection studies is mainly in the field of design of heat exchangers.

Heat transfer in a heat exchanger usually involves **convection** in each fluid and **conduction** through the wall separating the two fluids.

TYPES OF HEAT EXCHANGERS

In order to meet the widely varying applications, several types of heat exchangers have been developed which are classified on the basis of nature of heat exchange process, relative direction of fluid motion, design and constructional features, and physical state of fluids.

1. Nature of heat exchange process

Heat exchangers, on the basis of nature of heat exchange process, are classified as follows:

(i) Direct contact (or open) heat exchangers.
(ii) Indirect contact heat exchangers (a) Regenerators, (b) Recuperators.

(i) Direct contact (or open) heat exchangers.

In a direct contact or open heat exchanger the exchange of heat takes place by direct mixing of hot and cold fluids and transfer of heat and mass takes place simultaneously. The use of such units is made under conditions where mixing of two fluids is either harmless or desirable. Examples:
(i) Cooling towers; (ii) Jet condensers; (iii) Direct contact feed heaters.

Figure 1.1 shows a direct contact heat exchanger in which steam mixes with cold water, gives its latent heat to water and gets condensed. Hot water and non-condensable gases leave the container as shown in the figure.

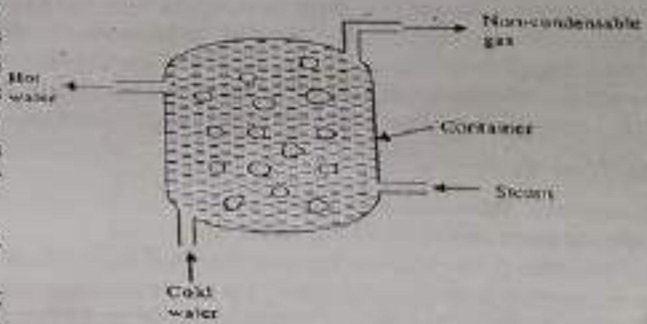

Fig. 1.1. Direct contact or open heat exchanger.

(ii) Indirect contact heat exchangers

In this type of heat exchanger, the heat transfer between two fluids could be carried out by transmission through wall which separates the two fluids. This type includes the following:
(a) Regenerators, (b) Recuperators or surface exchangers.

(a) **Regenerators:** In a regenerator type of heat exchanger the hot and cold fluids pass alternately through a space containing solid particles (matrix), these particles providing alternately a sink and a source for heat flow.

Examples : (i) I.C. engines and gas turbines; (ii) Open hearth and glass melting furnaces; (iii) Air heaters of blast furnaces.

(b) **Recuperators:** 'Recuperator' is the most important type of heat exchanger in which the flowing fluids exchanging heat are on either side of dividing wall (in the form of pipes or tubes generally). These heat exchangers are used when two fluids cannot be allowed to mix i.e., when the mixing is undesirable.

Examples: (i) Automobile radiators, (ii) Oil coolers, intercoolers, air preheaters, economisers, superheaters, condensers and surface feed heaters of a steam power plant, (iii) Milk chiller of pasteurising plant, (iv) Evaporator of an ice plant.

The flow through direct heat exchangers and recuperators may be treated as steady state while through regenerators the flow is essentially transient.

2. Relative direction of fluid motion

According to the relative directions of two fluid streams the heat exchangers are classified into the following three categories:

(i) Parallel flow or unidirectional flow, (ii) Counter-flow, (iii) Cross-flow.

(i) Parallel flow or unidirection flow

The simplest type of heat exchanger consists of two concentric pipes of different diameters, as shown in Figure 1.2, called the double-pipe heat exchanger. Two types of flow arrangement are possible in a double-pipe heat exchanger: in **parallel flow**, both the hot and cold fluids enter the heat exchanger at the same end and move in the same direction.

Examples: Oil coolers, oil heaters, water heaters etc.

As the two fluids are separated by a wall, this type of heat exchanger may be called parallel flow recuperator or surface heat exchanger.

(ii) Counter-flow

In **counter flow**, on the other hand, the hot and cold fluids enter the heat exchanger at opposite ends and flow in opposite directions.

In a counter-flow heat exchanger, the two fluids flow in opposite directions. The hot and cold fluids enter at the opposite ends. The flow arrangement and temperature distribution for such a heat exchanger are shown schematically in Fig. 1.2. The **temperature difference** between the two fluids remains more or less nearly constant. This type of heat exchanger, due to counter flow, gives maximum rate of heat transfer for a given surface area. Hence such heat exchangers are most favoured for heating and cooling of fluids.

(iii) Cross-flow

In cross-flow heat exchangers, the two fluids (hot and cold) cross one another in space, usually at right angles, which is specifically designed to realize a large heat transfer surface area per unit volume, is the **compact heat exchanger**. Fig. 1.3 shows a schematic diagram of common arrangements of cross-flow heat exchangers.

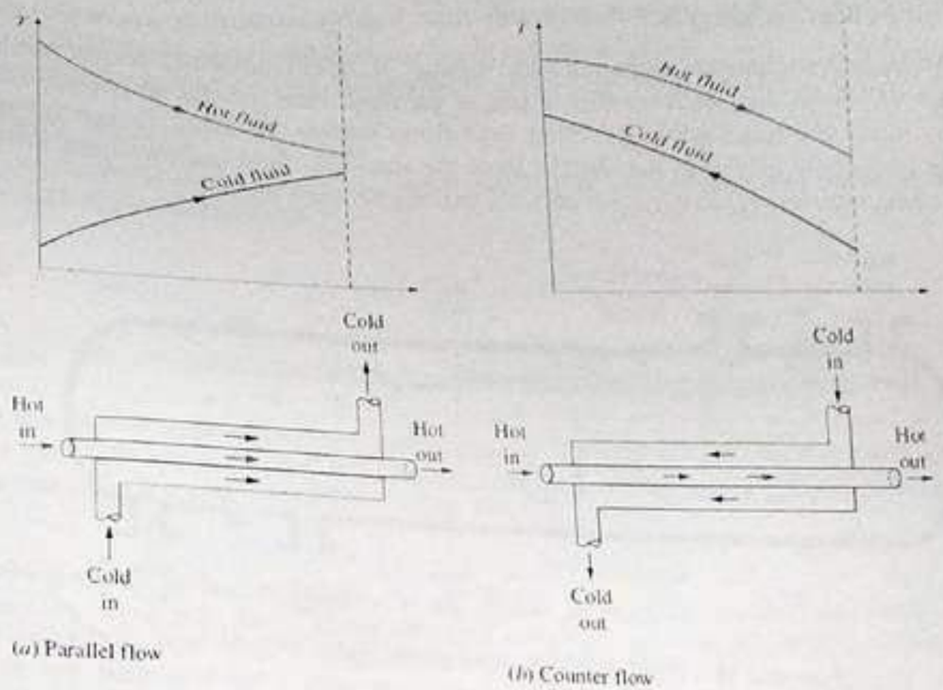

Figure 1.2:- Different flow regimes and associated temperature profiles in a double-pipe heat exchanger (a)Parallel flow and (b) Counter flow.

Compact heat exchangers enable us to achieve high heat transfer rates between two fluids in a small volume, and they are commonly used in applications with strict limitations on the weight and volume of heat exchangers. The large surface area in compact heat exchangers is obtained by attaching closely spaced thin plate or corrugated fins to the walls separating the two fluids. There are two types of cross flow heat exchangers

(a) **Unmixed**, Examples: Automobile radiator etc. and

(b) **Mixed flow**. Examples: The cooling unit of refrigeration system etc.

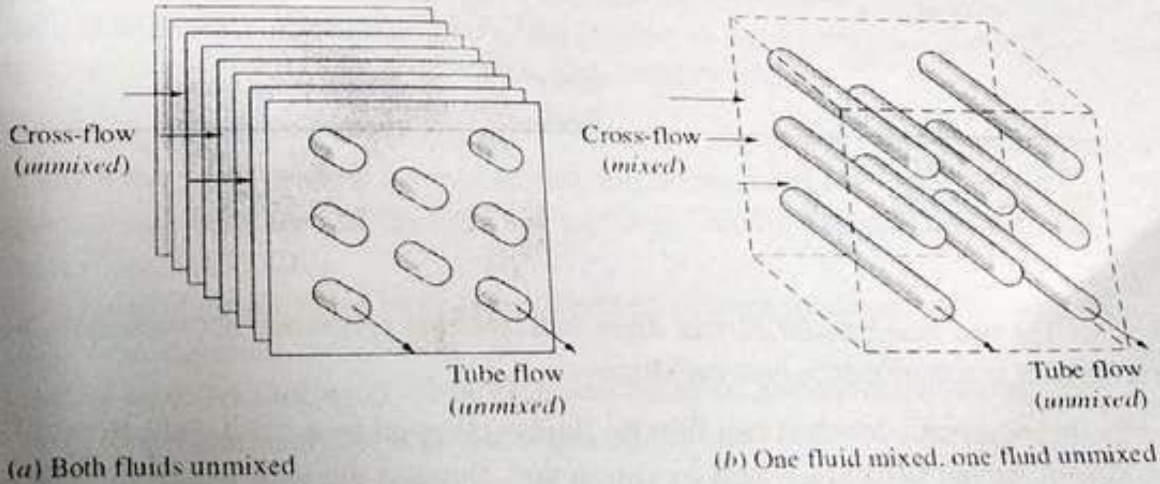

Figure 1.3:- Different flow configurations in cross-flow heat exchangers.

Shell-and-tube heat exchangers

Shell-and-tube heat exchangers contain a large number of tubes (sometimes several hundred) packed in a shell with their axes parallel to that of the shell. Heat transfer takes place as one fluid flows inside the tubes while the other fluid flows outside the tubes through the shell. Baffles are commonly placed in the shell to force the shell-side fluid to flow across the shell to enhance heat transfer and to maintain uniform spacing between the tubes.

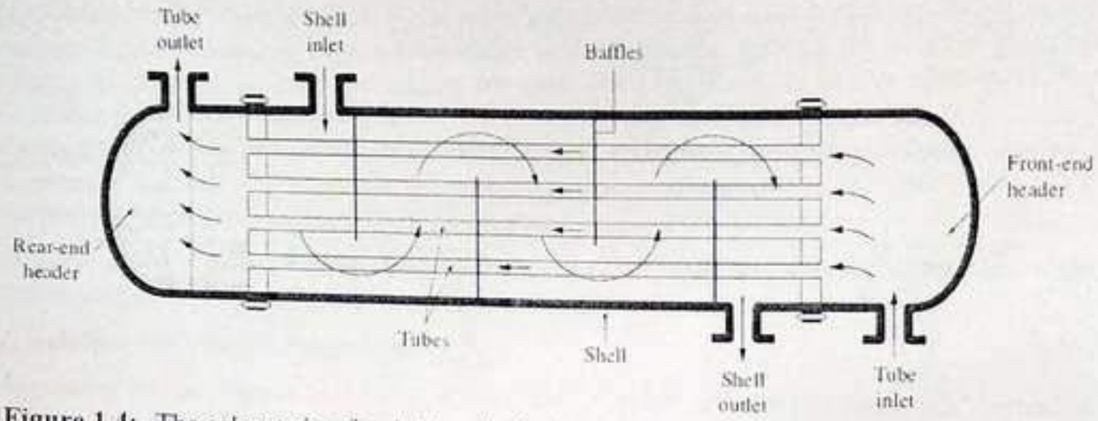

Figure 1.4:- The schematic of a shell-and-tube heat exchanger.

Common examples of heat exchangers are shell and tube exchangers, automobile radiators, condensers, evaporators, air preheaters, and cooling towers.

THE OVERALL HEAT TRANSFER COEFFICIENT

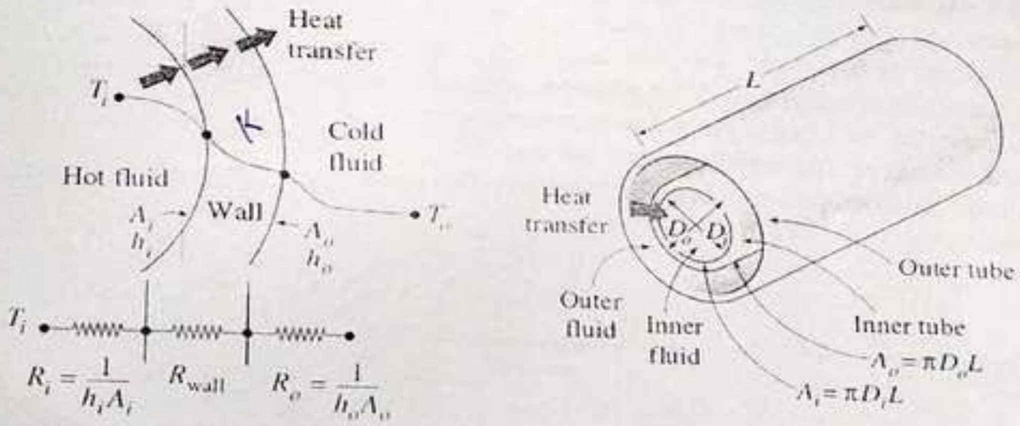

Figure 1.5:- The two heat transfer surface areas with thermal resistance network associated with heat transfer in a double-pipe heat exchanger.

A heat exchanger typically involves two flowing fluids separated by a solid wall. Heat is first transferred from the hot fluid to the wall by convection, through the wall by conduction, and from the wall to the cold fluid again by convection. Any radiation effects are usually included in the convection heat transfer coefficients.

The thermal resistance network associated with this heat transfer process involves two convection and one conduction resistances, as shown in Figure 1.3.
the thermal resistance of the tube wall in this case is

$$R_{wall} = \frac{\ln(D_o/D_i)}{2\pi kL}$$

where k is the thermal conductivity of the wall material and L is the length of the tube. Then the total thermal resistance becomes,

$$R = R_{total} = R_i + R_{wall} + R_o = \frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{1}{h_o A_o}$$

When one fluid flows inside a circular tube and the other outside of it, we have $A_i = \pi D_i L$ and $A_o = \pi D_o L$.
In the analysis of heat exchangers, it is convenient to combine all the thermal resistances in the path of heat flow from the hot fluid to the cold one into a single resistance R, and to express the rate of heat transfer between the two fluids as

$$Q = \frac{\Delta T}{R} = UA\,\Delta T = U_i A_i\,\Delta T = U_o A_o\,\Delta T$$

* The value of U is independant of flow direction and it is only dependant on flow velocity and till flow conditions remains constant, the U value does not change.

where U is the overall heat transfer coefficient, whose unit is W/m² °C
If the fluids are separated by a tube wall as shown in Fig. 1.5, the overall heat transfer coefficient for Inner surface is given by U_i and for Inner surface is given by U_o:

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{r_i}{k}\ln(r_o/r_i) + (r_i/r_o) \times \frac{1}{h_o}} \qquad U_0 = \frac{1}{(r_o/r_i)\frac{1}{h_i} + \frac{r_o}{k}\ln(r_o/r_i) + \frac{1}{h_o}}$$

Note that $U_i A_i = U_o A_o$

Fouling Factor

The performance of heat exchangers usually deteriorates with time as a result of accumulation of deposits on heat transfer surfaces. The layer of deposits represents additional resistance to heat transfer and causes the rate of heat transfer in a heat exchanger to decrease. The net effect of these accumulations on heat transfer is represented by a fouling factor R_f, which is a measure of the thermal resistance introduced by fouling.

Fouling can be caused by the following sources:

1) ***Scaling*** is the most common form of fouling and is associated with inverse solubility salts. Examples of such salts are $CaCO_3$, $CaSO_4$, $Ca_3(PO_4)_2$, $CaSiO_3$, $Ca(OH)_2$, $Mg(OH)_2$, $MgSiO_3$, Na_2SO_4, $LiSO_4$, and Li_2CO_3.
2) ***Corrosion fouling*** is caused by chemical reaction of some fluid constituents with the heat exchanger tube material.
3) ***Chemical reaction fouling*** involves chemical reactions in the process stream which results in deposition of material on the heat exchanger tubes. This commonly occurs in food processing industries.

4) ***Freezing fouling*** is occurs when a portion of the hot stream is cooled to near the freezing point for one of its components. This commonly occurs in refineries where paraffin frequently solidifies from petroleum products at various stages in the refining process. , obstructing both flow and heat transfer.
5) ***Biological fouling*** is common where untreated water from natural resources such as rivers and lakes is used as a coolant. Biological micro-organisms such as algae or other microbes can grow inside the heat exchanger and hinder heat transfer.
6) ***Particulate fouling*** results from the presence of microscale sized particles in solution. When such particles accumulate on a heat exchanger surface they sometimes fuse and harden. Like scale these deposits are difficult to remove.

The most common type of fouling is the **precipitation of solid deposits** in a fluid on the heat transfer surfaces. **Fouling increases with increasing temperature and decreasing velocity.**

$$R_f = \frac{1}{h_s} = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}} \qquad R_{si} = \frac{1}{A_i h_{si}} \qquad R_{so} = \frac{1}{A_o h_{so}}$$

where, fouling factor R_{si} and R_{so} for inside and outside surface, h_{si} and h_{so} heat transfer coefficient for the scale depositd on the inside and outside surface.

Heat transfer rate considering scale deposition is given by,

$$Q = \frac{(t_i - t_{o})}{\frac{1}{A_i h_i} + \frac{1}{A_i h_{si}} + \frac{1}{2\pi Lk} \ln (r_o / r_i) + \frac{1}{A_o h_{so}} + \frac{1}{A_o h_o}}$$

The overall heat transfer coefficient base on considering scale deposition for Inner surface is given by U_i and for Inner surface is given by U_o is given by,

$$U_i = \frac{1}{\frac{1}{h_i} + R_{f_i} + \frac{r_i}{k} \ln (r_o/r_i) + (r_i/r_o) R_{fo} + (r_i/r_o) \frac{1}{h_o}}$$

$$U_o = \frac{1}{(r_o/r_i) \frac{1}{h_o} + (r_o/r_i) R_{fi} + \frac{r_o}{k} \ln (r_o/r_1) + R_{fo} + \frac{1}{h_o}}$$

In case the tube is thin walled and the thermal resistances due to tube wall thickness and scale formed are neglected, then the overall heat transfer coefficient based on outer surface is given by:

$$U_o = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}}$$

When only fouling factors are neglected, we have

$$U_o = \frac{1}{(r_o / r_i)\frac{1}{h_i} + \frac{r_o}{k}\ln(r_o/r_i) + \frac{1}{h_o}}$$

LOGARITHMIC MEAN TEMPERATURE DIFFERENCE (LMTD)

Logarithmic mean temperature difference (LMTD) is defined as that temperature difference which, if constant, would give the same rate of heat transfer as actually occurs under variable conditions of temperature difference.

12.3.1. Determination of mean temperature difference: In the case of single tube parallel flow heat exchanger: Referring to Fig. 12.3. the cold fuid enters at a temperature T_{ci} and leaves at T_{co}. The hot fluid enters at T_{hi} and leaves at T_{ho}. The flow rates and specific heats are $m_c\, c_c$ and $m_h\, c_h$ respectively. The product **m c** is also known as heat capacity sometimes denoted as C_h and C_c.

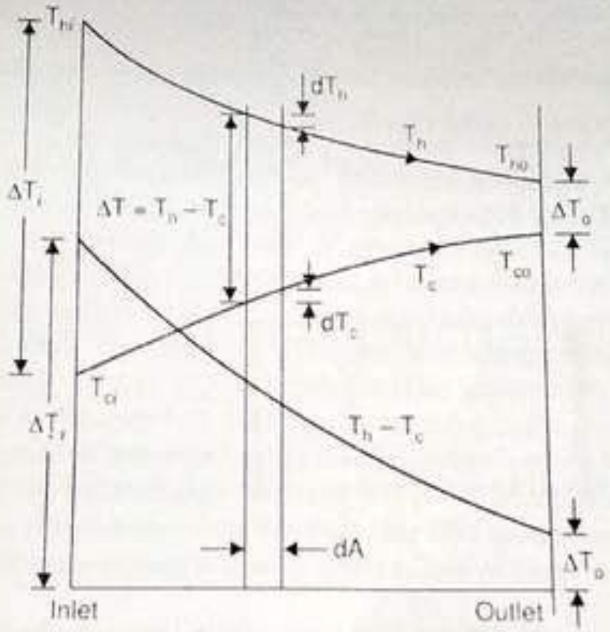

Fig. 12.3. Parallel flow-single tube heat exchanger–Temperature variation along flow.

Consider a small area dA at a location. In passing through the area let the hot fluid be cooled by dT_h and let the cold fluid be heated dT_c. Let the heat flow be dQ.

$$dQ = -m_h c_h\, dT_h = m_c c_c\, dT_c \qquad ...(12.5)$$

The –ve sign indicates temperature drop or dT_h is negative.

The total heat flow is calculated using.

$$Q = m_h c_h (T_{hi} - T_{ho}) = m_c c_c (T_{co} - T_{ci}) \qquad ...(12.6)$$

This equation is used to determine one of the unknown temperatures if mass flows are specified or the unknown mass flow if all temperatures are specified.

Using equation (12.5)

$$dT_h = -\frac{dQ}{m_h c_h},\; dT_c = \frac{dQ}{m_c c_c}$$

$$dT_h - dT_c = d(T_h - T_c) = -dQ\left[\frac{1}{m_h c_h} + \frac{1}{m_c c_c}\right] \quad ...(12.7)$$

...(12.8)

From equation (12.1)

$$dQ = U\,dA\,(T_h - T_c)$$

substituting in (12.7)

$$\frac{d(T_h - T_c)}{(T_h - T_c)} = -U\,dA\left[\frac{1}{m_h c_h} + \frac{1}{m_c c_c}\right] \quad ...(12.9)$$

Integrating from the inlet to the outlet

$$\ln(T_h - T_c)\Big|_i^o = -U\,A\left[\frac{1}{m_h c_h} + \frac{1}{m_c c_c}\right] \quad ...(12.10)$$

Substituting the following in equation (12.10),

$$m_h\,c_h = Q/(T_{hi} - T_{ho}) \quad \text{and} \quad m_c c_c = Q/(T_{co} - T_{ci})$$

$$\ln\frac{\Delta T_o}{\Delta T_i} = -\frac{UA}{Q}\left[(T_{hi} - T_{ho}) + (T_{co} - T_{ci})\right]$$

$$= \frac{UA}{Q}\left[(T_{ho} - T_{co}) - (T_{hi} - T_{ci})\right]$$

$$\therefore \quad Q = UA\left[\frac{\Delta T_o - \Delta T_i}{\ln\dfrac{\Delta T_o}{\Delta T_i}}\right] \quad ...(12.11)$$

Where ΔT_o and ΔT_i are the difference in temperatures between the hot and cold fluids at outlet and at inlet.

Comparing with equation (12.4), the mean temperature is given by

$$\Delta T_{mean} = \frac{\Delta T_o - \Delta T_i}{\ln\dfrac{\Delta T_o}{\Delta T_i}}$$ and this value is known as Log Mean

Temperature Difference or simply LMTD

$$\therefore \quad Q = UA\,(\text{LMTD}) \quad ...(12.12)$$

Q = UA (LMTD) where, LMTD = ΔT_{mean}

Similar derivation is worked out for counter flow.

LMTD = ΔT_{mean} for a counter-flow unit is always greater than that for a parallel flow unit; hence counter-flow heat exchanger can transfer more heat than parallel-flow one; in other words a counter-flow heat exchanger needs a smaller heating surface for the same rate of heat transfer. For this reason, the counter-flow arrangement is usually used.

When the temperature variations of the fluids are relatively small, then temperature variation curves are approximately straight lines and adequately accurate results are obtained by taking the **arithmatic mean temperature dijference (AMTD).**

$$AMTD = \frac{t_{h1} + t_{h2}}{2} - \frac{t_{c1} + t_{c2}}{2} = \frac{(t_{h1} - t_{c1}) + (t_{h2} - t_{c2})}{2} = \frac{\theta_1 + \theta_2}{2}$$

However, practical considerations suggest that the logarithmic mean temperature difference ($\theta_m = \Delta T_{mean}$) should be invariably used when $\frac{\theta_1}{\theta_2} > 1.7$

Under comparable conditions, more heat is transferred in a counter-flow arrangement than in a parallel flow heat exchanger.

The temperature profiles of the two heat exchangers indicate two major disadvantages in the **parallel-flow design**. First, the large temperature difference at the ends (Figure 1.2) causes large thermal stresses. The opposing expansion and contraction of the construction materials due to diverse fluid temperatures can lead to eventual material failure. Second, the temperature of the cold fluid exiting the heat exchanger never exceeds the lowest temperature of the hot fluid. This relationship is a distinct disadvantage if the design purpose is to raise the temperature of the cold fluid. The design of a parallel flow heat exchanger is advantageous when two fluids are required to be brought to nearly the same temperature.

The **counter-flow heat exchanger** has three significant advantages over the parallel flow design. First, the more uniform temperature difference between the two fluids minimizes the thermal stresses throughout the exchanger. Second, the outlet temperature of the cold fluid can approach the highest temperature of the hot fluid (the inlet temperature). Third, the more uniform temperature difference produces a more uniform rate of heat transfer throughout the heat exchanger.

The rate of heat transfer varies along the length of the exchanger tubes because its value depends upon the temperature difference between the hot and the cold fluid at the point being viewed. Hence, counter-flow heat exchanger can transfer more heat than parallel-flow one; in other words a counter-flow heat exchanger needs a smaller heating surface for the same rate of heat transfer. For this reason, the counter-flow arrangement is usually used.

HEAT EXCHANGER EFFECTIVENESS AND NUMBER OF TRANSFER UNITS (NTU)

A heat exchanger can be designed by the LMTD (logarithmic mean temperature difference) when inlet and outlet conditions are specified. However, when the problem is to determine the inlet or exit temperatures for a particular heat exchanger, the analysis is performed more easily, by using a method based on effectiveness of the heat exchanger (concept first proposed by Nusselt) and number of transfer units (NTU).

The heat exchanger effectiveness (ε) is defined as the ratio of actual heat transfer to the maximum possible heat transfer. Thus

$$\varepsilon = \frac{Q}{Q_{max}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}}$$

$$\text{effectiveness} = \frac{\textbf{change in temperature of the minimum heat capacity fluid}}{\textbf{Difference between the fluid inlet temperatures}}$$

It is obvious from above Eqn. that effectiveness ε is a function of several variables and as such it is inconvenient to combine them in a graphical or tabular form. However, by compiling a non-dimensional grouping, ε can be expressed as a function of three non-dimensional parameters. This method is known as **NTU** method. This method/approach facilitates the comparison between the various types of heat exchangers which may be used for a particular application.

Effectiveness relations of the heat exchangers typically involve the dimensionless group UA_s/C_{min}. This quantity is called the number of transfer units NTU and is expressed as

$$NTU = \frac{UA_s}{C_{min}} = \frac{UA_s}{(\dot{m}C_p)_{min}}$$

where U is the overall heat transfer coefficient and A_s is the heat transfer surface area of the heat exchanger. Note that **NTU is proportional to A_s**. Therefore, for specified values of U and C_{min}, the value of **NTU is a measure of the heat transfer surface area A_s**. Thus, the **larger the NTU, the larger the heat exchanger.**

K. D.K. College of Engineering, Nagpur

Department of Mechanical Engineering

Subject: - HEAT TRANSFER

Unit No-1

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UNIT I
CONDUCTION HEAT TRANSFER

Define heat transfer:

Heat transfer can be defined as the transmission of energy from one region to another due to temperature difference. The science that deals with the determination of the rates of such energy transfers is heat transfer.

Thermodynamics deals with equilibrium states and changes from one equilibrium state to another, and it is concerned with the amount of heat transfer as a system undergoes a process from one equilibrium state to another; it may be used to predict the amount of energy required to change a system from one equilibrium state to another; it may not be used to predict how fast a change will take place since the system is not in equilibrium during the process, and also, it gives no indication about how long the process will take. A thermodynamic analysis simply tells us how much heat must be transferred to realize a specified change of state to satisfy the conservation of energy principle.

Heat transfer, on the other hand, deals with systems that lack thermal equilibrium, and thus it is a *nonequilibrium* phenomenon. Therefore, the study of heat transfer cannot be based on the principles of thermodynamics alone.

FIGURE 1 Heat flows in the direction of decreasing temperature.

However, the laws of thermodynamics lay the framework for the science of heat transfer. The *first law* requires that the rate of energy transfer into a system be equal to the rate of increase of the energy of that system. The *second law* requires that heat be transferred in the direction of decreasing temperature (Fig. 1). This is like a car parked on an inclined road that must go downhill in the direction of decreasing elevation when its brakes are released. It is also analogous to the electric current flowing in the direction of decreasing voltage or the fluid flowing in the direction of decreasing total pressure.

Heat transfer equipment such as heat exchangers, boilers, condensers, radiators, heaters, furnaces, refrigerators, and solar collectors are designed primarily on the basis of heat transfer analysis.

The heat transfer problems encountered in practice can be considered in two groups:

(1) ***rating*** and (2) ***sizing*** problems.

The rating problems deal with the determination of the heat transfer rate for an existing system at a specified temperature difference. The sizing problems deal with the determination of the size of a system in order to transfer heat at a specified rate for a specified temperature difference.

A heat transfer process or equipment can be studied either *experimentally* (testing and taking measurements) or *analytically* (by analysis or calculations).

Difference between heat and temperature

In heat transfer problems, we often interchangeably use the terms heat and temperature. Actually, there is a distinct difference between the two. Temperature is a measure of the amount of energy possessed by the molecules of a substance. It manifests itself as a degree of hotness, and can be used to predict the direction of heat transfer. The usual symbol for temperature is T. The scales for measuring temperature in SI units are the Celsius (^{0}C) and Kelvin (K) temperature scales. Heat, on the other hand, is energy in transit. Spontaneously, heat flows from a hotter body to a colder one. The usual symbol for heat is Q. In the SI system, common units for measuring heat are the Joule and calorie.

What are the modes of heat transfer? Explain the different modes of heat transfer with appropriate expressions?

1. Conduction.
2. Convection.
3. Radiation

Dr. Vivek Vaidya
Assistant Professor

1. Conduction:

Heat conduction is a mechanism of heat transfer from a region of high temperature to a region of low temperature with in a medium (solid, liquid or gases) or different medium in directly physical contact. In conduction, energy exchange takes place by the kinematic motion or direct impact of molecules. Pure conduction is found only in solids.

Examples: Heating a Rod.

Q = heat flow, Watt (W)

$$Q = -k\,A\,\frac{dT}{dx}$$

k = Thermal conductivity, W/m°C
A = Area, m^2
dT = °C
dx = Thickness of wall, m

In gases and liquids, conduction is due to the *collisions* and *diffusion* of the molecules during their random motion. In solids, it is due to the combination of vibrations of the molecules in a lattice and the energy transport by free electrons.

2. Convection:

Convection is a process of heat transfer that will occur between a solid surface and a fluid medium (gas or liquid), and it involves the combined effects of conduction and fluid motion, when they are at different temperatures. Convection, h, (W/m^2°C) is possible only in the presence of fluid medium. The faster the fluid motion, the greater the convection heats transfer.

Example: Cooling of Hot Plate by air.

$$Q = h\,A\,(T_s - T_\infty)$$

h = convective heat transfer coefficient
= W/m^2°C
T_s = surface temp, T_∞ = fluid temp

The faster the fluid motion, the greater the convection heat transfer. Convection is called forced convection if the fluid is forced to flow over the surface by external means such as a fan, pump, or the wind. In contrast, convection is called natural or free convection if the fluid motion is caused by buoyancy force that is induced by density difference due to the variation of temperature in the fluid.

3. Radiation:

The heat transfer from one body to another without any transmitting medium is known as radiation. It is an electromagnetic wave phenomenon.

Example: Radiation sun to earth.

The maximum rate of radiation which can be emitted from a surface at an absolute temperature (T_s) is given by the Stefan-Boltzmann law as:

$$Q = \sigma\left[T_s^4 - T_\infty^4\right]$$

Where $\sigma = 5.67 \times 10^{-8}$ W/m^2.K^4 is the Stefan-Boltzmann constant and ε is the emissivity of the surface. An idealized surface, which emits radiation at a maximum rate has $\varepsilon = 1$, is known as a blackbody. The radiation emitted by actual surfaces is less than that emitted by the blackbody. The value of ε is in the range $0 \le \varepsilon \le 1$, is a measure of how closely a surface approximates a blackbody.

Another important radiation property of a surface is its absorptivity (α), which is the fraction of the radiation energy incident on a surface which is absorbed by the surface. Like emissivity, its value is in the range $0 \le \alpha \le 1$. A blackbody absorbs the entire radiation incident on it. That is, a black is a perfect absorber ($\alpha = 1$) as well as a perfect emitter.

Newton's Law of Cooling

Temperature difference in any situation results from energy flow into a system or energy flow from a system to surroundings. The former leads to heating, whereas latter leads to cooling of an object.

Newton's Law of Cooling states that the rate of temperature of the body is proportional to the difference between the temperature of the body and that of the surrounding medium.

Newton stated that the rate of loss of heat energy is proportional to the temperature difference between the hot surface and the surroundings.

Example: Cooling of Hot Plate by air.

$$Q = h\,A\,(T_s - T_\infty)$$

Newton's Law of Cooling is useful for studying water heating because it can tell us how fast the hot water in pipes cools off.

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It works on 2nd law of thermodynamics,
Energy flow from high to low temp.

State Fourier's law of conduction :

The rate of heat conduction is proportional to the area measured normal to the direction of heat flow and to the temperature gradient in that direction.

Consider a wall as shown; we know that the rate of heat transfer through the wall increases when:

- The temperatures difference between the left and right surfaces increase,
- The wall surface area increases,
- The wall thickness reduces,
- The wall is change from brick to aluminum.

If we measure temperatures of the wall from left to right and plot the temperature variation with the wall thickness, we get:

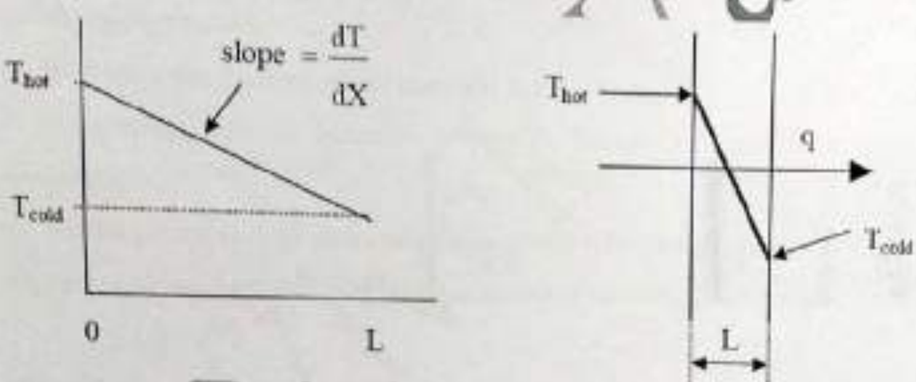

Figure 1.2: Heat conduction through a wall.

$$Q \alpha A \frac{dT}{dx}$$

$$Q = -K A \frac{dT}{dx}$$

Where, A – Area in m^2

$\frac{dT}{dx}$ – *Temperature gradient, K/m*

K – Thermal conductivity, W/mk

Heat is conducted in the direction of decreasing temperature, and the temperature gradient becomes negative when temperature decreases with increasing

dx. The *negative sign* ensures that heat transfer in the positive dx direction is a positive quantity. Negative sign introduced because heat flow in the dirn of decreasing temp.

Define Thermal conductivity :

Thermal conductivity is defined as the ability of a material or substance to conduct heat. Thermal conductivity, k, (W/m°C) is formed to vary with temperature. **In good conductors, thermal conductivity decreases with temperature** due to impedance to electron flow of higher electron densities. **In insulators,** as temperature increases, thermal atomic activity also increases and **hence thermal conductivity increases with temperature.** In the **case of gases,** thermal conductivity **increases with temperature** due to increased random activity of atoms and molecules.

1. What are the factors affect thermal conductivity?

1. Material structure. 2. Moisture content. 3. Density of material. 4. Pressure and temperature.

2. Explain variation in thermal conductivity with temperature.

The thermal conductivity vary with temperature according to relation

$$k = k_o(1 + \beta T)$$

Where k_o= thermal conductivity at 0°c

β= temperature co- efficient of thermal conductivity.

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Derive general heat conduction equation in Cartesian coordinates?

Consider a small rectangular element of sides dx, dy and dz as shown in figure 1.2.

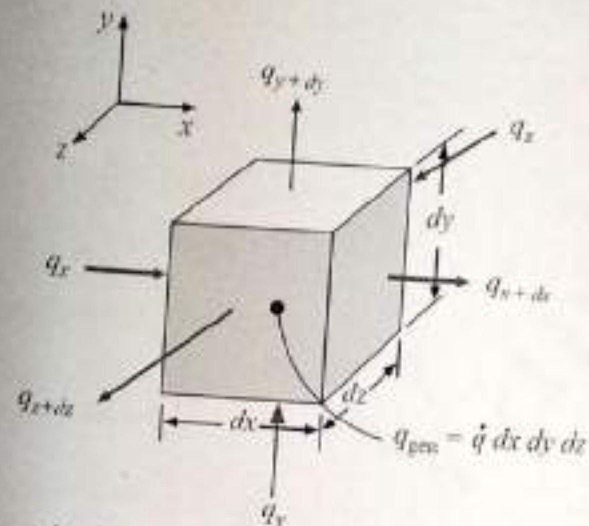

Figure 1.3: Elemental volume for three-dimensional heat-conduction analysis: Cartesian coordinates

The energy balance of this rectangular element is obtained from first law of thermodynamics

Net heat conducted into element from all the coordinate directions + **Heat generated within the element** = **Heat stored in the element -----1**

Net heat conducted into element from all the coordinate directions:

As per the Fourier law of heat conduction ($Q = -K\,A\,\frac{dT}{dx}$) the rate of heat flow into the element in X, Y, Z directions through face ABCD, ABEF, ADHE are

$$\left.\begin{aligned} Q_x &= -k\,(dy\,dz)\,\frac{\partial T}{\partial x}dx \\ Q_y &= -k\,(dx\,dz)\,\frac{\partial T}{\partial y}dy \\ Q_z &= -k\,(dy\,dx)\,\frac{\partial T}{\partial z}dz \end{aligned}\right\} \text{-----------------2}$$

Where, $\frac{dT}{dx}$ = Rate of temp. change along x direction and

$\frac{dT}{dx}dx$ = Rate of temp. change through distance dx.

The rate of heat flow out of the element in X direction through the face EFGH is

$$Q_{x+dx} = Q_x + \frac{\partial}{\partial x}(Qx)dx \text{ ------3}$$

Sub Equ 2 in Equ 3

$$Q_x - Q_{x+dx} = Q_x - \{Q_x + \frac{\partial}{\partial x}(Qx)dx\}$$

$$= -\frac{\partial}{\partial x}\left[-kdydz\frac{dT}{dx}\right]dx$$

$$Q_x - Q_{x+dx} = \frac{\partial}{\partial x}\{kdydzdx\frac{\partial T}{\partial x}\}$$

$$Q_x - Q_{x+dx} = kdxdydz\frac{\partial^2 T}{\partial x^2} \text{ -------4}$$

Similarly

$$Q_y - Q_{y+dy} = kdxdydz\frac{\partial^2 T}{\partial y^2} \text{ ------5}$$

$$Q_z - Q_{z+dz} = kdxdydz\frac{\partial^2 T}{\partial z^2} \text{ ------6}$$

$$Q = \{ (Q_x - Q_{x+dx}) + (Q_y - Q_{y+dy}) + (Q_z - Q_{z+dz}) \} \text{ ---- 7}$$

Sub Eq^n 4 , 5, 6 in Equ 7

$$kdxdydz \{ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \} \text{ ---8}$$

Heat generated in the element internally: $Q_g = q\ dxdydz$ ------9

Heat stored in the element: mass of the body ($\rho dxdydz$) x specific heat (C) of the body material x rise in temperature of body ($\frac{dT}{dt}$)

$$Q_{IE} = \rho C \frac{\partial T}{\partial t} dxdydz \text{ -----10}$$

Sub Eq^n 8, 9, 10 in Eq. 1

$$kdxdydz \{ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \} + q\ dxdydz = \rho Cp \frac{\partial T}{\partial t} dxdydz \text{ ---11}$$

Take dxdydz commonly and divide by k

$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{\rho Cp}{k}\frac{\partial T}{\partial t}$ ---- **Three dimensional heat conduction equation for Cartesian coordinates**

Case 1: No heat generation : $\frac{q}{k} = 0$ $\frac{dT}{dt} \neq 0$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{\rho \mathrm{Cp}}{\mathrm{k}}\frac{\partial T}{\partial t}=\frac{1}{\alpha}\frac{dT}{dt}$$ ----- **Fourier Equation** ***(unsteady state without internal heat generation i.e.*** $\frac{dT}{dt} \neq 0$ ***)***

Case 2: Steady state condition : $\frac{\partial T}{\partial t}=0$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q}{k}=0$$ --- **Poisson Equation** ***(steady state without internal heat generation i.e.*** $\frac{dT}{dt}=0$ ***)***

Case 3 : No heat generation and steady state : $\frac{q}{k}=0 \quad \frac{\partial T}{\partial t}=0$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=0$$ --- **Laplace Equation**

1. Explain about Fourier equation.

This Fourier equation is used to find out the conduction heat transfer. According to this equation, heat transfer is directly proportional to surface area and temperature gradient. It is indirectly proportional to the thickness of the slab.

$$Q \alpha \frac{A \Delta T}{L}$$

$$Q=\frac{-K A \Delta T}{L}$$

2. Explain about Poisson's equation.

When the temperature is not varying with respect to time, then the conduction is called as steady state conduction.

$$\frac{\partial T}{\partial t}=0$$

3. Explain about Laplace equation.

When the conduction is steady state conduction, and there is no heat generation, the general equation becomes,

$$\frac{\partial^2 T}{\partial^2 x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=0$$

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Thermal Diffusivity :

The product ρCp, which is frequently encountered in heat transfer analysis, is called the **heat capacity** of a material. Both the specific heat Cp and the heat capacity ρCp represent the heat storage capability of a material. But Cp expresses it *per unit mass* whereas ρCp expresses it *per unit volume*, as can be noticed from their units J/kg · °C and J/m^3 · °C, respectively.
Another material property that appears in the transient heat conduction analysis is the **thermal diffusivity,** which represents how fast heat diffuses through a material and is defined as

$$\alpha = \frac{\text{Heat conducted}}{\text{Heat stored}} = \frac{k}{\rho C_p} \qquad (\text{m}^2/\text{s})$$

Note that the thermal conductivity k represents how well a material conducts heat, and the heat capacity ρCp represents how much energy a material stores per unit volume. Therefore, the thermal diffusivity of a material can be viewed as the ratio of the *heat conducted* through the material to the *heat stored* per unit volume. A material that has a high thermal conductivity or a low heat capacity will obviously have a large thermal diffusivity. The larger the thermal diffusivity, the faster the propagation of heat into the medium.
A small value of thermal diffusivity means that heat is mostly absorbed by the material and a small amount of heat will be conducted further.

1. Explain the significance of thermal diffusivity.

The physical significance of thermal diffusivity is that it tells us how fast heat is propagated or it diffuses through a material during changes of temperature with time.

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General heat conduction equation in Cylindrical coordinates :

While dealing with problems of conduction of heat through systems having cylindrical geometrics it is convenient to use cylindrical coordinates. Consider an elemental volume having the coordinates (r, φ, z) for three dimensional heat conduction analysis as shown in figure 1.4.

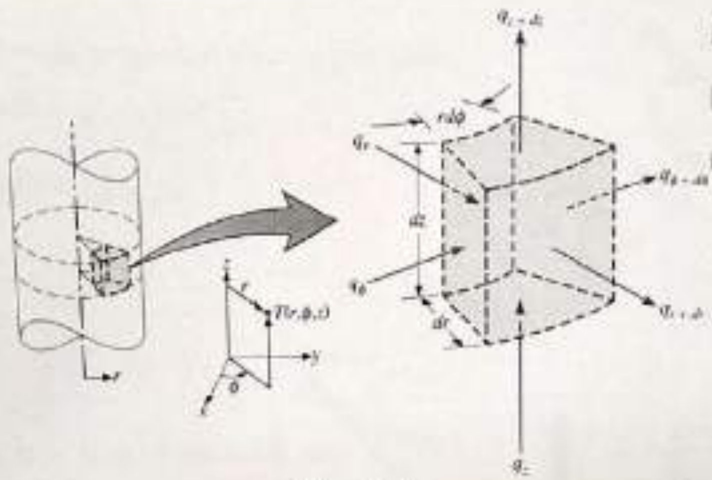

Figure 1.4: Elemental volume for three-dimensional heat-conduction analysis: cylindrical coordinates.

The volume of the element, V = rdφ dr dz

Net heat conducted into element from all the coordinate directions	+	**Heat generated within the element**	=	**Heat stored in the element -----1**

a. Net heat accumulated in the element due to conduction of heat from all the coordinate direction considered:

Heat flow in radial direction (r plane):

Heat influx $Qr = -krd\phi dz \frac{\partial T}{\partial r} dr$

Heat efflux $Qr + dr = Qr + \frac{\partial}{\partial r} Qr dr$

$Qr - Qr + dr = Qr - (Qr + \frac{\partial}{\partial r} Qr dr)$

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$$= -\frac{\partial}{\partial r}\left\{-krd\phi dz\frac{\partial T}{\partial r}dr\right\}$$

$$= kd\phi drdz\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

$$= kd\phi drdz\left(r\frac{\partial^2 T}{\partial r^2}+\frac{\partial T}{\partial r}\right)$$

$$\boxed{Qr - Qr + dr \quad = kd\phi drdz\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right) \text{---2}}$$

Heat flow in tangential direction (φ plane):

Heat influx: $Q_\phi = -kdrdz\frac{\partial T}{r\partial\phi}$

Heat efflux: $Q_{\phi+d\phi} = Q_\phi + \frac{\partial}{r\partial\phi}Q_\phi rd\phi$

$$Q_\phi - Q_{\phi+d\phi} = Q_\phi - \left[Q_\phi + \frac{\partial}{r\partial\phi}\left(-kdrdz\frac{\partial T}{r\partial\phi}\right)rd\phi\right]$$

$$\boxed{Q_\phi - Q_{\phi+d\phi} \quad = kdrrd\phi dz\left[\frac{1}{r^2}\frac{\partial^2 T}{\partial\phi^2}\right] \text{---3}}$$

Heat flow in axial direction (z plane):

Heat influx: $Q_z = -kdrrd\phi\frac{\partial T}{\partial z}$

Heat efflux: $Q_{z+dz} = Q_z + \frac{\partial}{\partial z}Q_z dz$

$$\boxed{Q_z - Q_{z+dz} \quad = kdrrd\phi dz\left[\frac{\partial^2 T}{\partial z^2}\right] \text{---4}}$$

b. Heat generated within the element:

$Qg = qrd\phi drdz$ ------------ 5

c. Energy stored in the element: mass of the body ($\rho r d_\phi drdz$) x specific heat (C) of the body material x rise in temperature of body ($\frac{dT}{dt}$)

$Q_{st} = \rho C\frac{\partial T}{\partial t}rd\phi drdz$ ------6

Sub Equations 2, 3, 4, 5, 6 in Equation 1

$$krd\phi drdz\left[\frac{\partial^2 T}{dr^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial\phi^2}+\frac{\partial^2 T}{\partial z^2}\right]+qrd\phi drdz = \rho Cp\frac{\partial T}{\partial t}rd\phi drdz$$

$$\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial z^2}\right]+\frac{q}{k}=\frac{\rho C p}{k}\frac{\partial T}{\partial t}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$ **Heat conduction equation for cylindrical coordinates**

THERMAL CONTACT RESISTANCE :

When two different layers (or multi-layer) of conducting materials are placed in thermal contact, a thermal resistance develops at the interface. This is termed as contact resistance. A significant temperature drop develops at the interface and this has to be taken into account in heat transfer calculation. The contact resistance depends on the surface roughness to a great extent. The pressure holding the two surfaces together also influences the contact resistance. When the surfaces are brought together the contact is partial and air may be trapped between the other points as shown in Fig. 1.4.

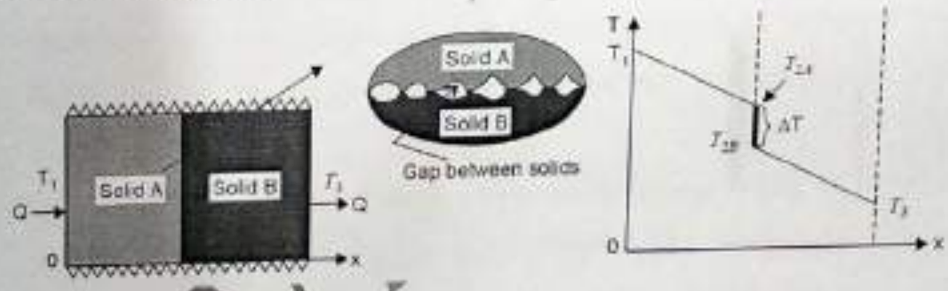


Fig. 1.5: Contact resistance temperature drop

There are two principal contributions to the heat transfer at the joint:

1. The solid-to-solid conduction at the spots of contact
2. The conduction through entrapped gases in the void spaces created by the contact

The second factor is believed to represent the major resistance to heat flow, because the thermal conductivity of the gas is quite small in comparison to that of the solids.

Performing an energy balance on the two materials, we obtain

$$q=k_A A\frac{T_1-T_{2A}}{\Delta x_A}=\frac{T_{2A}-T_{2B}}{1/h_c A}=k_B A\frac{T_{2B}-T_3}{\Delta x_B}$$

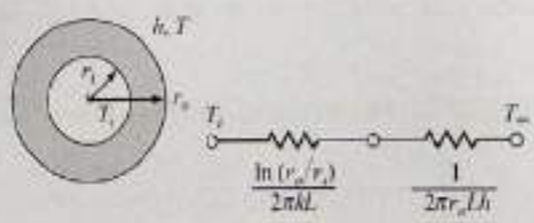
$$q=\frac{T_1-T_3}{\Delta x_A/k_A A+1/h_c A+\Delta x_B/k_B A}$$

where the quantity $1/h_c A$ is called the thermal contact resistance and h_c is called the contact coefficient. This factor can be extremely important in a number of applications because of the many heat-transfer situations that involve mechanical joining of two materials.

What do you understand by critical radius of insulation, obtain the expression for the same?

The addition of insulation always increases the conductive thermal resistance. But when the total thermal resistance is made of conductive thermal resistance and convective thermal resistance, the addition of insulation in some cases may reduce the convective thermal resistance due to increase in outer surface area for convection as in case of a cylinder and sphere and the total thermal resistance may actually decrease resulting in increased heat flow. It may be shown that the thermal resistance actually decreases and then increases in some cases. The thickness up to which heat flow increases and after which heat flow decreases is termed as critical thickness. In case of cylinders and spheres it is called as Critical radius.

For Cylinders: Let us consider a layer of insulation which might be installed around a circular pipe, as shown in Figure. The inner temperature of the insulation is fixed at T_i, and the outer surface is exposed to a convection environment at T_∞. From the thermal network the heat transfer is



$$q = \frac{2\pi L (T_i - T_\infty)}{\frac{\ln(r_o/r_i)}{k} + \frac{1}{r_o h}}$$

Now let us manipulate this expression to determine the outer radius of insulation r_o, which will maximize the heat transfer. The maximization condition is

$$\frac{dq}{dr_o}=0=\frac{-2\pi L\,(T_i-T_\infty)\left(\frac{1}{kr_o}-\frac{1}{hr_o^2}\right)}{\left[\frac{\ln(r_o/r_i)}{k}+\frac{1}{r_oh}\right]^2}$$

which gives the result

$$r_o=r_c=\frac{k}{h_o}$$

If the outer radius is less than the value given by this equation, then the heat transfer will be increased by adding more insulation. For outer radii greater than the critical value an increase in insulation thickness will cause a decrease in heat transfer. The central concept is that for sufficiently small values of h the convection heat loss may actually increase with the addition of insulation because of increased surface area.

Note that the critical radius of insulation depends on the thermal conductivity of the insulation k and the external convection heat transfer coefficient h. The value of the critical radius r_c will be the largest when k is large and h is small.

The critical radius would be much less in forced convection, often less than 1 mm, because of much larger h values associated with forced convection. Therefore, we can insulate hot water or steam pipes freely without worrying about the possibility of increasing the heat transfer by insulating the pipes.

The radius of electric wires may be smaller than the critical radius. Therefore, the plastic electrical insulation may actually enhance the heat transfer from electric wires and thus keep their steady operating temperatures at lower and thus safer levels.

For Sphere: The following relationship can be derived, using similar method

$$R=\frac{1}{4\pi k}\left[\frac{1}{r_1}-\frac{1}{r}\right]+\frac{1}{4\pi r^2h} \qquad \frac{dR}{dr}=0=\frac{1}{4\pi k}[1]-\frac{2}{4\pi rh}$$

$$r=r_c=\frac{2k}{h_o}$$

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1. **What is critical radius of insulation?**

Critical radius (r_c): it is defined as outer radius of insulation for which the heat transfer rate is maximum.

Critical thickness: it is defined as the thickness of insulation for which the heat transfer rate is maximum.

2. **What is super insulation and give its application.**

Super insulation is a process which is used to keep the cryogenic liquids at very low temperature. The super insulation consists of multiple layers of highly reflective material separated by insulating spacers. The entire system is evacuated to minimize air conduction.

State Newton's law of cooling or convection law.

What is meant by Newtonian heating or cooling process?

The process in which the internal resistance is assumed as negligible in comparison with its surface resistance is known as Newtonian heating or cooling process. Heat transfer by convection is given by Newton's law of cooling

$Q = hA(T_s - T_a)$

THE OVERALL HEAT-TRANSFER COEFFICIENT

Consider the plane wall shown in Figure 1 exposed to a hot fluid A on one side and a cooler fluid B on the other side. The heat transfer is expressed by

$$q = h_1 A (T_A - T_1) = \frac{kA}{\Delta x}(T_1 - T_2) = h_2 A (T_2 - T_B)$$

The heat-transfer process may be represented by the resistance network in Figure, and the overall heat transfer is calculated as the ratio of the overall temperature difference to the sum of the thermal resistances:

$$q = \frac{T_A - T_B}{1/h_1 A + \Delta x/kA + 1/h_2 A}$$

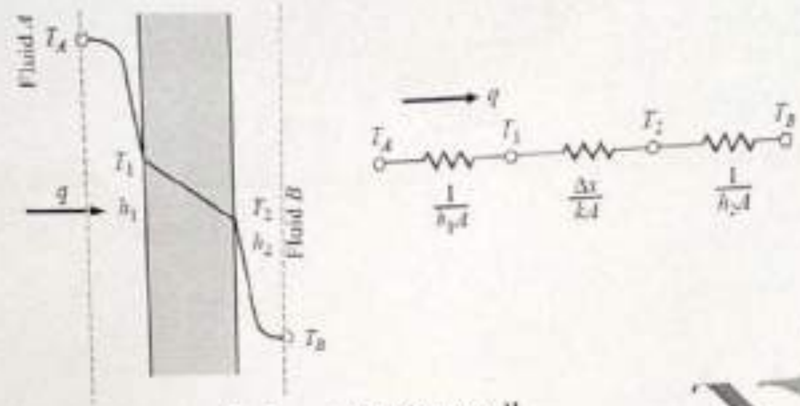

Figure 1: Overall heat transfer through a plane wall.

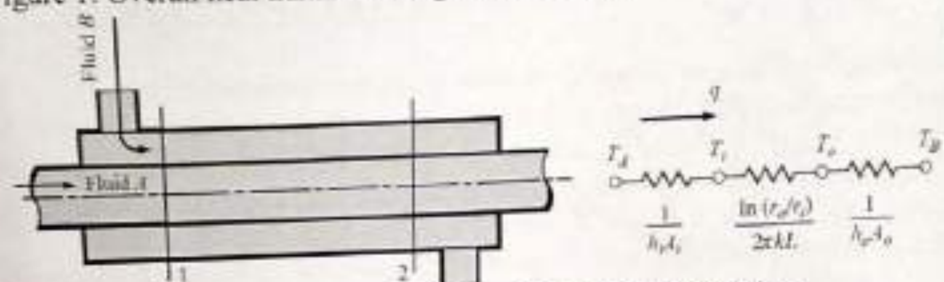

Figure 2: Resistance analogy for hollow cylinder with convection boundaries.

Observe that the value $1/hA$ is used to represent the convection resistance. The overall heat transfer by combined conduction and convection is frequently expressed in terms of an overall heat-transfer coefficient U, defined by the relation $q = UA\,\Delta T_{\text{overall}}$ where A is some suitable area for the heat flow.

For a hollow cylinder exposed to a convection environment on its inner and outer surfaces, the electric-resistance analogy would appear as in Figure 2 where, again, T_A and T_B are the two fluid temperatures. Note that the area for convection is not the same for both fluids in this case, these areas depending on the inside tube diameter and wall thickness. The overall heat transfer would be expressed by

$$q = \frac{T_A - T_B}{\dfrac{1}{h_i A_i} + \dfrac{\ln(r_o/r_i)}{2\pi kL} + \dfrac{1}{h_o A_o}}$$

in accordance with the thermal network shown in Figure 2. The terms A_i and A_o represent the inside and outside surface areas of the inner tube. The overall heat-transfer coefficient may be based on either the inside or the outside area of the tube. Accordingly,

$$U_i = \frac{1}{\dfrac{1}{h_i} + \dfrac{A_i \ln(r_o/r_i)}{2\pi kL} + \dfrac{A_i}{A_o}\dfrac{1}{h_o}}$$

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$$U_o = \frac{1}{\frac{A_o}{A_i}\frac{1}{h_i} + \frac{A_o \ln(r_o/r_i)}{2\pi k L} + \frac{1}{h_o}}$$

The general notion, for either the plane wall or cylindrical coordinate system, is that $UA = 1/\Sigma R_{th} = 1/R_{th,overall}$

BOUNDARY CONDITIONS :

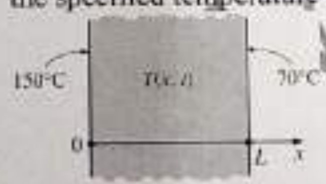
The heat conduction equations were developed using an energy balance on a differential element inside the medium, and they remain the same regardless of the thermal conditions on the surfaces of the medium. The *mathematical expressions* of the thermal conditions at the boundaries are called the **boundary conditions**.

From a mathematical point of view, solving a differential equation is essentially a process of removing derivatives, or an integration process, and thus the solution of a differential equation typically involves arbitrary constants,

For example, the differential equation is $\frac{d^2T}{dx^2} = 0$ after solving derivatives, or an integration, we get, general solution,
$T(x) = c_1 x + c_2$ where, c_1 and c_2 are arbitrary constants.

1. Specified Temperature Boundary Condition

For one-dimensional heat transfer through a plane wall of thickness L, for example, the specified temperature boundary conditions can be expressed as figure



$T(0, t) = 150°C$
$T(L, t) = 70°C$

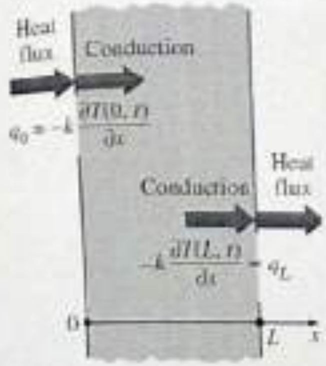
$$T(0, t) = T_1$$
$$T(L, t) = T_2$$

where T1 and T2 are the specified temperatures at surfaces at x = 0 and x = L, respectively. The specified temperatures can be constant, which is the case for steady heat conduction, or may vary with time.

2. Specified Heat Flux Boundary Condition

To determine the rate of heat transfer and thus the heat flux q (heat transfer rate per unit surface area, W/m^2) on that surface, and this information can be used as one of the boundary conditions. The heat flux in the positive x-direction anywhere in the medium, including the boundaries, can be expressed by Fourier's law of heat conduction as

$$q = -k\frac{\partial T}{\partial x} = \begin{pmatrix} \text{Heat flux in the} \\ \text{positive } x\text{-direction} \end{pmatrix} \qquad (\text{W/m}^2)$$



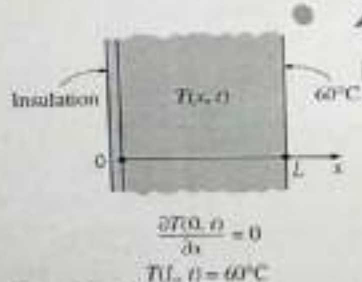
Then the boundary condition at a boundary is obtained by setting the specified heat flux equal to $-k(dT/dx)$ at that boundary. The sign of the specified heat flux is determined by inspection: positive if the heat flux is in the positive direction of the coordinate axis, and negative if it is in the opposite direction.

Note that it is extremely important to have the correct sign for the specified heat flux since the wrong sign will invert the direction of heat transfer and cause the heat gain to be interpreted as heat loss, shown in figure.

3. Insulated Boundary

Some surfaces are commonly insulated in practice in order to minimize heat loss (or heat gain) through them. Insulation reduces heat transfer but does not totally eliminate it unless its thickness is infinity. However, heat transfer through a properly insulated surface can be taken to be zero since adequate insulation reduces heat transfer through a surface to negligible levels. Therefore, a well-insulated surface can be modeled as a surface with a specified heat flux of zero. Then the boundary condition on a perfectly insulated surface (at x = 0, for example) can be expressed as shown in figure.

$$k\frac{\partial T(0,t)}{\partial x} = 0 \quad \text{or} \quad \frac{\partial T(0,t)}{\partial x} = 0$$



That is, on an insulated surface, the first derivative of temperature with respect to the space variable (the temperature gradient) in the direction normal to the insulated surface is zero. This also means that the temperature function must be perpendicular to an insulated surface since the slope of temperature at the surface must be zero.

4. Convection Boundary Condition

The convection boundary condition is based on a surface energy balance expressed as

$$\begin{pmatrix}\text{Heat conduction}\\ \text{at the surface in a}\\ \text{selected direction}\end{pmatrix} = \begin{pmatrix}\text{Heat convection}\\ \text{at the surface in}\\ \text{the same direction}\end{pmatrix}$$

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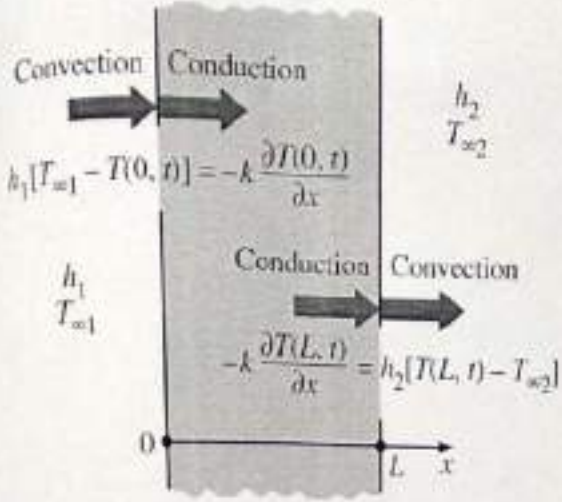


For one-dimensional heat transfer in the x-direction in a plate of thickness L, the convection boundary conditions on both surfaces can be expressed as

$$-k\frac{\partial T(0,t)}{\partial x} = h_1[T_{\infty1} - T(0,t)] \quad (1a)$$

And

$$-k\frac{\partial T(L,t)}{\partial x} = h_2[T(L,t) - T_{\infty2}] \quad (1b)$$

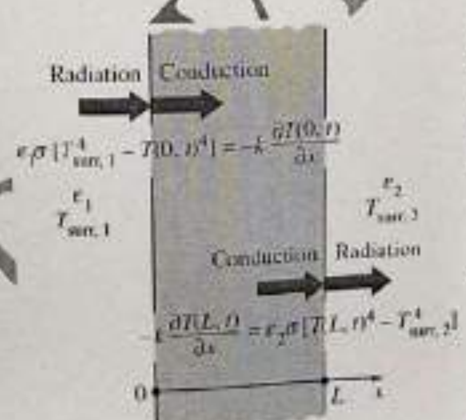
where h_1 and h_2 are the convection heat transfer coefficients and T_1 and T_2 are the temperatures of the surrounding mediums on the two sides of the plate, as shown in Figure.

The convection boundary condition simply states that heat continues to flow from a body to the surrounding medium at the same rate

5. Radiation Boundary Condition

In some cases, such as those encountered in space and cryogenic applications, a heat transfer surface is surrounded by an evacuated space and thus there is no convection heat transfer between a surface and the surrounding medium. In such cases, radiation becomes the only mechanism of heat transfer between the surface under consideration and the surroundings. Using an energy balance, the radiation boundary condition on a surface can be expressed as

$$\begin{pmatrix}\text{Heat conduction}\\ \text{at the surface in a}\\ \text{selected direction}\end{pmatrix} = \begin{pmatrix}\text{Radiation exchange}\\ \text{at the surface in}\\ \text{the same direction}\end{pmatrix}$$



For one-dimensional heat transfer in the x-direction in a plate of thickness L, the radiation boundary conditions on both surfaces can be expressed as shown in figure,

$$-k\frac{\partial T(0,t)}{\partial x} = \varepsilon_1\sigma[T^4_{surr,1} - T(0,t)^4] \quad (2a)$$

And

$$-k\frac{\partial T(L,t)}{\partial x} = \varepsilon_2\sigma[T(L,t)^4 - T^4_{surr,2}] \quad (2b)$$

Where, ε_1 and ε_2 are the emissivities of the boundary surfaces,
$\alpha = 5.67 \times 10^{-8}\ W/m^2.K^4$ is the Stefan–Boltzmann constant, and $T_{surr,1}$ and $T_{surr,2}$ are the average temperatures of the surfaces surrounding the two sides of the plate,

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respectively. Note that the temperatures in radiation calculations must be expressed in K or R (not in °C or °F).

The radiation boundary condition involves the fourth power of temperature, and thus it is a nonlinear condition. As a result, the application of this boundary condition results in powers of the unknown coefficients, which makes it difficult to determine them. Therefore, it is tempting to ignore radiation exchange at a surface during a heat transfer analysis in order to avoid the complications associated with nonlinearity. This is especially the case when heat transfer at the surface is dominated by convection, and the role of radiation is minor.

HEAT TRANSFER FROM EXTENDED SURFACES

FINS

The rate of heat transfer from a surface at a temperature Ts to the surrounding medium at T_∞ is given by Newton's law of cooling as

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$

There are two ways to increase the rate of heat transfer:

1. To increase the convection heat transfer coefficient h, but this approach may or may not be practical. or,
2. To increase the surface area A_s. The alternative is to increase the surface area by attaching to the surface extended surfaces called fins made of highly conductive materials such as aluminum.

Finned surfaces are manufactured by extruding, welding, or wrapping a thin metal sheet on a surface. Fins enhance heat transfer, and they often increase the rate of heat transfer from a surface by exposing a larger surface area to convection and radiation. The value of h is usually much lower at the fin base than it is at the fin tip because the fluid is surrounded by solid surfaces near the base.

Common examples of the use of extended surfaces are in cylinder heads of air cooled engines and compressors and on electric motor bodies. In air conditioners and radiators tubes with circumferential fins are used to increase the heat flow.

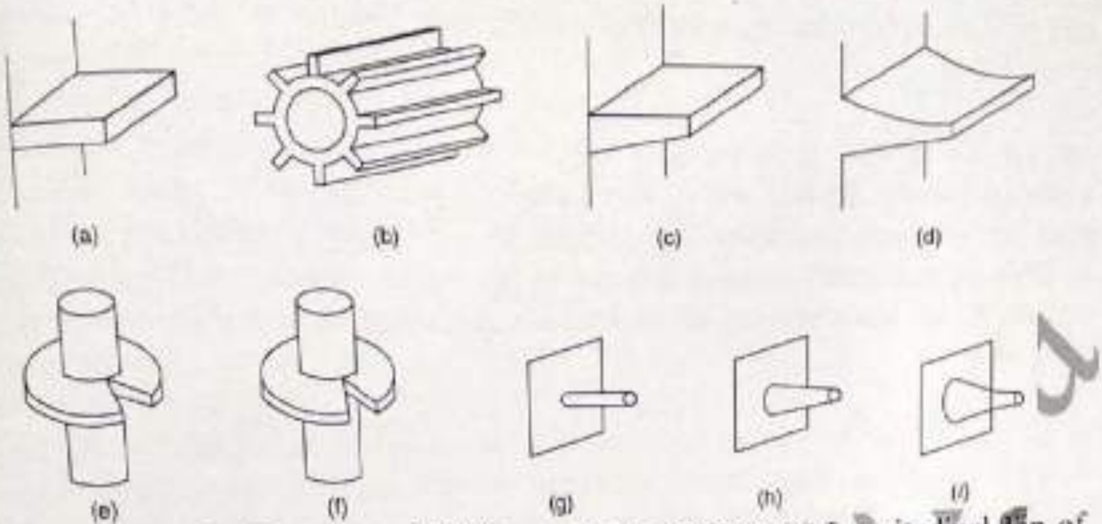

Fig.: Schematic Diagrams of Different Types of Fins: (a) Longitudinal Fin of Rectangular Profile; (b) Cylindrical Tube with Fins of Rectangular Profile; (c) Longitudinal Fin of Trapezoidal Profile; (d) Longitudinal Fin of Parabolic Profile; (e) Cylindrical Tube with Radial Fin of Rectangular Profile; (f) Cylindrical Tube with Radial Fin of Truncated Conical Profile; (g) Cylindrical Pin Fin; (h) Truncated Conical Spine; (i) Parabolic Spine.

Why aluminium used as a fin material?

The main aim of the study is to design fins to optimize the use of a given amount of material to maximize heat transfer. For this purpose it will be desirable that the fin surface temperature is closer to the base surface temperature. This can be achieved by the use of materials of high thermal conductivity like copper or aluminium. In terms of weight and ease of lubrication aluminium will score over copper though its thermal conductivity will be lower.

Thermal conductivity of the fin material should be high to give large fin effectiveness. This leads to the choice of aluminium and its alloys.

Density

Lightness is the outstanding and best known characteristic of aluminium. The metal has an atomic weight of 26.98 and a specific gravity of 2.70, approximately **one-third the weight of other commonly used metals**; As with most metals the density decreases with increasing temperature. The addition of other metals in the amounts commonly used in aluminium alloys does not appreciably change the density (plus 3%, minus 2%). Weight is important for all applications involving motion. Saving weight results in more payload or greater economy of operation. Saving weight also saves energy, reduces vibration forces, and improves the performance of reciprocating and moving parts.

Electrical Conductivity and Resistivity

The electrical conductivity of 99.99% pure aluminium at 20°C. Because of its low specific gravity, the mass electrical conductivity of pure aluminium is more than twice that of annealed copper and greater than that of any other metal. The resistivity at 20^0C is 2.69 μΩ-cm.

The thermal conductivity

The thermal conductivity, κ, of 99.99% pure aluminium is 244 W/mK for the temperature range 0-100^0C, and again because of its low specific gravity its **mass thermal conductivity is twice that of copper.** The combined properties of high thermal conductivity, low weight and good formability make aluminium an obvious choice for use in heat exchangers, car radiators as fin materials and for I/C engine cylinder heads.

Corrosion Resistance

Aluminium has a higher resistance to corrosion than many other metals owing to the protection conferred by the thin but tenacious film of oxide. This oxide layer is always present on the surface of aluminium in oxygen atmospheres.

Specific and Latent Heats

Aluminium has a relatively high specific heat when compared with other metals on a weight basis, i.e. 921 J/kg at 1000C which is higher than that of any common metal.

Heat Flow Through "RECTANGULAR FIN"

Consider the one-dimensional rectangular fin exposed to a surrounding fluid at a temperature T_∞ as shown in Figure. The temperature of the base of the fin is T_0. We approach the problem by making an energy balance on an element of the fin of thickness dx as shown in the figure. Thus

Energy in left face = Energy out right face + Energy lost by convection

The defining equation for the convection heat-transfer coefficient is recalled as

$q = hA\,(T_w - T_\infty)$ ---------- 1

Where the area in this equation is the surface area for convection. Let the cross-sectional area of the fin be A and the perimeter be P. Then the energy quantities are

$$\text{Energy in left face} = q_x = -kA\frac{dT}{dx}$$

$$\text{Energy out right face} = q_{x+dx} = -kA\left.\frac{dT}{dx}\right]_{x+dx}$$

$$= -kA\left(\frac{dT}{dx} + \frac{d^2T}{dx^2}dx\right)$$

$$\text{Energy lost by convection} = h\,P\,dx\,(T - T_\infty)$$

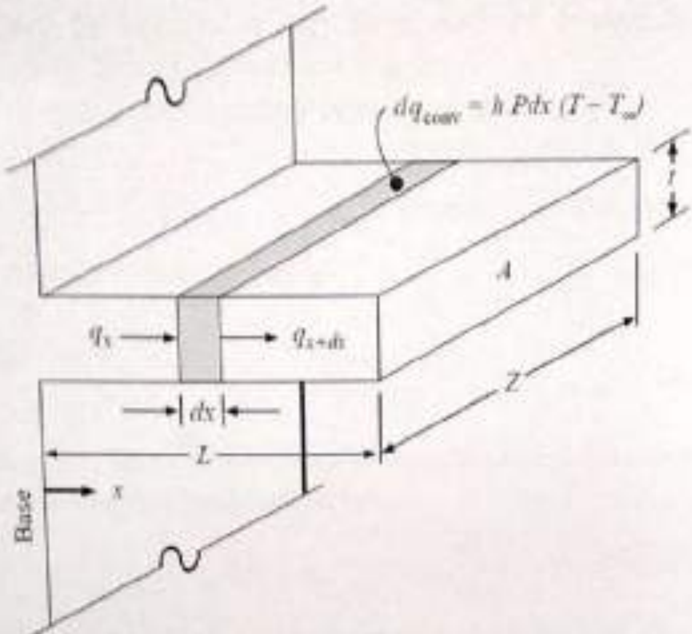

Fig.: Rectangular fin of uniform cross-section.

Applying an energy balance on the element,

Energy in left face = Energy out right face + Energy lost by convection

$$Q_x = Q_{(x+dx)} + Q_{convection}$$

Here it is noted that the differential surface area for convection is the product of the perimeter of the fin and the differential length dx. When we combine the quantities of Q_x, $Q_{(x+dx)}$, $Q_{convection}$, the energy balance yields

$$\frac{d^2T}{dx^2} - \frac{hP}{kA}(T - T_\infty) = 0 \quad \text{---------} \quad 2$$

Let $\theta = T - T_\infty$. Then Equation (2) becomes

$$\frac{d^2\theta}{dx^2} - \frac{hP}{kA}\theta = 0 \quad \text{---------} \quad 3$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \quad \text{-----------} \quad 4$$

Where, $m^2 = hP/kA$

One boundary condition is $\theta = \theta_0 = T_0 - T_\infty$ at $x = 0$

The other boundary condition depends on the physical situation. Several cases may be considered:

If we let $m^2 = hP/kA$, the general solution for Equation (4) may be written

$$\theta = C_1 e^{-mx} + C_2 e^{mx}$$

All of the heat lost by the fin must be conducted into the base at x=0. Using the equations for the temperature distribution, we can compute the heat loss from

$$q = -kA \left.\frac{dT}{dx}\right]_{x=0}$$

Case 1: The fin is very long, and the temperature at the end of the fin is essentially that of the surrounding fluid.

For case 1 the boundary conditions are

$$\theta = \theta_0 \quad \text{at } x = 0$$

$$\theta = 0 \quad \text{at } x = \infty$$

and the solution becomes $$\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} = e^{-mx}$$

$$\frac{T(x) - T_\infty}{T_0 - T_\infty} = e^{-mx} = e^{-x\sqrt{hp/kA_c}}$$

The steady rate of *heat transfer* from the entire fin can be determined from Fourier's law of heat conduction

$$\dot{Q}_{\text{long fin}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_0 - T_\infty)$$

Case 2: The fin is of finite length and loses heat by convection from its end.

For case 2 the boundary conditions are, At $x = 0, \theta = \theta_0$

dT/dx=0 at x=L

The solution for case 2 is more involved algebraically, and the result is

$$\frac{T - T_\infty}{T_o - T_\infty} = \frac{\cosh m\,(L - x) + (h/mk)\sinh m\,(L - x)}{\cosh mL + (h/mk)\sinh mL}$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = \frac{\tanh(ml) + \frac{h}{mk}}{1 + \frac{h}{mk}\tanh(ml)}$$

$$Q_{fin} = \sqrt{PhkA}(T_0 - T_\infty)\frac{\tanh(ml) + \frac{h}{mk}}{1 + \frac{h}{mk}\tanh(ml)}$$

Case 3: The end of the fin is insulated so that dT/dx=0 at x=L.

For case 3 the boundary conditions are $\theta = \theta_0$ at $x = 0$

$$\frac{d\theta}{dx} = 0 \quad \text{at } x = L$$

Thus

$$\theta_0 = C_1 + C_2$$
$$0 = m(-C_1 e^{-mL} + C_2 e^{mL})$$

Solving for the constants C_1 and C_2, we obtain

$$\frac{\theta}{\theta_0} = \frac{e^{-mx}}{1+e^{-2mL}} + \frac{e^{mx}}{1+e^{2mL}} \quad \text{—— } 5$$

$$= \frac{\cosh[m(L-x)]}{\cosh mL} \quad \text{—— } 6$$

$$\boldsymbol{Q_{fin\ insulated} = \sqrt{PhkA_{cs}}(T_0 - T_\infty)\tanh(ml)}$$

The hyperbolic functions are defined as

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Fin Efficiency

The efficiency of a fin is defined as

$$\eta_{fin} = \frac{Q_{fin,Actual}}{Q_{fin,max}} = \frac{Actual\ heat\ transferred\ by\ the\ fin}{Maximum\ heat\ that\ that\ would\ be\ transferred\ if\ whole\ surface\ of\ the\ fin\ is\ maintained\ at\ the\ base\ temp}$$

$$\eta_{fin} = \frac{\sqrt{PhkA}(T_0 - T_\infty)}{Phl(T_0 - T_\infty)} = \sqrt{\frac{kA}{Phl^2}} = \frac{1}{ml} \quad \text{------------ } 1$$

where A_{fin} is the total surface area of the fin. This relation enables us to determine the heat transfer from a fin when its efficiency is known. For the cases of constant cross section of *very long fins* and *fins with insulated tips*, the fin efficiency can be expressed as

For a fin with insulated tips:

$$\eta_{fin} = \frac{\sqrt{PhkA}(T_0 - T_\infty)\tanh(ml)}{Phl(T_0 - T_\infty)} = \frac{\tanh(ml)}{ml}$$

Effectiveness of fin (ϵ_{fin}):

$$\epsilon_{fin} = \frac{Q_{with\ fin}}{Q_{without\ fin}} = \frac{\sqrt{PhkA}(T_0 - T_\infty)}{PA(T_0 - T_\infty)} = \sqrt{\frac{Pk}{Ah}}$$

Relation between η_{fin} and ϵ_{fin}:

The performance parameters of η_{fin} and ϵ_{fin}, in case of a fin insulated at the tip, are related to each other by the following expressions:

Efficiency of fin,

$$\eta_{fin} = \frac{\sqrt{PhkA}(T_0 - T_\infty)\tanh(ml)}{Phl(T_0 - T_\infty)} = \frac{\tanh(ml)}{ml} \quad \text{.............. (i)}$$

Effectiveness of fin,

$$\epsilon_{fin} = \frac{\sqrt{PhkA}(T_0 - T_\infty)\tanh(ml)}{PA(T_0 - T_\infty)} \quad \text{........................(ii)}$$

Dividing eqn. (ii) by eqn. (i), we have

$$\frac{\epsilon_{fin}}{\eta_{fin}} = \frac{Pl}{A} \quad or \quad \epsilon_{fin} = \eta_{fin}\frac{Pl}{A} = \eta_{fin} \times \frac{Surface\ area\ of\ the\ fin}{Cross-sectional\ area\ of\ the\ fin}$$

It is evident from the above equations that an increase in fin effectiveness can be obtained by increasing the length of the fin but it decreases the efficiency of the fin on the other hand.

An effectiveness of $\epsilon_{fin} = 1$ indicates that the addition of fins to the surface does not affect heat transfer at all. That is, heat conducted to the fin through the base area *A* is equal to the heat transferred from the same area *Ab* to the surrounding medium. An effectiveness of $\epsilon_{fin} < 1$ indicates that the fin actually acts as *insulation*, slowing down the heat transfer from the surface. This situation can occur when fins made of low thermal conductivity materials are used. An effectiveness of $\epsilon_{fin} > 1$ indicates that fins are *enhancing* heat transfer from the surface, as they should. However, the use of fins cannot be justified unless _fin is sufficiently larger than 1. Finned surfaces are designed on the basis of *maximizing* effectiveness for a specified cost or *minimizing* cost for a desired effectiveness.

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Estimation of Error in Temperature Measurement in a Thermometer Well:

For estimating error in the value of temperature measured by a thermometer dipped in a thermometer well, the theory of extended surfaces is very helpful. A *thermometer well* is defined as a *small tube welded radially into a pipeline through which a fluid whose temperature is to be measured is flowing.*

Refer to Fig.

Let, l = Length of the well/tube,

d = Internal diameter of the well/tube,

δ = Thickness of well/tube,

t_f = Temperature of the fluid flowing through the pipe (which is to be measured), and

t_o = Temperature of the pipe-wall.

When the temperature of the fluid flowing through the pipeline is higher than the ambient temperature, the heat flows from the fluid towards the tube walls along the well. Consequently the temperature at the bottom of well becomes colder than the fluid flowing around, obviously the temperature shown by the thermometer will not be the true temperature of the fluid. This error may be calculated by assuming the well to be a spine protruding from the wall of a pipe in which fluid is flowing. It may be assumed, for simplicity, that there is no flow of heat from the tip of the well (*i.e.,* the tip of the well is *insulated*). The temperature distribution at any distance x measured from pipe wall along the temperature well is given by

$$\frac{\theta_x}{\theta_o} = \frac{t_x - t_f}{t_o - t_f} = \frac{\cosh[m(l-x)]}{\cosh(ml)} \qquad \text{———} \ 1$$

At $x = l$, we have

$$\frac{t_l - t_f}{t_o - t_f} = \frac{\cosh[m(l-l)]}{\cosh(ml)} = \frac{1}{\cosh(ml)} \qquad \text{———} \ 2$$

[Thermometeric error]

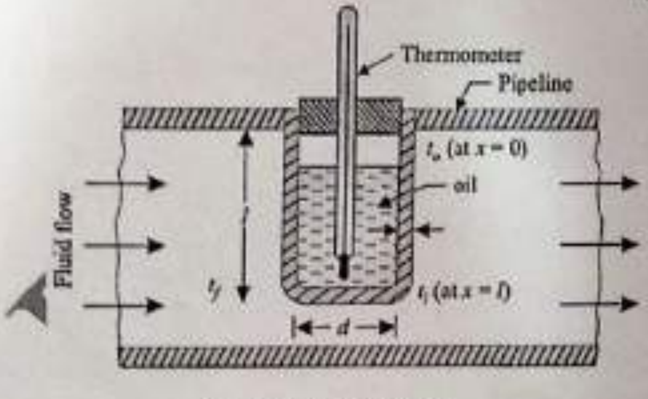


Fig: Thermometer Well

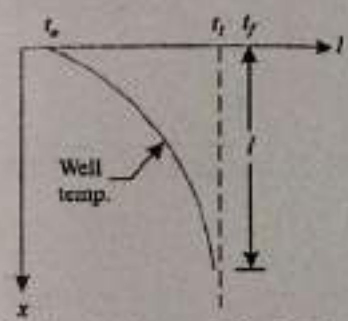


Fig: Temperature variation in well

(where, t_l = Temperature recorded by the thermometer at the bottom of the well.)

Now, perimeter of the well, $P = \pi(d + 2\delta) = \pi d$,

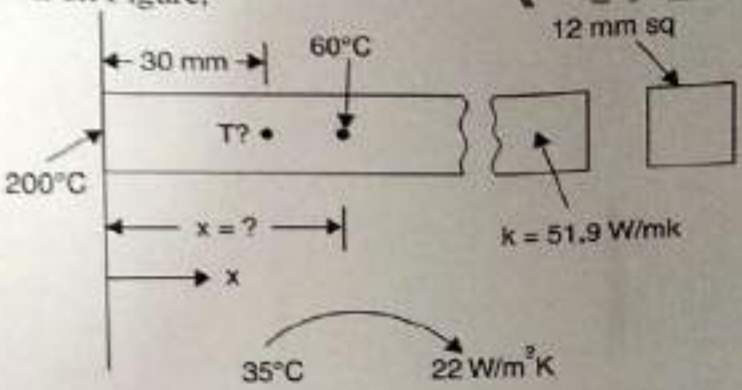
and cross-sectional area, $A_{cs} = \pi d \delta$

$$\therefore \qquad \frac{P}{A_{cs}} = \frac{\pi d}{\pi d\,\delta} = \frac{1}{\delta}$$

Then,
$$m = \sqrt{\frac{hP}{k A_{cs}}} = \sqrt{\frac{h}{k\delta}}$$

Numerical on Fins:

Example 01: A long rod 12 mm square section made of low carbon steel protrudes into air at 35°C from a furnace wall at 200°C. The convective heat transfer coefficient is estimated at 22W/m^{2}K. The conductivity of the material is 51.9 W/mK. Determine the location from the wall at which the temperature will be 60°C. Also calculate the temperature at 80 mm from base.
The data are shown on Figure.



Solution: The long fin model will suit the problem situation.

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{60 - 35}{200 - 35} = e^{-mx}$$

The distance x is to be calculated:

$$m = \sqrt{hp/kA} = \sqrt{(22 \times 4 \times 0.012/51.9 \times 0.012 \times 0.012)} = 11.89$$

$$0.1515 = e^{-11.89x}$$

Taking logarithm and solving $x = 0.159$ m or 159 mm.

Temperature at 80 mm: $\frac{T_x - T_\infty}{T_b - T_\infty} = \frac{T_x - 35}{200 - 35} = e^{-11.89 \times 0.08}$

Therefore, **$T = 98.74^0C$**

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Problem 02: One end of a long rod of 35 mm diameter is inserted into a furnace with the other end projected in the outside air. After the steady state reached, the temp. of rod is measured at two points 180 mm apart and found to be 180^0 C and 145^0 C. The atmospheric temp. is 25^0 C. A convection coefficient of 65 W/m^{2}K. Determine the thermal conductivity of the material.

Solution: Dia. Of rod, $d = 35$ mm = 0.035 m

$Ta = 25^0$ C and $h = 65$ W/m^{2}K

Starting point is at $x = 0$ and outer point is at $x = l$

Assume end of the fin is insulated, we have,

$$\frac{T-T_\infty}{T_b-T_\infty} = \frac{\cos[m(l-x)]}{\cosh(ml)}$$

At $x = l$, the eqn reduced to

$$\frac{T-T_\infty}{T_b-T_\infty} = \frac{1}{\cosh(ml)} = \frac{145-25}{180-25}$$

$$\cosh(ml) = 1.292$$

$$m = \frac{0.747}{l}$$

$$m = \sqrt{\frac{hp}{kA}} = \frac{0.747}{l}$$ where, $A = \frac{\pi d^2}{4}$ and $p = \pi d$ and $l = 180$ mm

Solving the eqn, we get, k= **431.34** W/m^0C

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