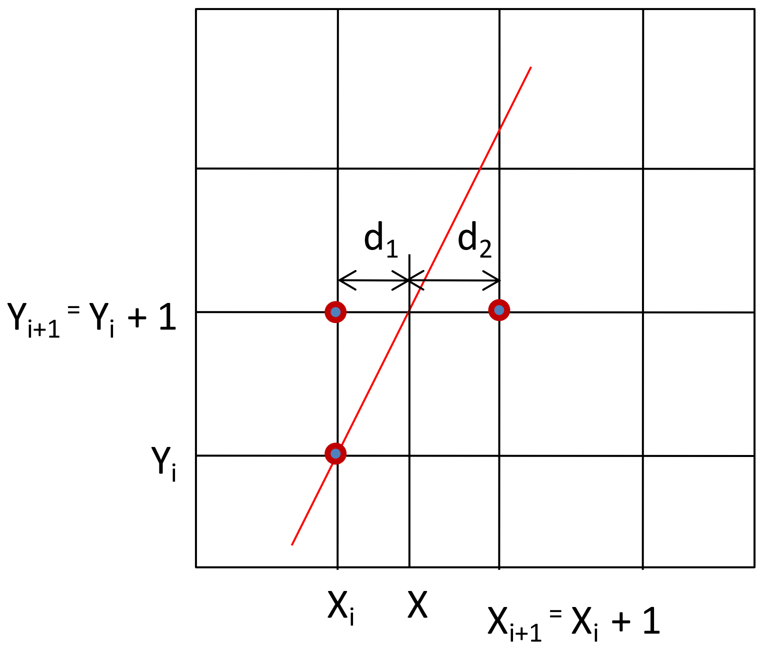
Bresenham's Algorithm for line having Slope, m>1

Solution: Input first point & Last point



As m>1

$Y_{i+1} = Y_i + 1$

we need decide

$X_{i+1} = X_i$ OR $X_{i+1} = X_i + 1$

$d_1 = X - X_i$ & $d_2 = X_{i+1} - X = X_i + 1 - X$

If $(d_1 - d_2) < 0$

then $d_1 < d_2$

$X_{i+1} = X_i$

If $(d_1 - d_2) > 0$

then $d_1 > d_2$

$X_{i+1} = X_i + 1$

$d_1 = X - X_i$ & $d_2 = X_{i+1} – X = X_i +1 –X$

$d_1 - d_2 = X - X_i – (X_i +1 –X)$

$= X - X_i – X_i -1 + X$

$d_1 - d_2 = 2X – 2X_i -1$ ----(1)

Here we need to consider eq of line

$Y = mX + C$

$X = (Y/m) – (C/m)$

$X = (Y/m) – K$ where, $K = C/m$

$X = [(Y_i + 1)/m] – K$

$d_1 - d_2 = 2\{[(Y_i + 1)/m] – K \} – 2X_i -1$

$d_1 - d_2 = [(2Y_i)/m] + (2/m) – 2K – 2X_i -1$

$d_1 - d_2 = [(2Y_i)/m] – 2X_i + N$ Where, $N = (2/m) – 2K – 1$

$d_1 - d_2 = [(2Y_i)/(dy/dx)] – 2X_i + N$

$(d_1 - d_2)\, dy = (2Y_i dx) – 2X_i\, dy + Ndy$

consider, $(d_1 - d_2)\, dy = P_i$

$P_i = 2Y_i dx – 2X_i\, dy + Ndy$ ---(2)

if $P_i < 0$, then, $d1<d2$, $X_{i+1} = X_i$

if $P_i > 0$, then, $d1>d2$, $X_{i+1} = X_i + 1$

decision parameter for (i+2)nd pixel

$P_{i+1} = 2Y_{i+1}\, dx – 2X_{i+1}\, dy + Ndy$ --- (3)

(3) – (2)

$P_{i+1} - P_i = 2Y_{i+1}\,dx - 2X_{i+1}\,dy + Ndy - [2Y_i dx - 2X_i\,dy + Ndy]$

$P_{i+1} - P_i = 2Y_{i+1}\,dx - 2X_{i+1}\,dy$ **+ Ndy** $- 2Y_i dx + 2X_i\,dy$ **– Ndy**

$P_{i+1} - P_i = 2dx(Y_{i+1} - Y_i) - 2dy(X_{i+1} - X_i)$

$P_{i+1} = P_i + 2dx(Y_{i+1} - Y_i) - 2dy(X_{i+1} - X_i)$ --- (4)

If $P_i < 0$, then $Y_{i+1} = Y_i + 1$ & $X_{i+1} = X_i$

$P_{i+1} = P_i + 2dx(Y_i + 1 - Y_i) - 2dy(X_i - X_i)$

$P_{i+1} = P_i + 2dx$ ----(5)

If $P_i > 0$, then $Y_{i+1} = Y_i + 1$ & $X_{i+1} = X_i + 1$

$P_{i+1} = P_i + 2dx(Y_i + 1 - Y_i) - 2dy(X_i + 1 - X_i)$

$P_{i+1} = P_i + 2dx - 2dy$ ---(6) $P_{i+1} = P_i + 2(dx-dy)$

for first decision parameter

Pt.(X_A, Y_A)

$Y_A = mX_A + C$

$X_A = (Y_A/m) - (C/m)$

$X_A = (Y_A/m) - K$

$X_A = (Y_A/[dy/dx]) - K$

$X_A dy = Y_A dx - Kdy$

Consider equation (2)

$P_0 = 2Y_A\,dx - 2X_A\,dy + N\,dy$

$$P_0 = 2Y_A\ dx - 2X_A\ dy + [(2/m) - 2K - 1]\ dy$$

$$P_0 = 2Y_A\ dx - 2X_A\ dy + [(2dx/dy) - 2K - 1]\ dy \qquad \text{since, } m = dy/dx$$

$$P_0 = 2Y_A\ dx - 2X_A\ dy + 2dx - 2Kdy - dy$$

$$P_0 = 2Y_A\ dx - 2[Y_A dx - Kdy] + 2dx - 2Kdy - dy$$

$$P_0 = 2Y_A\ dx - 2Y_A dx + 2Kdy + 2dx - 2Kdy - dy$$

$$P_0 = 2dx - dy \qquad (7)$$

K. D. K. College of Engineering, Nagpur

Mechanical Engineering Department

Subject: Computer Aided Design (CAD)

Semester: VII

Session: 2020 -21

V. D. Dhopte

Syllabus

UNIT – I

Introduction of CAD, Difference between Conventional & CAD design, Rasterisation techniques frame buffer, N-bit plane buffers, Simple color frame buffer algorithm for the generation of basic geometric entities like line, circle & ellipse by using parametric & non-parametric equations.

UNIT – II

Introduction to windowing & clipping (excluding algorithm), Window and Viewport, line clipping & polygon clipping

2D transformation: Translation, Scaling, Rotation, Reflection & Shear, Concept of homogeneous representation & concatenation. Inverse Transformation (enumeration of entity on graph paper)

3D Transformation ; Translation, Scaling, Rotation, Reflection etc.

UNIT – III

Techniques for Geometric Modeling:

Graphic standards, parametric representation of geometry, Bezier curves, Cubic spline curves, Bspline curves, constructive solid geometry, Feature Based modeling, Feature recognition, Design by feature, Wire frame modeling, solid modeling of basic entities like box, cone, cylinder. CSG & B- representation technique using set theory.

Assembly modeling: Representation, mating conditions, representation schemes, generation of
assembly sequences and importance of precedence diagram.

UNIT – IV

Finite Element Analysis:

One Dimensional Problem: Fundamental concept of finite element method, Plain stress and strain, Finite Element Modeling, Potential Energy Approach, Galerkin Approach, Coordinate and Shape function, Assembly of Global Stiffness Matrix and Load Vector, Properties of Stiffness Matrix, Finite Element Equations, Quadratic Shape Function, Temperature Effects, Torsion of a circular shaft.

UNIT – V [8 Hrs.]

Truss & Two Dimensional FEM:

Plane truss problems, two dimensional problems using Constant strain triangle. Derivation of shape functions for CST element. Formulation of stiffness matrices for Truss and CST element. Preprocessing and Post processing.

UNIT – VI [8 Hrs.]

Optimization in Design:

Objectives of optimum design, adequate and optimum design, Johnson's Method of optimum design, primary design equation, subsidiary design equations and limit equations, optimum design with normal and redundant specifications of simple machine elements like: tension bar, transmission shaft and helical spring.

Course Outcome

On the completion of this course students will be able to

CO703.1 Know basic concept of CAD, Comparison between CAD and conventional design, generation of algorithms for basic geometric entities.

CO703.2 Understand Introduction to windowing & clipping, 2D transformation, 3D transformation.

CO703.3 Know Techniques for geometric modeling and assembly modeling.

CO703.4 Understand Finite element analysis of one dimensional problem, Finite element modeling and Potential energy approach.

CO703.5 Analyze Truss and Two dimensional Problems, Derivation of shape functions for CST element, Pre processing and Post processing.

CO703.6 Know optimization in Design, objectives of optimum design, Johnson's method of optimum design, Optimum design with normal and redundant specifications of simple machine elements.

Overall Product Development

↓

Market Survey – Customer Requirement

↓

Study of Functional Requirement or expected output from the product or machine

↓

Decide Mechanism to meet the Functional Requirement

↓

Synthesis of Mechanism – To decide some dimensions of elements as length

↓

Carry out Design – To decide dimensions of elements as cross section based on strength and rigidity criteria

↓

After all calculations a production drawing is need to be prepared

↓

Production of parts

↓

Assembly

↓

Conduction of Test on Product – To check whether it meets functional requirements or not

↓

If it meets all functional requirements then fine.

↓

If the product does not meet any of the Functional Requirement, then above whole process is repeated . Design is iterative process.

↓

i.e., correction in mechanism which leads to some change in dimensions – then correction in drawing – then production of parts – Assembly of parts - Test

↓

Also sometime customer feedback about product need to be implemented, which again leads to repeat whole process.

↓

Further value addition in products are Aesthetic requirements, Ergonomic requirements, cost effective, etc.

↓

Final Product

Note: Time is the important factor in the whole process.

Questions

1) What is CAD?

2) Which are the various CAD Softwares?

a) Creo b) CATIA c) Solidworks d) AutoCad e) NX

3) Which are the various applications of CAD?

4) What are the various benefits of CAD?

5) What are the various disadvantages of CAD?

6) What is CIM and CAE?

7) Which are the various softwares used for CIM and CAE?

a) ANSYS b) NASTRAN c) FEAST d) LS-DYNA e) Hypermesh f) PATRAN g) FEMAP

8) What are the various applications of FEM?

Definitions of CAD

1) CAD can be defined as the use of computer systems to assist in the creation, modification, analysis or optimization of a design.

2) By CAD, we mean the development and use of special computer programs (Softwares) to help the designer in carrying out routine computations for the design of product.

Introduction:

Design is a key feature of any production system and its importance is growing. It is becoming a major determinant of competitiveness due to emphasis on design and product innovation stressing novel features, frequent changes, greater response to customer requirements and more rapid new product development cycles. All this necessitates the use of not only new design tools such as Computer Aided Design (CAD), but also approaches to integrate design with manufacturing.

CAD helps in getting the analytical results very quickly. This will enable the designer to evaluate more than one design alternatives which otherwise is not possible. Also optimum design solutions can be obtained by using sophisticated programs. This will result in significant savings in unit costs.

When the design is finalized, all the data concerning the product, namely, the component and part numbers, material for each part, dimensions, tolerances and surface finish requirements are stored in a master file.

Benefits of CAD

1) Increased productivity of the designer through the visualization of product and its components, sub assemblies and related parts.

2) Reduction in time necessary for the development of a conceptual designing, analysis and documentation.

3) Integration between designing, analysis and manufacturing through the provision of a common data base.

4) Design error can be eliminated at the early stages of design.

5) CAD improves documentation and standardization of engineering drawing.

6) CAD provides the opportunity for more alternative designs to be produced before a final production version is selected. Thus a better design product is the result.

Benefits of CAD over Conventional Design:

1) By using computer aided design process tedious and repetitive task are handled by the computers. Therefore design engineer gets more free time for thinking. This is especially successful in design synthesis process, which may lead to economical and efficient design.

2) By using computer, computer design engineer can work out with number of alternatives in design which may helps to select the best alternatives.

3) By using CAD software it is possible to see the part before it is manufactured. This is specially useful in case of aesthetic design of parts.

4) In the design evaluation by using computer saving of material is effective, specially for large and critical part.

5) Design optimization of complicated problems where more constraints are impose on the design parameter can only be handled by using computers.

6) The computer can be use extensively in drafting and the following advantages achieved by using computer

a) The drawing can be edited faster and accurately.

b) The manipulation in the drafting can be effectively carried out.

c) The life of document is infinite.

Rasterization Techniques / Raster Scan System

The geometrical data of the image to be displayed on the screen is read from CPU and is converted to pixel data by rasterizer. This process of conversion of geometrical data into pixel data is called Rasterization.

In raster scan system, the display screen area is divided horizontally and vertically into a matrix of small elements called picture elements or pixels. The unique property of pixel is that, it is addressable.

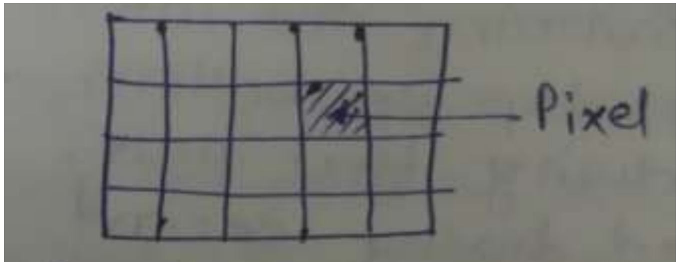


fig: Pixel Matrix

The element beam is swept across the screen, from left to right, top to bottom all the time. As the electron beam moves across each row, the beam intensity is turned “on” or “off” to create a pattern of illuminated pixels. The combinations of illuminated pixels produce a recognizable image as shown in below figure.

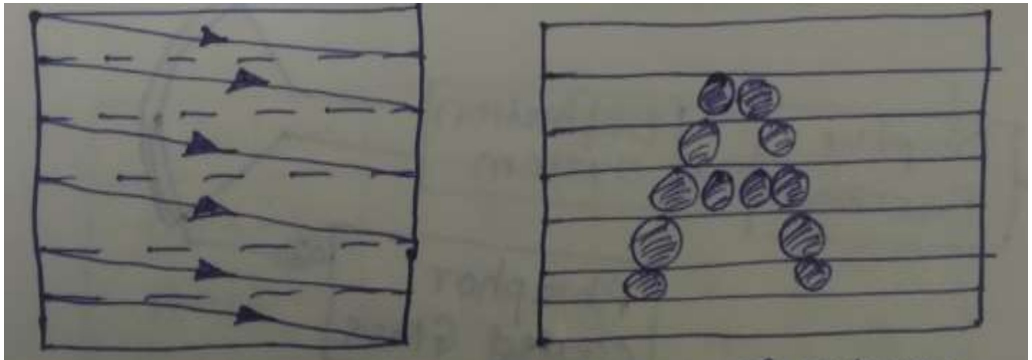


fig.: Raster Scan System

The resolution of a raster developed images depends on the number of pixel on the screen.

Simple Color Frame Buffer:

The color graphics monitor are usually referred to as RGB monitors since the different colors are obtained by mixing three primary colors, RED (R), GREEN (G) and BLUE (B).

A simple color frame buffer can be implemented with three bit planes, one for each primary color.

Each bit plane drives as individual color given for each of the three primary colors. This arrangement can lead to eight colors as given in below table.

Color Table

Color			
Black	0	0	0
Red	1	0	0
Green	0	1	0
Blue	0	0	1
Yellow	1	1	0
Cyan	0	1	1
Magenta	1	0	1
White	1	1	1

Additional bit planes can be used for each of the three color guns. In a 24 bit plane buffer 8 bit planes are used for each color. Each group drives a 8 bit Digital to Analog converter (DAC) and can generate 2^8=256 intensities of Red, Green & Blue. This when combined together can result in $(2^8)^3$=16,77,7,216 or 16.7 million possible color.

K. D. K. College of Engineering, Nagpur
Department of Mechanical Engineering
Computer Aided Design
Unit – V: Trusses (Plane Truss)

- V. D. Dhopte

Introduction:
1) The members of truss are subjected to the direct tension or compression.
2) All loads and reactions are applied only at the joints.
3) FEM is applicable to statically determinate of indeterminate structures.
4) FEM also provides joint deflections, effect of temperature change and reactions at the support.

Local and Global Coordinate Systems:

The main difference between the one-dimensional structures and trusses is that the elements of a truss have various orientations. To account for these different orientations, local and global coordinate systems are introduced.

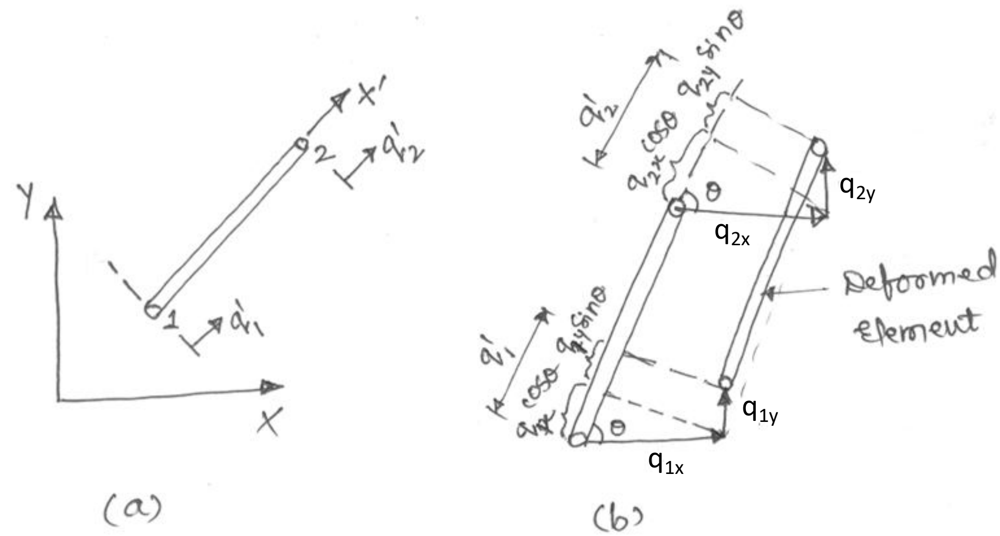


Fig.: A two dimensional truss element (a) in a local coordinate system and (b) in a global coordinate system

Let q_1' and q_2' be the displacements of nodes 1 and 2 respectively in the local coordinate system.
The element displacement vector in the local coordinate system is,

$q' = [q_1', q_2']^T$

The element displacement vector in the global coordinate system is a (4×1) vector denoted by,

$q = [q_{1x}, q_{1y}, q_{2x}, q_{2y}]^T$

The relationship between q' and q is developed as follows,

$q_1' = q_{1x}cos\theta + q_{1y}sin\theta$

$q_2' = q_{2x}cos\theta + q_{2y}sin\theta$

The direction cosines 'l' and 'm' are introduced as,

$l = cos\theta; m = sin\theta$

These direction cosines are the cosines of the angles that the local $X' - axis$ makes with the global X & Y axis respectively.

$\therefore q_1' = q_{1x}l + q_{1y}m$

$q_2' = q_{2x}l + q_{2y}m$

$$\begin{bmatrix} q_1' \\ q_2' \end{bmatrix} = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} \begin{bmatrix} q_{1x} \\ q_{1y} \\ q_{2x} \\ q_{2y} \end{bmatrix}$$

$$q' = Lq$$

Where,

L = Transformation Matrix

$$L = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix}$$

Note:-

Que. What do you mean by transformation matrix? Derive the transformation matrix for truss element. (5m, S-03)

Direction Cosines (l & m)

Direction cosines are calculated from the nodal data.

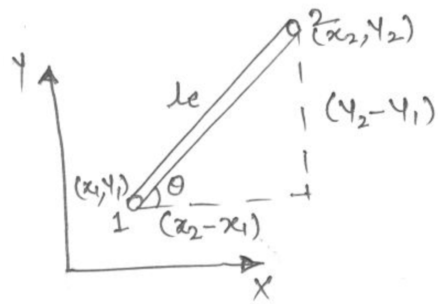


Fig.

$$l = \cos\theta;\ m = \sin\theta$$

OR

$$l = \frac{x_2 - x_1}{l_e};\ m = \frac{y_2 - y_1}{l_e}$$

$$l_e = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

Stiffness Matrix of Truss Element

The truss element is a 1-D element when viewed in the local coordinate system. The element stiffness matrix for a truss element in the local coordinate system is given by,

$$K' = \frac{A_e E_e}{l_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad (1)$$

Where,

A_e – Elemental cross-section area

E_e – Young's Modulus

l_e – Elemental length

The element strain energy in local coordinate is given by,

$$U_e = \frac{1}{2} q'^T K' q' \qquad (2)$$

Where,

$q' = Lq$

$q'^T = L^T q^T$

$\therefore U_e = \frac{1}{2} q^T [L^T K' L] q$

The strain energy in global coordinate system can be written as,

$U_e = \frac{1}{2} q^T K q$

Where,

K is the element stiffness matrix in global coordinates.

$K = L^T K' L$

$$K = \begin{bmatrix} l & 0 \\ m & 0 \\ 0 & l \\ 0 & m \end{bmatrix} \frac{A_e E_e}{l_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix}$$

$$K = \begin{bmatrix} l & 0 \\ m & 0 \\ 0 & l \\ 0 & m \end{bmatrix} \frac{A_e E_e}{l_e} \begin{bmatrix} l & m & -l & -m \\ -l & -m & l & m \end{bmatrix}$$

$$\therefore K = \frac{A_e E_e}{l_e} \begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

Que.

1) Derive an expression of element stiffness matrix in two-dimensional truss element. **[S-07, 3m]**

2) What is transformation matrix for truss element? Find out element stiffness matrix of truss element with the help of stiffness matrix of 1-D element. **[W-01, 6m], [S-01, 8m]**

Elemental Stress Calculation

A truss element in local coordinates is a simple two-force member. Thus, the stress ‘σ’ in a truss element is given by,

$\sigma = E_e \epsilon$

Since the strain $'\epsilon'$ is the change in length per unit original length,

$$\sigma = E_e \frac{q_2' - q_1'}{l_e}$$

$$\sigma = \frac{E_e}{l_e} [-1 \;\; 1] \begin{Bmatrix} q_1' \\ q_2' \end{Bmatrix}$$

This equation can be written in terms of the global displacement ‘q’ using the transformation

$q' = Lq$, as

$$\sigma = \frac{E_e}{l_e} [-1 \;\; 1] L q$$

$$\sigma = \frac{E_e}{l_e} [-1 \;\; 1] \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} q$$

$$\therefore \sigma = \frac{E_e}{l_e} [-l \; -m \;\; l \;\; m] q$$

Also, we can write ‘σ’ as,

$$\sigma = \frac{E_e}{l_e} [-l \; -m \;\; l \;\; m] \begin{bmatrix} q_{1x} \\ q_{1y} \\ q_{2x} \\ q_{2y} \end{bmatrix} \qquad \text{Since, } q = \begin{bmatrix} q_{1x} \\ q_{1y} \\ q_{2x} \\ q_{2y} \end{bmatrix}$$

Que. Determine the element stiffness matrix and element load vector for the plane truss element if it is subjected to tractional force of intensity P_o over its length.
Also explain how you will obtain stress and strain in the element from the element displacement vector. **[S-02, 13m]**

Problems

1) Consider the 3 bar truss shown in figure. Take $E = 200 \times 10^9$ N/m^2 and area of each member as 250 mm^2. Force P=25 KN is acting as shown in figure.
Determine:
i) Displacement at nodes ii) Stresses in each element iii) Reactions at supports **[S-09]**

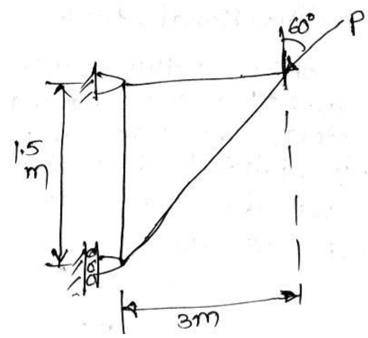


Fig.:

2) For the pin jointed configuration as shown in figure subjected to load P=50 KN, determine:
i) Nodal displacement ii) Stress in each element iii) Reaction at fixed support
Take Young's modulus, $E = 210\ GPa$ **[S-10, 14m]**

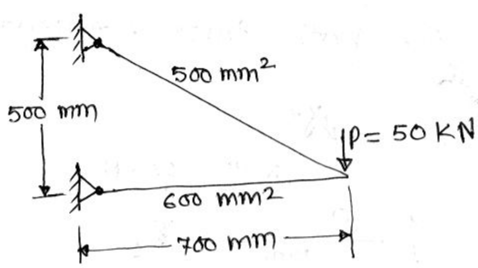


Fig.

3) For the plane truss shown in figure, determine the horizontal and vertical displacement of node 1 and the stresses in each element. All elements have $E = 210\ GPa$ and $A = 4.0 \times 10^{-4}\ m^2$. **[W-09, 13]**

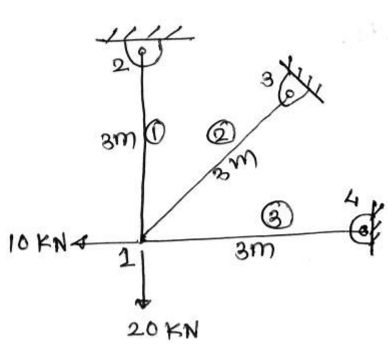


Fig.

4) For the plane truss supported by the spring at node 1 in figure, determine the nodal displacements and stresses in each element.
Let $E = 210\ GPa$ and $A = 5.0 \times 10^{-4}\ m^2$ for both truss elements. **[W-08]**

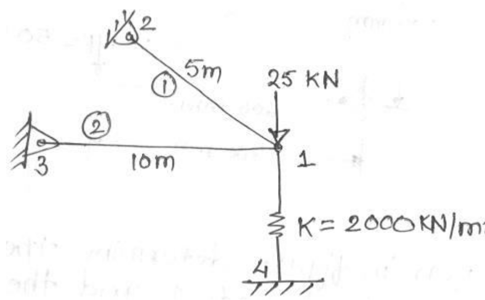


Fig.

5) Figure shows a truss consisting of three elements, cross-sectional area of each bar equals to 200 mm^2 and modulus of elasticity, $E = 210\ GPa$. Calculate deflection of nodes and stresses in each element. Also determine reactions at supports. **[W-07, 13m]**

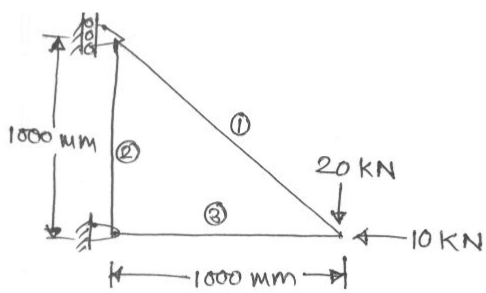


Fig.

6) Figure shows a two-dimensional simple truss with two members 1 and 2 having circular cross-section of diameters, d_1=30 mm and d_2=50 mm respectively. If the truss is pin jointed, determine the nodal displacement. The members are subjected to load P=100 N. Take E=200 GPa. **[S-07, 10m]**

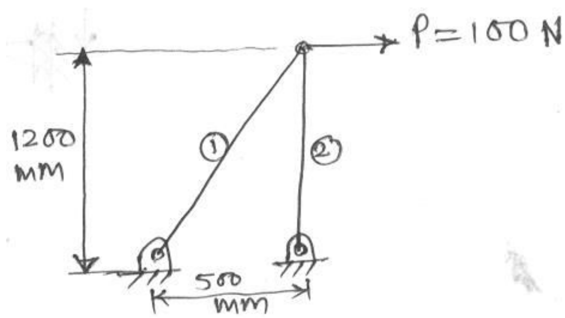


Fig.

7) A truss is shown in the figure. For that truss find the nodal displacements and element stresses.
A=200 mm^2 **[W-06, 13m]**
E=200 GPa.

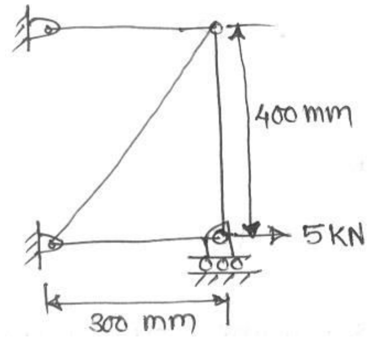


Fig.

8) The truss is shown in figure. Determine stresses in each member and reactions at support. Area of cross-section of each member is 200 mm^2 and $E = 200 \times 10^3\, MPa$. **[W-05, 14m]**

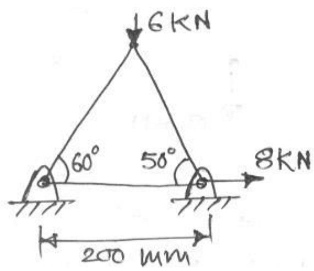


Fig.

9) A truss as shown in figure is subjected to the load of 3 KN. Determine the stresses in truss members. **[S-05]**

A=150*150 mm^2
$E = 120 \times 10^3\, N/mm^2$.

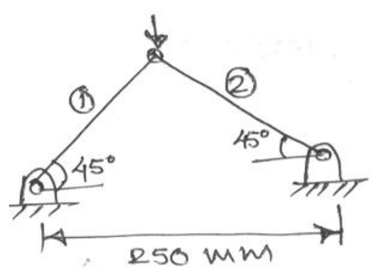


Fig.

10) For the truss shown in figure, find the displacement of each node and reaction at supports. E= 200 GPa and area of each element, A=200 mm^2. **[W-04]**

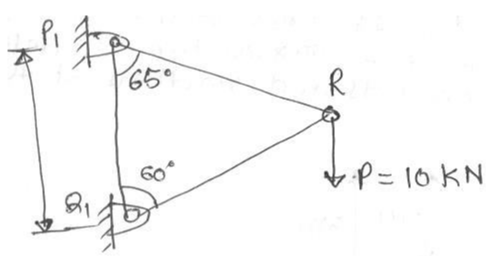


Fig.

11) Determine the stresses in the truss members as shown in figure. **[S-04]**
A=80 mm^2
$E = 210 \times 10^3 N/mm^2$.

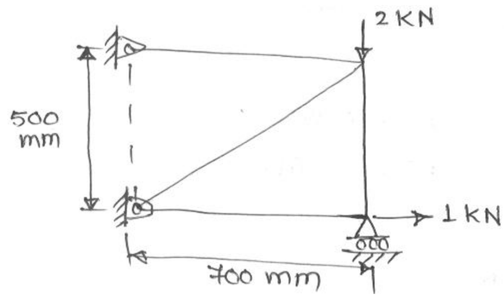


Fig.

12) A truss is shown in figure. Determine stresses and strains in each member. **[W-03, 13m]**
A=200 mm^2
$E = 200\ GPa$

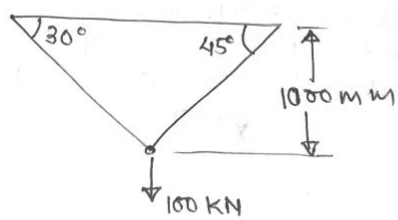


Fig.

13) A truss is shown in figure is subjected to the load of 7 KN. Determine deflection and stresses in the elements. **[S-03, 15m]**
A=400 mm^2
$E = 200 \times 10^3 N/mm^2$.

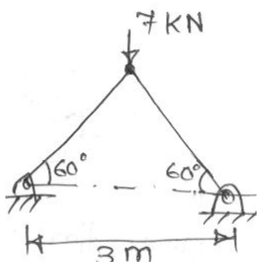


Fig.

14) A 2-D truss as shown in figure is subjected to loads F_1=20 KN and F_2= 5 KN. If cross-section area, A=300 mm^2 and modulus of elasticity, $E = 200 \times 10^3 N/mm^2$, determine the stresses in the truss elements and reactions at the supports. **[W-02, 16m]**

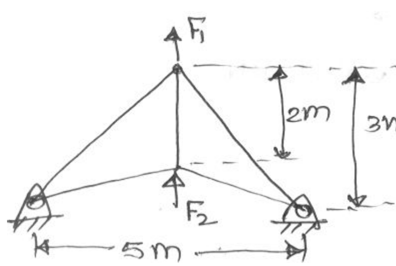


Fig.

15) A simple plane truss as shown in figure is subjected to load P=5000N. Determine the stress in the elements and reactions at the supports. **[W-01, 16m]**

Assume,

A = 300 mm^2

$E = 20 \times 10^4 N/mm^2$

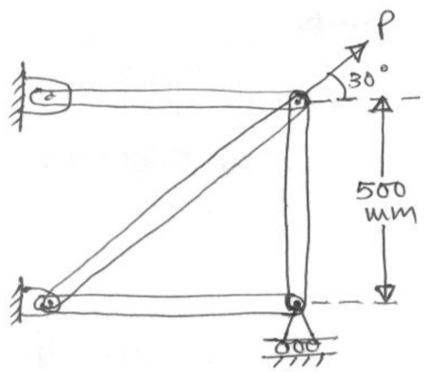


Fig.

16) A simple plane truss as shown in figure is made of two identical bars, and loaded as shown in the figure. Determine displacement and stresses. Cross-section area of each element is 100 mm^2.

$E = 200 \times 10^3 N/mm^2$.

Length of each bar = 150 mm

P_1=5 KN & P_2=3 KN **[S-01, 10m]**

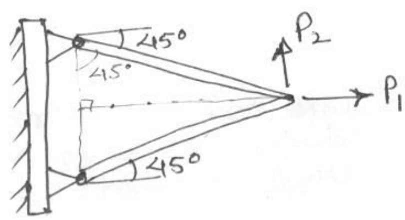


Fig.

Unit – 3

Parametric Representation of Synthetic Curves:

Analytic curves are not sufficient to meet geometric design requirements of mechanical parts. Product such as car bodies, ships hulls, airplane fuselage and wings, propeller blades and bottles are a few examples that require free-form or synthetic curves and surfaces.

Mathematically, synthetic curves represent a curve fitting problem to construct a smooth curve that passes through given data points. Major CAD/CAM systems provide three types of synthetic curves

1) Hermite Cubic Spline Curves

2) Bezier Curves

3) B-Spline Curves

The Cubic Spline curve passes through the data points and therefore is an interpolant. Bezier curve approximate the data points, i.e they do not pass through them. B-Spline curve can be interpolant as well as approximate the given data points.

Both Cubic Spline and Bezier curves have a first order continuity and the B-Spline curve has a second order continuity. A first-order continuity curve is the minimum acceptable curve for engineering design. First and second order continuities imply slope and curvature continuous curves respectively.

Properties of Bezier Curves

Bezier curves have the following properties −

1. They generally follow the shape of the control polygon, which consists of the segments joining the control points.
2. They always pass through the first and last control points.
3. They are contained in the convex hull of their defining control points.
4. The degree of the polynomial defining the curve segment is one less that the number of defining polygon point. Therefore, for 4 control points, the degree of the polynomial is 3, i.e. cubic polynomial.
5. The direction of the tangent vector at the end points is same as that of the vector determined by first and last segments.
6. Bezier curves exhibit global control means moving a control point alters the shape of the whole curve.

Requirements of Geometric Models:

Represent the realistic 3D geometric models in the computer. These models describe the geometry as well as its characteristics. Good geometric model is designed based on the following important guidelines,

1) Modeling method must be easy to use

2) Completeness of part representation

3) Model should not create any ambiguity to users.

4) A geometric model must be very accurate. There should not be any approximation.

5) The various transformations such as move (translate), rotate, scale etc. and manipulations such as union, intersect, subtract etc. should be able to be performed on geometric model.

6) Geometric model should have shading and rendering capability to give realistic effect to the model.

3D Modeling Techniques:

It is done in three different ways,

1) Wireframe Modeling

2) Surface Modeling and

3) Solid Modeling

Wireframe Modeling:

Wireframe modeling is the oldest and simplest method of geometric modeling which can be used to store model mathematically in the computer memory. It contains information about the locations of all the points (vertices) and edges in space coordinates. Various wireframe entities are points, lines, planner arcs, circle, curves etc. Each vertex is defined by (x, y, z) coordinates, edges are defined by a pair of vertices and faces are defined as three or more edges. Thus wireframe is a collection of edges, there is no skin defining the area between the edges. This is the lowest level of modeling and has serious limitations. But in some applications such as tool path simulation it is very convenient to use wireframe models.

Advantages:

1) It is simplest method and requires less memory space.

2) It forms the basis for surface and solid modeling.

3) Manipulation in the model can be done easily and quickly.

Disadvantages:

1) One of the serious limitation is the ambiguity of orientation and viewing plane. Due to this it is very difficult to interpret the object from a particular viewing plane and creates confusion.

2) Physical properties such as mass, surface area, volume, centre of gravity etc. are not possible to calculate.

3) Wireframe model has no knowledge of surface faces, therefore it will not detect interference between two mating components and this is serious drawback in component assembly.

4) Cannot model complex curved surfaces.

Solid Modeling:

In the solid modeling the solid definitions include vertices (points), edges, surfaces, weight, and volume. This model consist the complete description of the solid. It is the most ideal representation, as all the information required at every stage of product cycle can be obtained with this technique. The model is a complete and unambiguous representation of a precisely enclosed and filled volume. Solid modeling is most widely used method and a number of different techniques available to represent the solid. The two popular methods to represent solid model are Constructive solid geometry (CSG) and Boundary representation (B-Rep).

Advantages:

1) 2D standard drawings, assembly drawing and exploded drawings are generated from the model.
2) Can easily be exported to different FEM programs for analysis.
3) Can be used in newly manufacturing techniques as CIM, CAM and design for manufacturing (DFM).
4) Volumetric and mass properties of an object can be easily obtained.

Disadvantages:

1) More intensive computation than wireframe and surface modeling.
2) Requires more powerful computers.

Constructive Solid Geometry

Constructive Solid Geometry is the process of building solid objects from other solids.

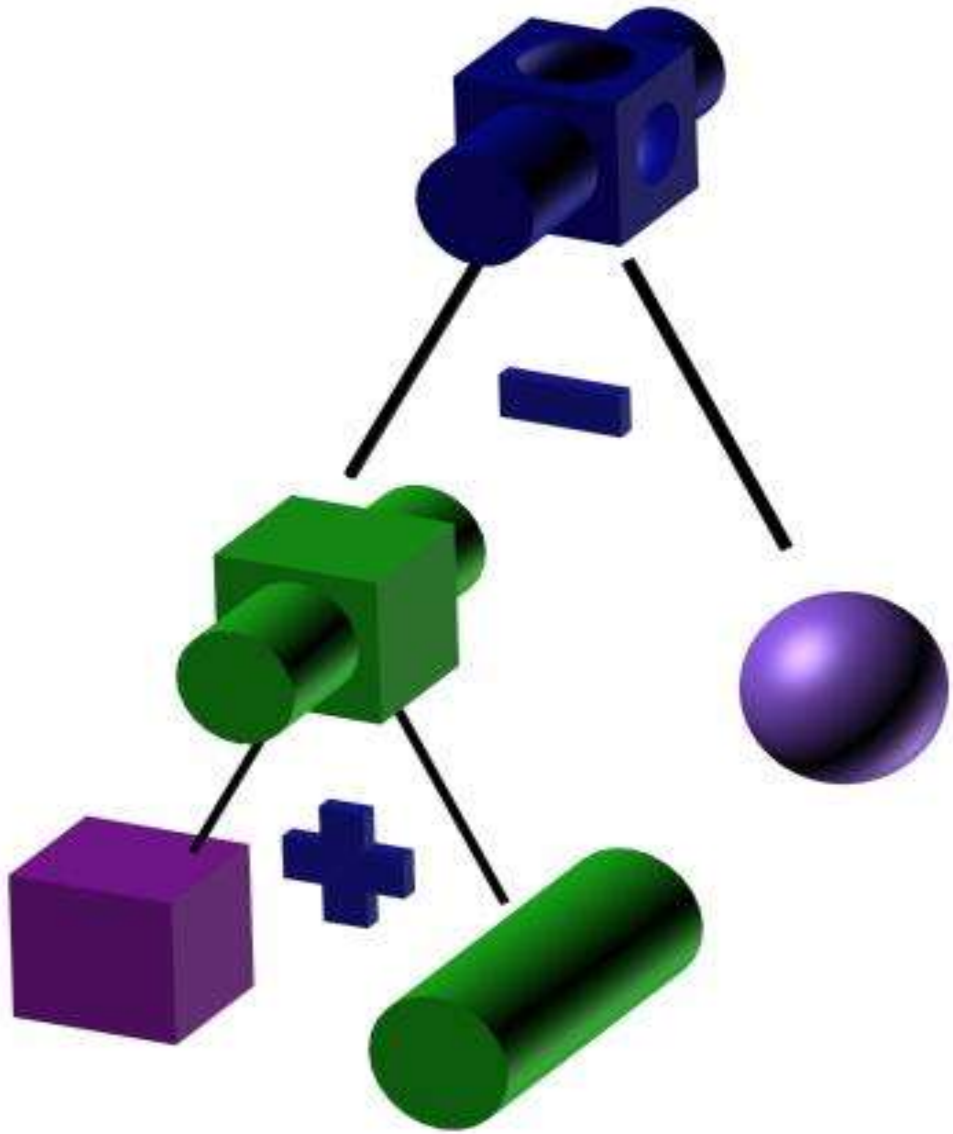
The three CSG operators are Union, Intersection, and Difference.

Each operator acts upon two objects and produces a single object result.

By combining multiple levels of CSG operators, complex objects can be produced from simple primitives.

The **union** of two objects results in an object that encloses the space occupied by the two given objects.

Intersection results in an object that encloses the space where the two given objects overlap. **Difference** is an order dependent operator; it results in the first given object minus the space where the second intersected the first.



CSG Primitives

The standard CSG primitives consist of the block (*i.e.*, cube), triangular prism, sphere, cylinder, cone and torus. These six primitives are in some *normal* or *generic* form and must be *instantiated* by the user to be used in design. Moreover, the instantiated primitive may require transformations such as scaling, translation and rotation to be positioned at the desired place.

Boolean Operators

We can combined two instantiated and perhaps transformed primitives into one with set union, set intersection and set difference operators.

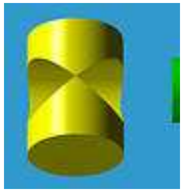
Let us just use set operations,

Given two sets, ***A*** and ***B***, In the following, ***A*** is the vertical cylinder and ***B*** is the horizontal cylinder.

1) Its union consists of all points from either ***A*** or ***B***;

2) Its intersection consists of all points in *both* sets;

3) Its difference, written as ***A* - *B*** (*resp*., ***B* - *A***), consists of all points in ***A*** but not in ***B*** (*resp*., in ***B*** but not in ***A***).



A* - *B

B* - *A

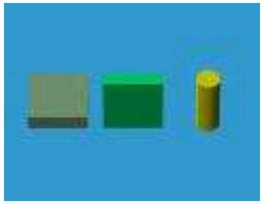
Therefore, a solid can be considered as the result of applying Boolean operators to a set of instantiated and transformed CSG primitives.

Example:

Design a bracket-like shape with a hole as shown below figure.

Start with two instantiations of blocks and one instantiation of a cylinder.

Then, the two blocks are scaled and one of them is rotated to a vertical position. The cylinder is also scaled so that its radius matches that of the hole. These three instantiations are then transformed to their desired positions.



The final product is obtained by computing the union of the two blocks and then subtracting from it the cylinder.

Note: The design of the above solid is ***not*** unique. For example, the L shape can be constructed from subtracting a cube from another one.

CSG Expressions

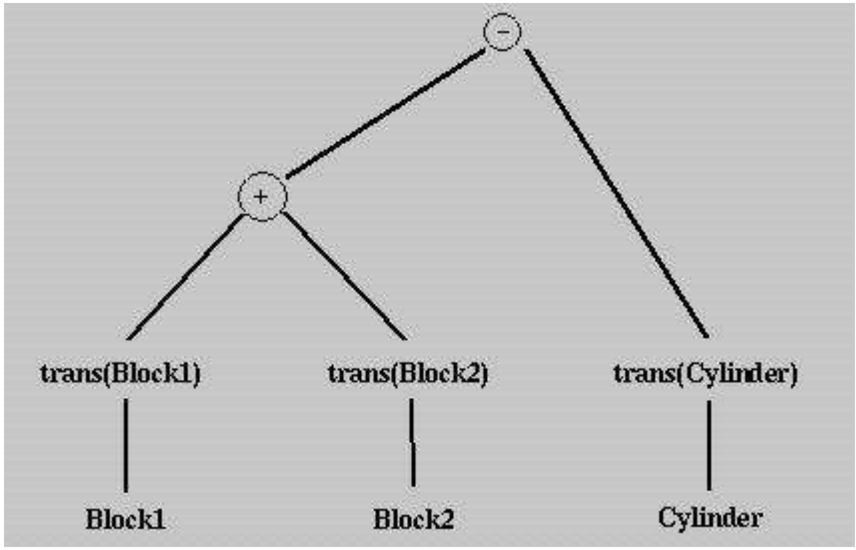
The design procedure of the above bracket can be written as an expression:

diff(union(trans(Block1), trans(Block2)), trans(Cylinder))

where **union(*A*,*B*)** and **diff(*A*,*B*)** are the union and difference of ***A*** and ***B***, and **trans()** indicates appropriate transformations. Or, if we use **+**, ^ and **-** for set union, intersection and difference, the above function calls can be rewritten as a set expression as follows:

(trans(Block1) + trans(Block2)) - trans(Cylinder)

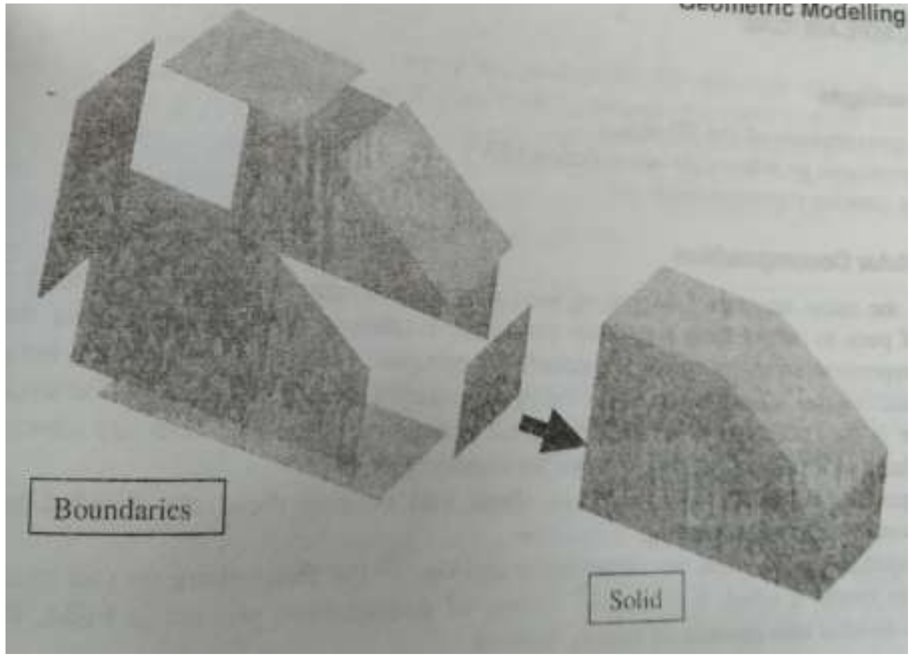
This expression can be converted to an expression tree, the *CSG Expression*, of the design:



In fact, every solid constructed using the CSG technique has a corresponding CSG expression which in turn has an associated CSG tree. The expression of the CSG tree is a representation of the final design. Recall that the same solid may have different CSG expressions/trees. For example, one might punch a hole from **Block1** first and then compute the union of this result with **Block2**. As a result, *CSG representations are not unique*.

Boundary Representation (B-Rep)

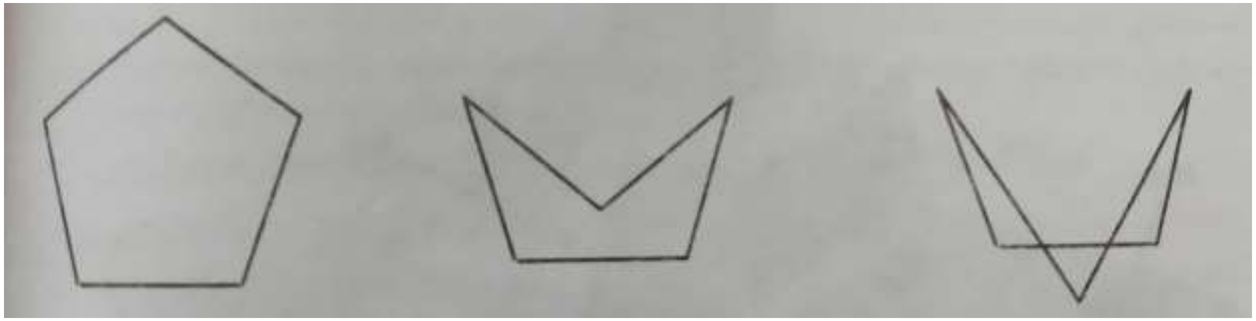
The following figure shows an example of object in boundary representation, where boundaries are converted into 3D model.



In solid modeling, boundary representation is abbreviated as B-rep. It is a method for representing shapes using the limits where solids are defined and stored by their boundaries. A solid is represented as a collection of connected surface elements, the boundary between solid and non-solid. Boundary representation models are composed of two parts: topology and geometry (consists of surfaces, curves and points).

In geometry each face is bounded by edges and each edge is bounded by vertices. The topology and geometry are interrelated and cannot be separated entirely. Both must be compatible otherwise meaningless object may result.

Following figure shows a pentagon which may produce a true object or a meaningless object (intersecting lines) depending on the position of vertices of the geometry.



Given 2D Object   Modified Object   Modified meaningless Object

Topology is created by performing Euler operations and geometry is created by performing Euclidean calculations.

Advantages:

1) Easy and efficient rendering (face plane)

2) Simple algorithm for solid volume computation

3) Point inside or outside of the solid

4) Formal definition of the 3D object

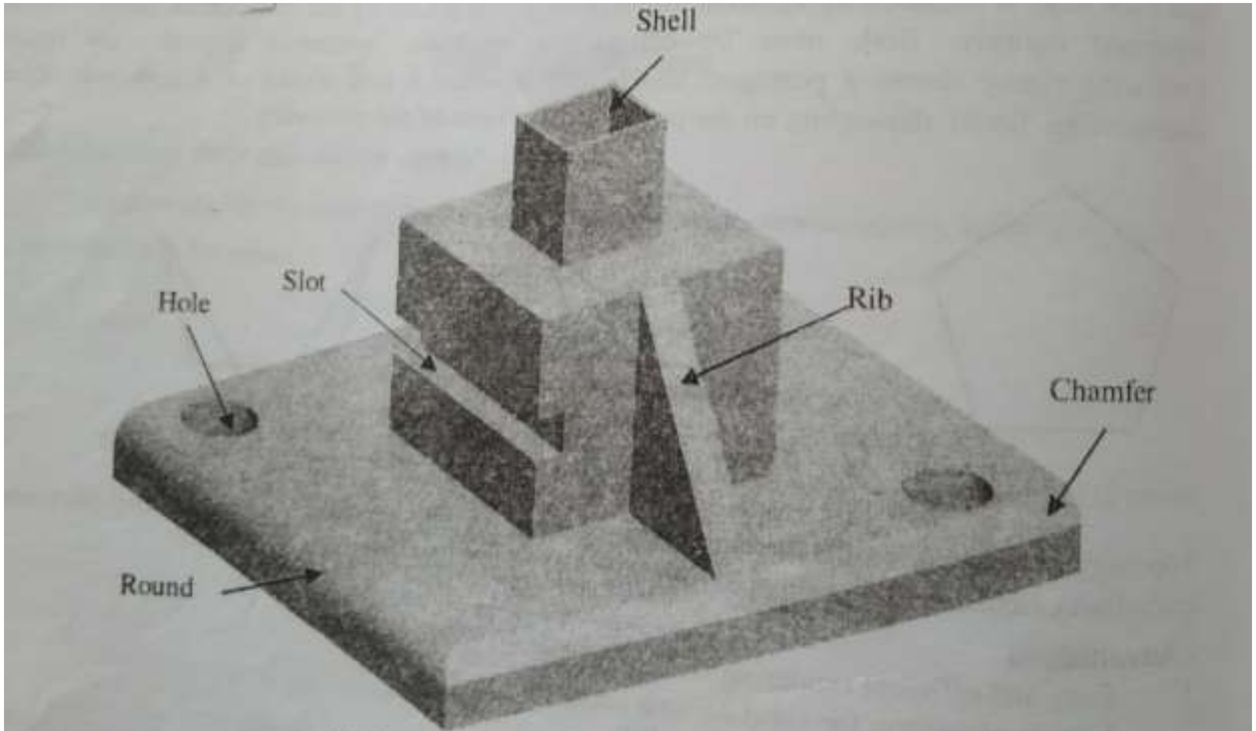
Disadvantages:

1) Approximation of 3D object

2) Not concise representation

Feature Based Modeling

The feature modeler contains not only a geometric and topological structure but also support geometric characteristics of a part.



Various features used in 3D modelling

Feature is one of the latest approaches to convert 2D drawings into 3D models. It is combinations of creating 2D shape and then performing various operations to build a 3D model. 3D solids are generated with the help of 2D cross-sectional planner profiles and dragging functions, such as extrude, revolve, sweeps, blend, fillet, chamfer etc. In solid modeling we have various which ultimately results in the creation of a complex model.

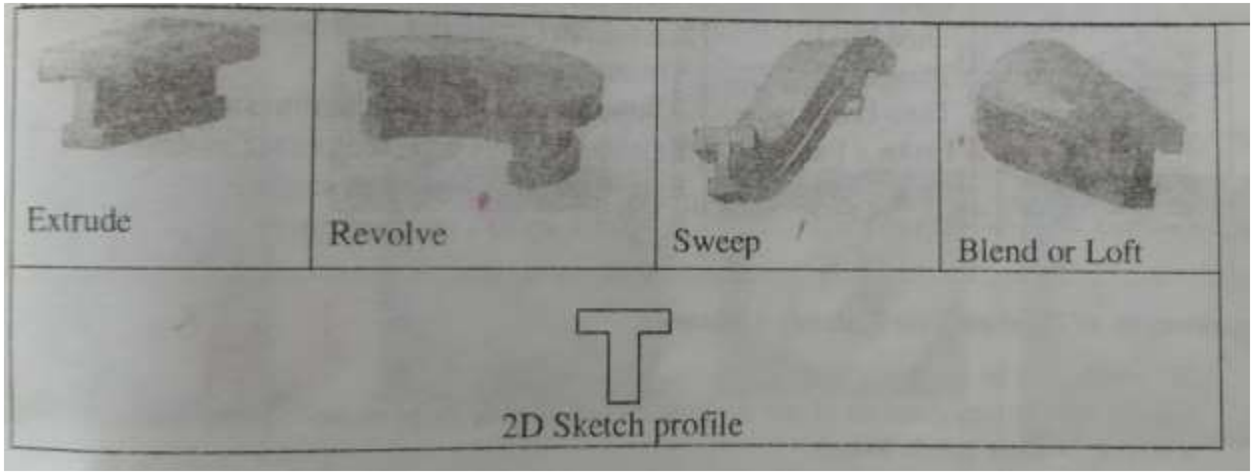
Following feature operations are mostly used

Extrude: Create a feature by extruding its section. Direction of extrusion is normal to the sketch plane.

Revolve: Create a feature by revolving its section. The revolved section must have a centerline. 2D sketch is rotated about the rotation axis to get symmetric object. In addition to 2D sketch here we need rotation axis.

Sweep: Create a feature by sweeping a section along a trajectory. Here we need 2D sketch and a path along which sketch is swept.

Blend: Create a feature by blending multiple parallel sections. Here we have specify various cross sections on different parallel and offset sketch planes.



By combining above basic features, we can develop advance features such as Sweep-Blend which is combination of Sweep and Blend feature.

Thus to create 3D model using feature options we follow three steps

1) Initially create 2D profile on the selected sketch plane.

2) Depending upon the shape of 3D model select appropriate feature and apply feature operation to convert 2D profile into 3D object.

3) Use various Boolean operations such as Add, Subtract or Intersect to create complicated objects from the base features to generate final shape object.

Advantages of feature based modeling

Designing with features provides a number of advantages:

1) Insert intelligent parametric geometry into a model without having o create intermediate construction geometry.

2) Associate a variety of information to portions of a model.

3) Create features with attributes unique to your design requirements.

K. D. K. College of Engineering, Nagpur

Mechanical Engineering Department

Subject: Computer Aided Design (CAD)
Unit: 5

Semester: VII

Session: 2020 -21

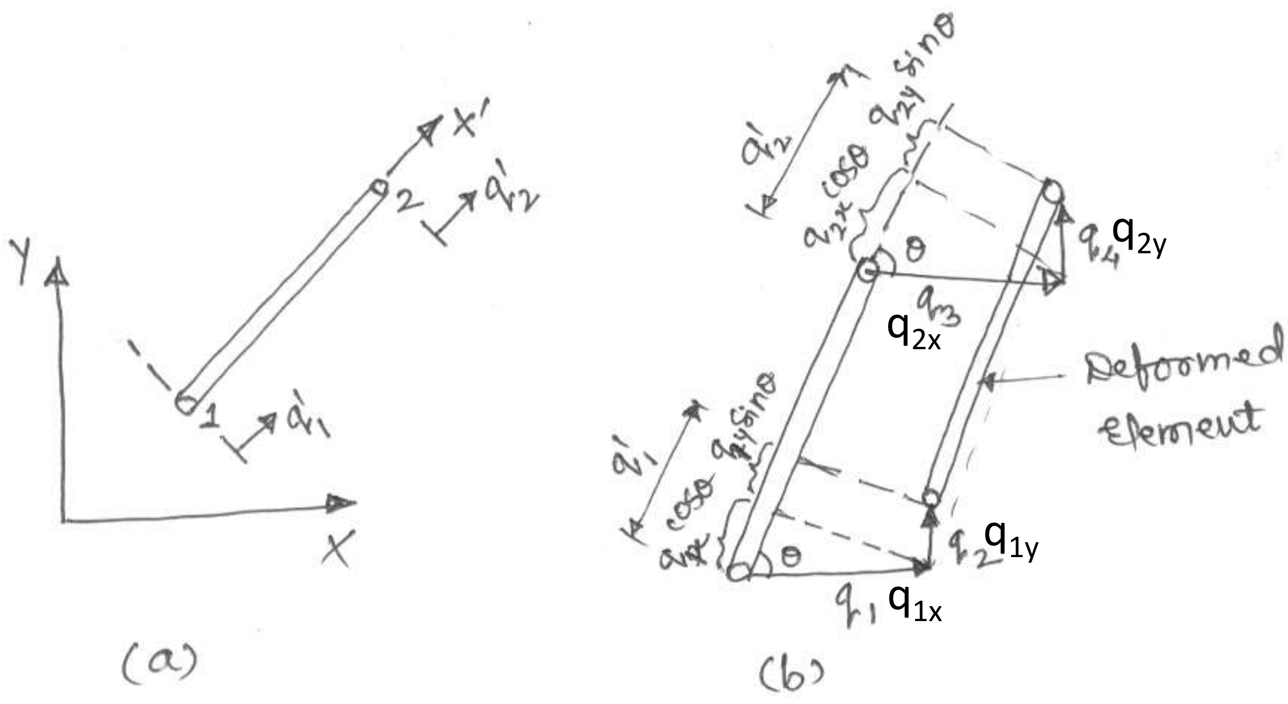
V. D. Dhopte

Introduction:

1) The members of truss are subjected to the direct tension or compression.
2) All loads and reactions are applied only at the joints.
3) FEM is applicable to statically determinate of indeterminate structures.
4) FEM also provides joint deflections, effect of temperature change and reactions at the support.

Local and Global Coordinate Systems:

The main difference between the one-dimensional structures and trusses is that the elements of a truss have various orientations. To account for these different orientations, local and global coordinate systems are introduced.



Truss element (a) in a local coordinate system and (b) in a global coordinate system

Let q_1' and q_2' be the displacements of nodes 1 and 2 respectively in the local coordinate system.
The element displacement vector in the local coordinate system is,

$$q' = [q_1', q_2']^T$$

The element displacement vector in the global coordinate system is a (4×1) vector denoted by,

$$q = [q_{1x}, q_{1y}, q_{2x}, q_{2y}]^T$$

The relationship between q' and q is developed as follows,

$$q_1' = q_{1x} cos\theta + q_{1y} sin\theta$$

$$q_2' = q_{2x} cos\theta + q_{2y} sin\theta$$

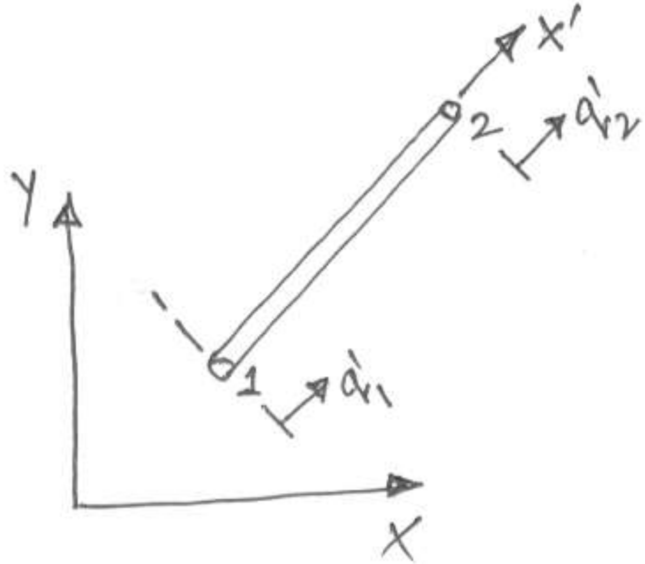
The direction cosines 'l' and 'm' are introduced as,

$$l = cos\theta;\ m = sin\theta$$

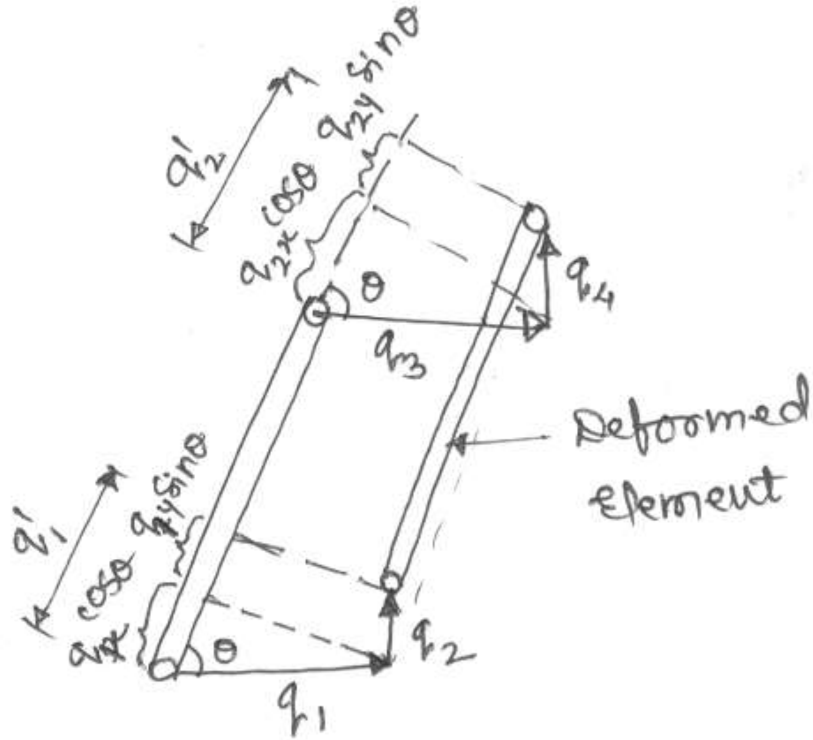
These direction cosines are the cosines of the angles that the local $X' - axis$ makes with the global X & Y axis respectively.

$$\therefore q_1' = q_{1x} l + q_{1y} m$$

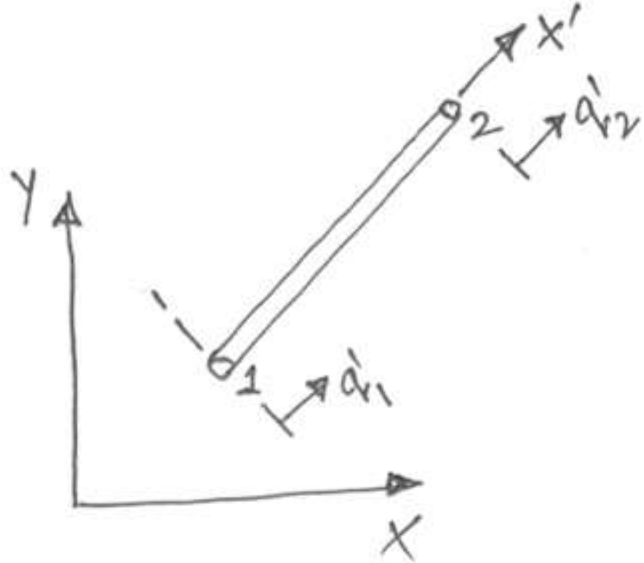
$$q_2' = q_{2x} l + q_{2y} m$$


X'
2
q'2
Y
1
q'1
X

(a)

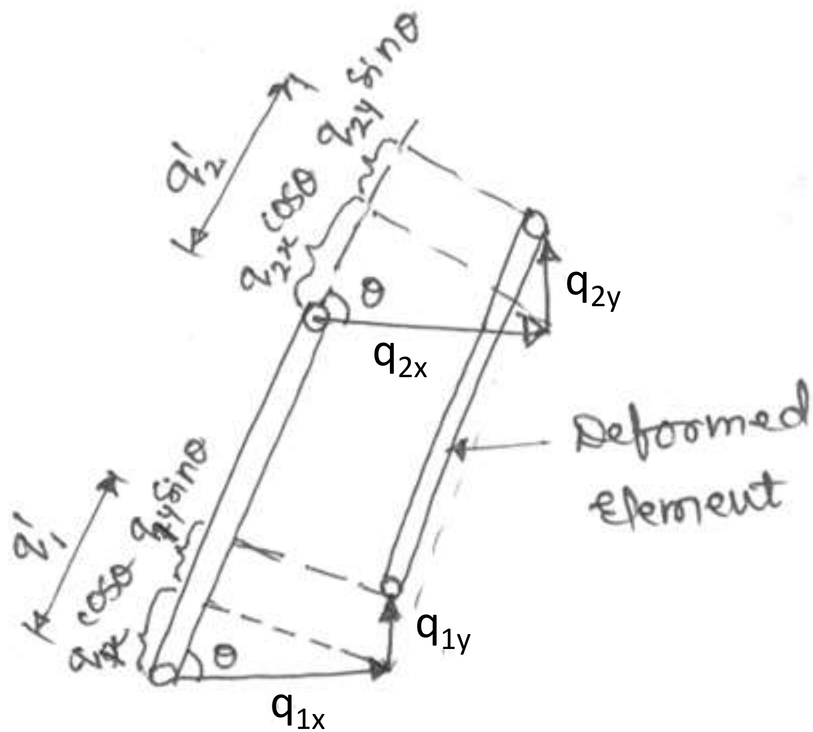

q'2
q2x cosθ
q2y sinθ
θ
q4
q3
Deformed
Element
q'1
q1x cosθ
q1y sinθ
θ
q2
q1

(b)



Y
X
X'
1
2
q'1
q'2

(a)



q'2
q2x cosθ
q2y sinθ
θ
q2y
q2x
Deformed Element
q'1
q1x cosθ
q1y sinθ
θ
q1y
q1x

(b)